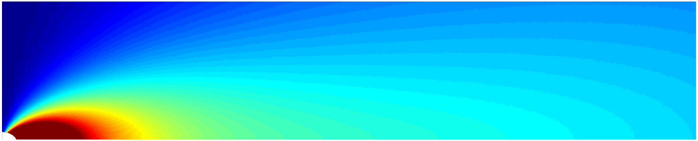
Magnetodisc Theory

Nicholas Achilleos
University College London
July 2, 2018



Theory of Magnetodisc Structure

- We wish to *quantify* some aspects of how plasma disc properties in rapidly rotating magnetospheres affect field structure.
- Our main reference point is Caudal's theory (JGR, 1986) which solves equation of force balance between magnetic '$\mathbf{J} \times \mathbf{B}$' force, centrifugal force and pressure gradient. Here, $\mathbf{J}$ denotes *current density* (vectors in bold typeface) and $\mathbf{B}$ is magnetic field.
- The system is assumed to be *axisymmetric* with parallel rotation and magnetic axes, and poloidal field (i.e azimuthal component $B_\phi = 0$).

Theory of Magnetodisc Structure

- Axisymmetric assumption also means that azimuthal gradients in all quantities (field components, plasma pressure, plasma properties) are zero: $\frac{d}{d\phi} \equiv 0$.
- Although a strong assumption, axisymmetry is commonly used in modelling the so-called 'middle magnetospheres' of Jupiter and Saturn which, from an observational point of view, show a structure which is well approximated by axisymmetry about the planet's magnetic (dipole) axis.
- The axisymmetric assumption implies that the only *non-zero* component of $\mathbf{J}$ is the azimuthal current density J_ϕ. (Why?)

Theory of Magnetodisc Structure

- Return to our simple, axisymmetric disc. We start in spherical polar coordinates, and define the radial and meridional field components by using magnetic *Euler* potentials α and β:

$$\mathbf{B} = \nabla\alpha \times \nabla\beta \tag{1}$$

$$\beta = a\phi, \alpha = \alpha(r, \theta)$$

  a = planet radius (length scale).

- Hence:

$$B_r = \frac{a}{r^2 \sin\theta} \frac{d\alpha}{d\theta} \tag{2}$$

$$B_\theta = \frac{-a}{r \sin\theta} \frac{d\alpha}{dr} \tag{3}$$

Theory of Magnetodisc Structure

- Link field structure and current with Ampère's Law: $\nabla \times \mathbf{B} = \mu_o \mathbf{J}$
-

$$(\nabla \times \mathbf{B})_\phi = \frac{a}{r} \left[\frac{\partial}{\partial r} \left(\frac{-1}{\sin\theta} \frac{\partial \alpha}{\partial r} \right) - \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin\theta} \frac{\partial \alpha}{\partial \theta} \right) \right] \quad (4)$$

- Bridge between **J** and plasma properties comes from force balance:

$$\mathbf{J} \times \mathbf{B} \approx \nabla P - (P/(2kT)) m_i \rho \omega^2, \quad (5)$$

  where P denotes plasma pressure, T is temperature, m_i is mean ion mass (constant along field line), ρ is cylindrical radial coordinate and ω is ang. vel.
- Ideal gas law means that $P/(2kT)$ is half the total particle number density (i.e. ion num. dens. in quasi-neutral plasma).

Theory of Magnetodisc Structure

- Caudal nicely demonstrates that (given without proof here):

$$\frac{\partial^2\alpha}{\partial r^2} + \frac{(1-x^2)}{r^2}\frac{\partial^2\alpha}{\partial x^2} = -\frac{\mu_o}{a^2}\rho^2 \exp\left(\frac{\rho^2-\rho_0^2}{2l^2}\right) \cdot \left[\frac{dP_0}{d\alpha} + \frac{P_0 a}{l^2 B_{\theta 0}}\right] \quad (6)$$

- Susbcript '0' denotes equatorial quantities magnetically conjugate to the point with coordinates (r, x) (with $x = \cos\theta$).
- Scale length l given by $l^2 = 2kT/(m_i\omega^2)$. Illustrates the competition between thermal energy and centrifugal confinement of 'hot' and 'cold' plasma.
- Right side of equation is source function $g(r, x, \alpha)$.
- Analytical form for solution may be derived, but since g depends on field structure, actual solution obtained numerically, starting with pure dipole as first 'iteration'.

Theory of Magnetodisc Structure

- In practice, one 'builds' g on the equator from spacecraft observations, then integrates throughout volume to obtain a field / plasma model.

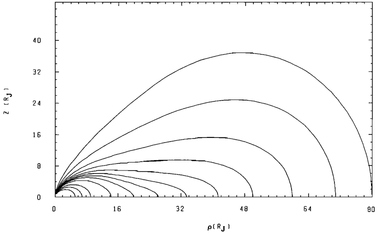
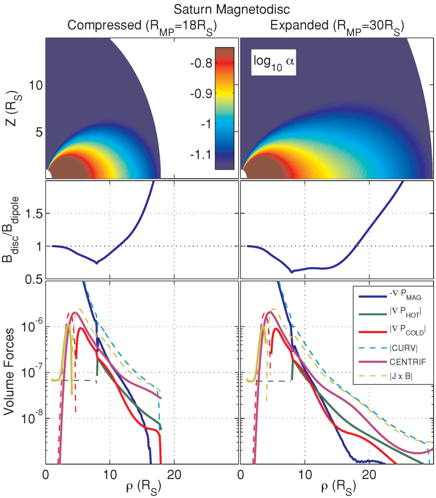


Fig. 6. Meridian plane projection of magnetospheric field lines computed by the present model, in cylindrical coordinates (ρ, z), assuming a magnetopause subsolar point distance $R_S = 80\ R_J$. The model is valid for the noon meridian and not suitable for latitudes higher than $\sim 50°$.

- Caudal's solution shows disc-like field shape in a 'middle magnetosphere'.

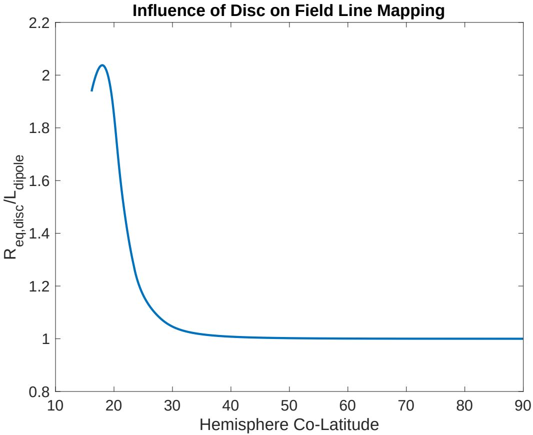
Theory of Magnetodisc Structure



- Advantage: can model 'hot' and 'cold' plasma populations - 'hot' pressure is uniform along field. (Why?)
- Achilleos, Guio and Arridge (2010) repeated for Saturn, using data from *Cassini*. Studied effect of system size on field.
- Hot plasma pressure varies significantly - can change 'competition' between pressure grad. and centrif. force.

Theory of Magnetodisc Structure

Example of quantifying the ‘stretching’ of the dipole field by the current sheet / disc.

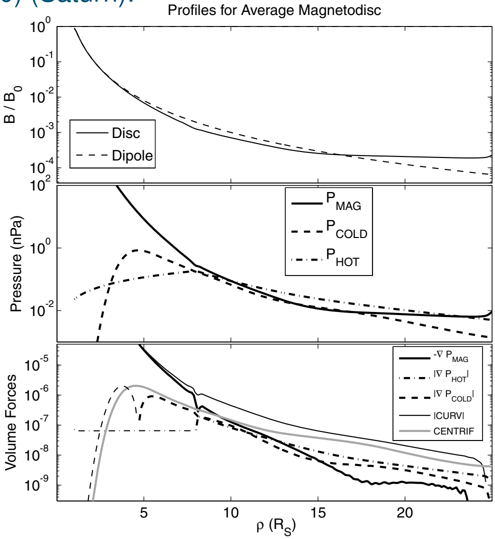


Theory of Magnetodisc Structure

- **Toy models** can be useful - 'Sacrifice realism, gain insight'.
- Achilleos, Guio and Arridge considered a toy model of a rigidly corotating disc with distinct hot (carries pressure) and cold (isothermal, carries mass) components.
- By considering the toy source function, they identified a transition distance ρ_T, for $\rho >> \rho_T$ centrif. force $>>$ pressure gradients.
- $\rho_T^2 = 2\chi l^2 \beta_{hot}/\beta_{cold}$.
- Equatorial $B \propto \rho^{-\chi}$.

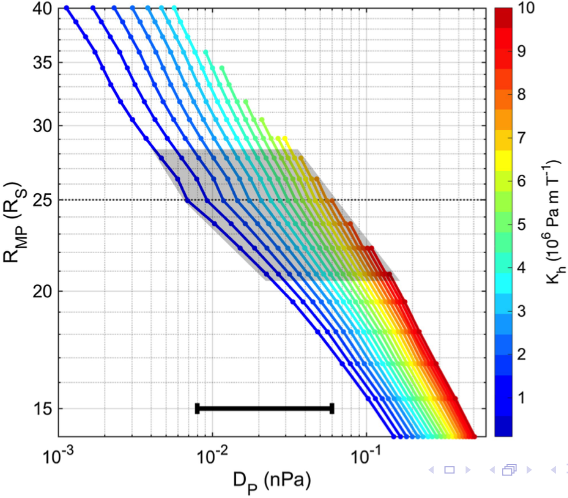
Theory of Magnetodisc Structure

Connecting χ and force balance, Achilleos, Guio and Arridge (MNRAS, 2010) (Saturn):



Theory of Magnetodisc Structure

- From *Sorba et al.* (JGR, 2017) - Saturn model used to estimate solar wind pressure, and behaviour of system size. Follow-up study for further modelling of the observations made by *Pilkington et al.* (JGR, 2015).



What about pressure anisotropy?

- Non-zero $P_{||} - P_{\perp}$ makes pressure force depend on 'shape' of field, as well as pressure gradients.
- Non-isotropic plasma also experiences a force related to the field structure and the 'averaged' mirror force.
- For more detail in this context, see *Nichols, Achilleos and Cowley (JGR, 2015)*.

Comparing Astrophysical Systems

	MAGNETO-DISCS	POLARS / IPS	GALACTIC DISCS
DISC FORMATION	Plasma loading / outflow	Accretion onto white dwarf	Star formation / kinematics
MAIN FORCES	Centrifugal Magnetic Pressure	Magnetic Gravity	Gravity
TRANSPORT MODES	Flux Tube Interchange	Viscous Spread Blobby Accretion	Orbital Motion Disc Heating
SCALE	Tens of planet. radii	Tens of white dwarf radii	Tens of kiloparsecs

Comparing Astrophysical Systems

	MAGNETO-DISCS	POLARS / IPS	GALACTIC DISCS
ORIGIN OF DISC ANG. VEL.	Planet Rot'n; Magn. field M-I coupling	Gravity; Magnetic coupling; Spin-Orbit Equilib.	Grav. potential of large-scale mass distrib'n

More details in notes on 'Astrophysical Discs' (http://www.ucl.ac.uk/$\sim$ucapnac).

Thank You for Listening

... and enjoy the Lab exercises! (Visualizing and interpreting magnetodisc models).