EFT Approaches in Cosmology

: Applications to Gravitational Waves

Vicharit Yingcharoenrat

Chulalongkorn University

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Outline

- Basic cosmology and dark energy
- EFT of Inflation/Dark Energy
- EFT of BH perturbations on arbitrary backgrounds

Friedmann’s Universe

- Physics — General Relativity

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$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + L_m\right] \quad \Rightarrow \quad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{T_{\mu\nu}}{M_{\rm Pl}^2}$$

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- Observations — Isotropy & Homogeneity in space — Cosmological Principles

  on super large scales $\sim 200 - 300$ Mpc $\quad (1$ pc $\sim 10^{13}$ km$)$

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Friedmann-Robertson-Walker (FRW) metric:

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 {\rm d}\Omega^2\right)$$

Translational symmetry: everywhere in space looks the same — T_4: $x^i \to x^i + c^i$

Rotational symmetry: every direction looks the same — SO(3): $x^i \to O^i{}_j x^j$

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Dust ($w = 0$): $\rho_{\mathrm{M}} \propto a^{-3}$

Cosmological Constant ($w = -1$): $\rho_{\Lambda} = const.$

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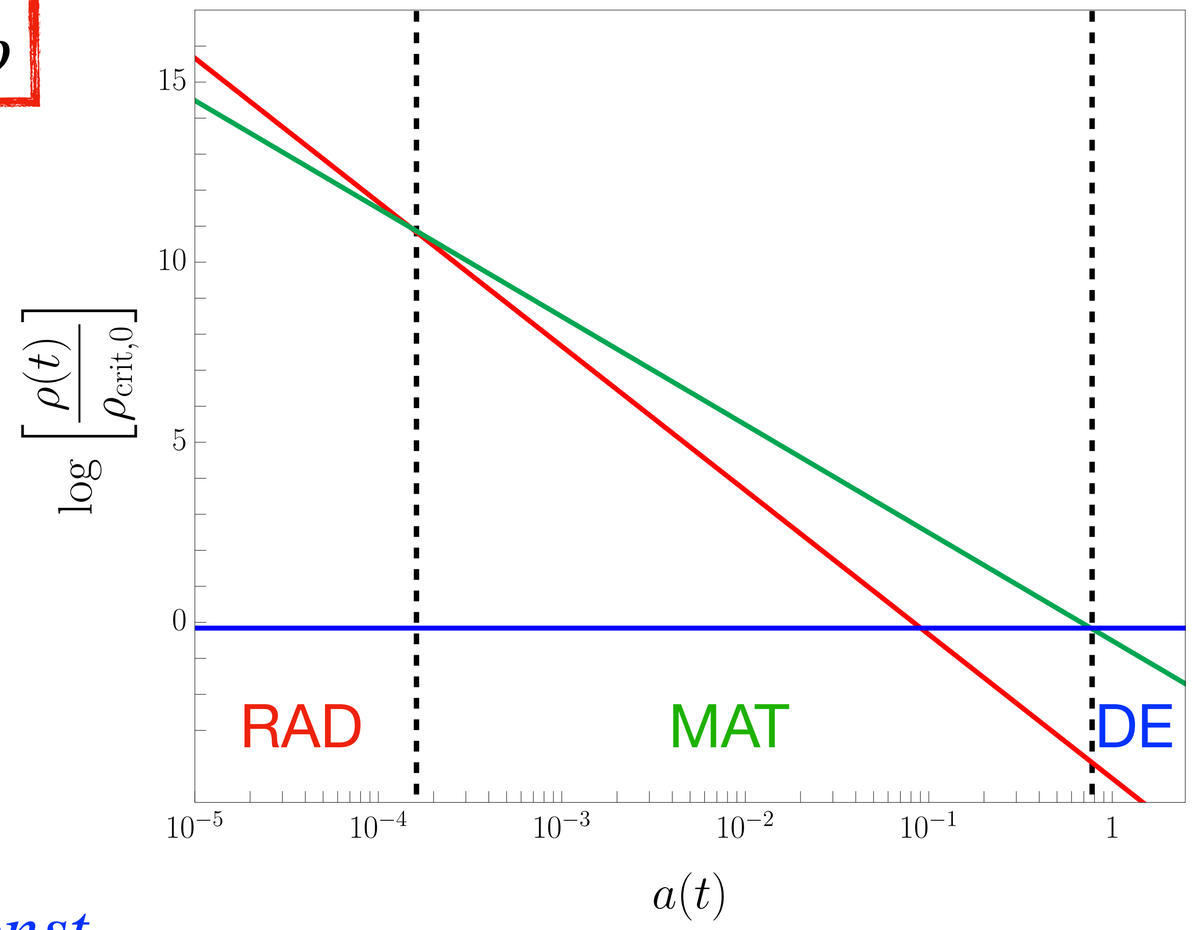
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Hubble always decreasing

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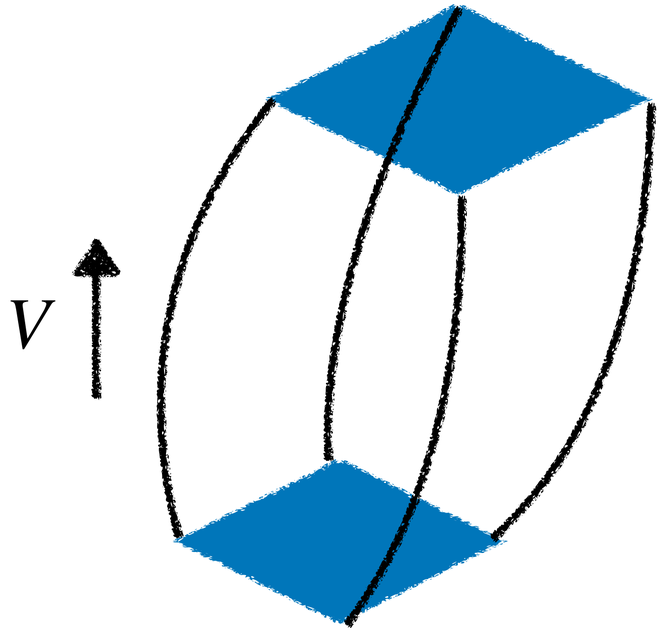
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Energy cons. important role in the Raychaudhuri eq.

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Dark Energy as Λ

Λ (> 0): Cosmological constant = Dark Energy

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Cosmological Constant Problem

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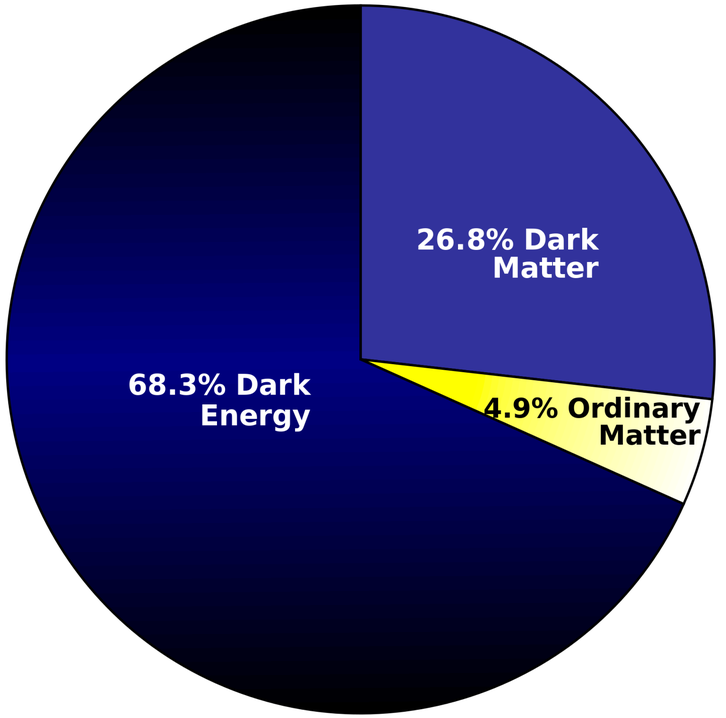
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Cosmological Constant Problem

Present Energy Budget



CDM: Cold Dark Matter — Non-relativistic

Formation of large scale structure

WIMPs (Weakly Interacting Massive Particles)

PBHs (Primordial Black Holes)

Axions ・・・

} Long-lived and No interaction with light

Measuring w of Dark Energy

- Friedmann's eq. $\Omega_i \equiv \rho_i/\rho_c$ $w = w(z)$

$$\frac{H^2}{H_0^2} = \Omega_{\rm M}(1+z)^3 + \Omega_{\rm R}(1+z)^4 + \Omega_k(1+z)^2 + \Omega_{\rm DE}\frac{\rho_{\rm DE}(z)}{\rho_{\rm DE,0}}$$

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w_0w_aCDM model

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BAO + CMB + SNe data

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$$H(z) = \left[\frac{\rm d}{{\rm d}z}\left(\frac{d_{\rm L}(z)}{1+z}\right)\right]^{-1}$$

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$d_{\rm L}$ = Luminosity distance

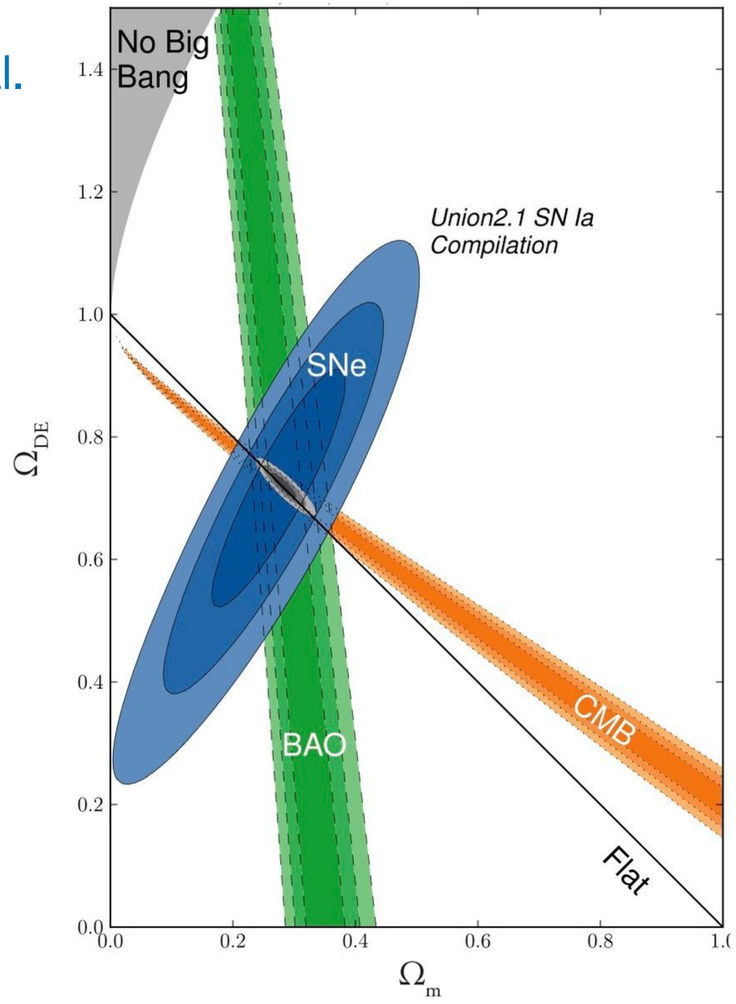
w_0w_aCDM model

Tight constraints on $\Omega_{\rm DE}$ and $w(z)$

Dark Energy Evidences

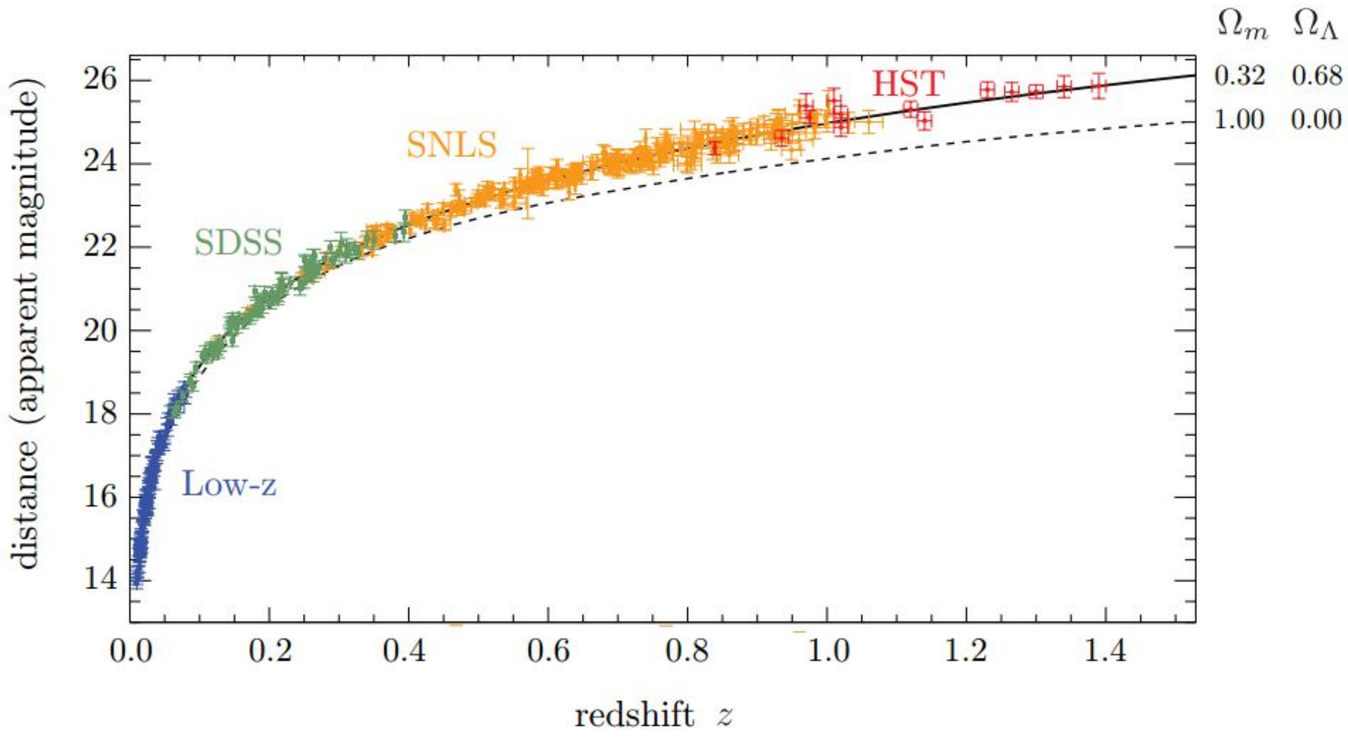
- Responsible for accelerating expansion of the Universe today

Suzuki et al. 2012



2011 Nobel Prize — Perlmutter, Schmidt and Riess

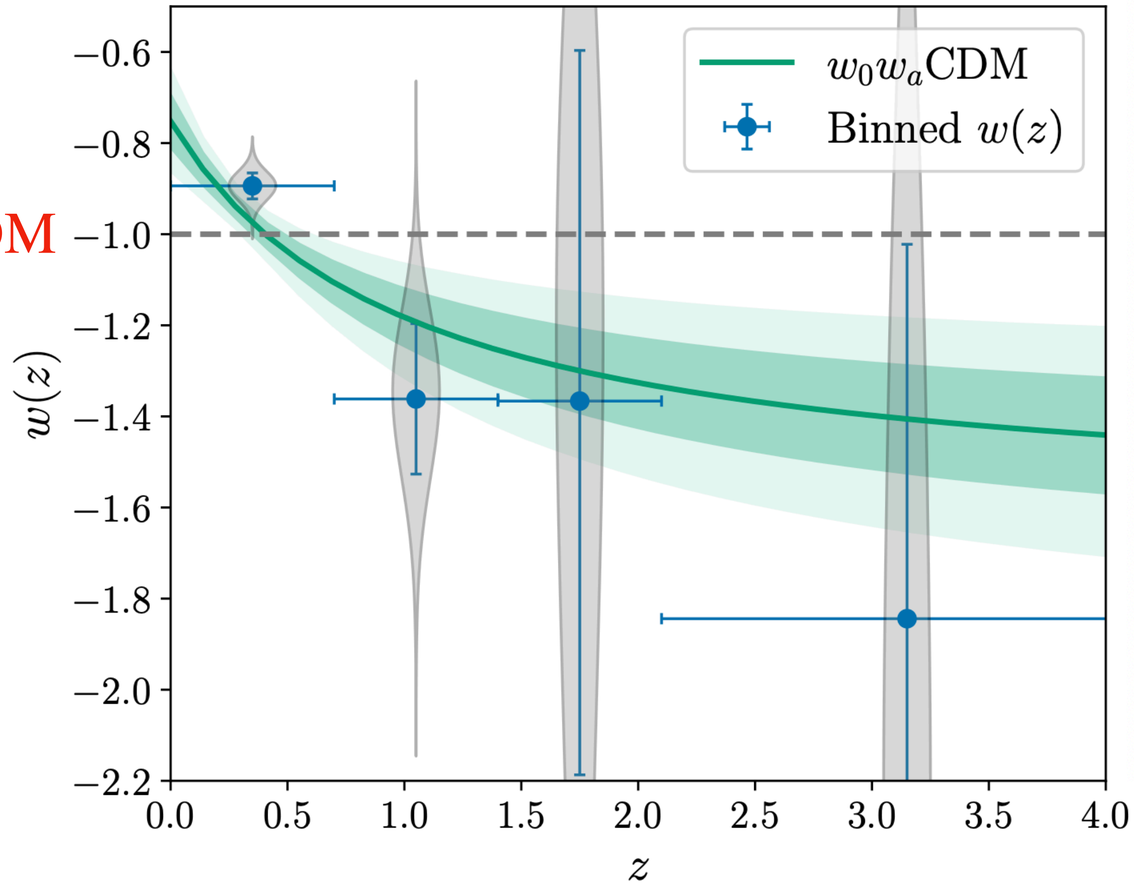
$$\Omega_{DE} \sim 0.68 \Rightarrow \rho_{DE} \sim 7 \cdot 10^{-30}\ \mathrm{g/cm^3} \simeq 10^{-12}\ \mathrm{eV}^4$$

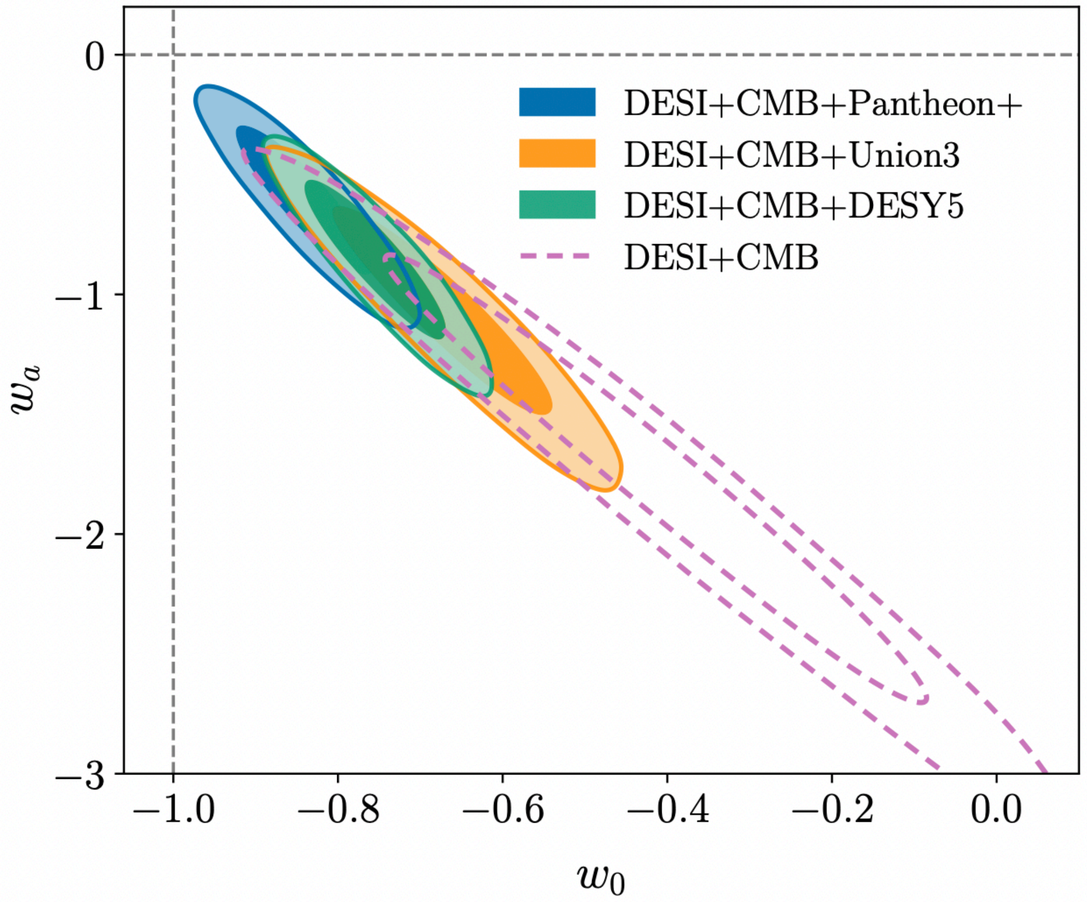


Perhaps DE is dynamical…

- DESI DR2 Results (DESI 25)

ΛCDM





$$w_0 = -0.752 \pm 0.057$$

$$w_a = -0.86^{+0.23}_{-0.20}$$

DESI + CMB + DESY5

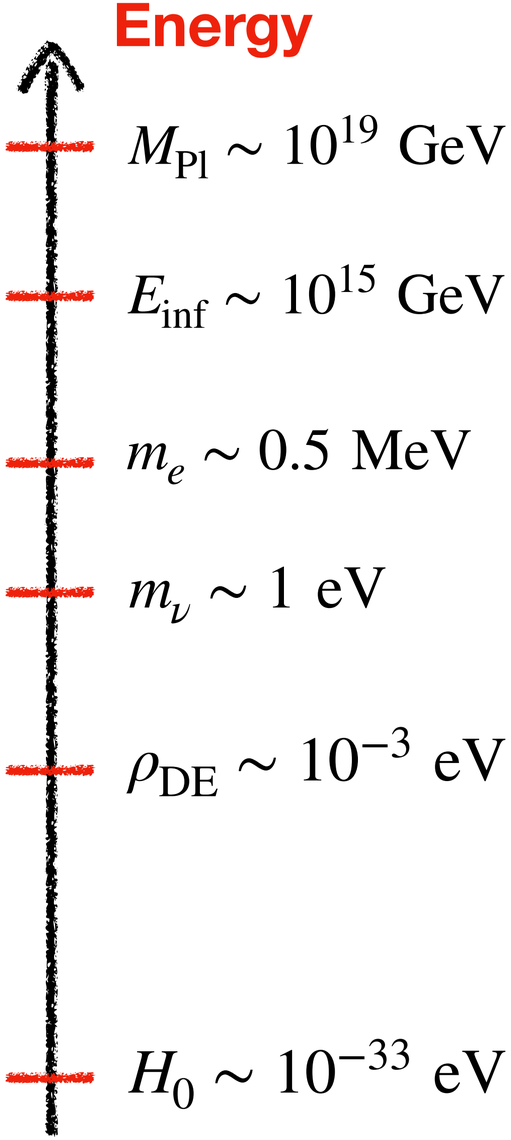
Blue: no assuming a functional form of $w(z)$

Dark Energy…

$$H_0 = (67 - 74)\,\frac{\mathrm{km}}{\mathrm{s}\cdot\mathrm{Mpc}}$$

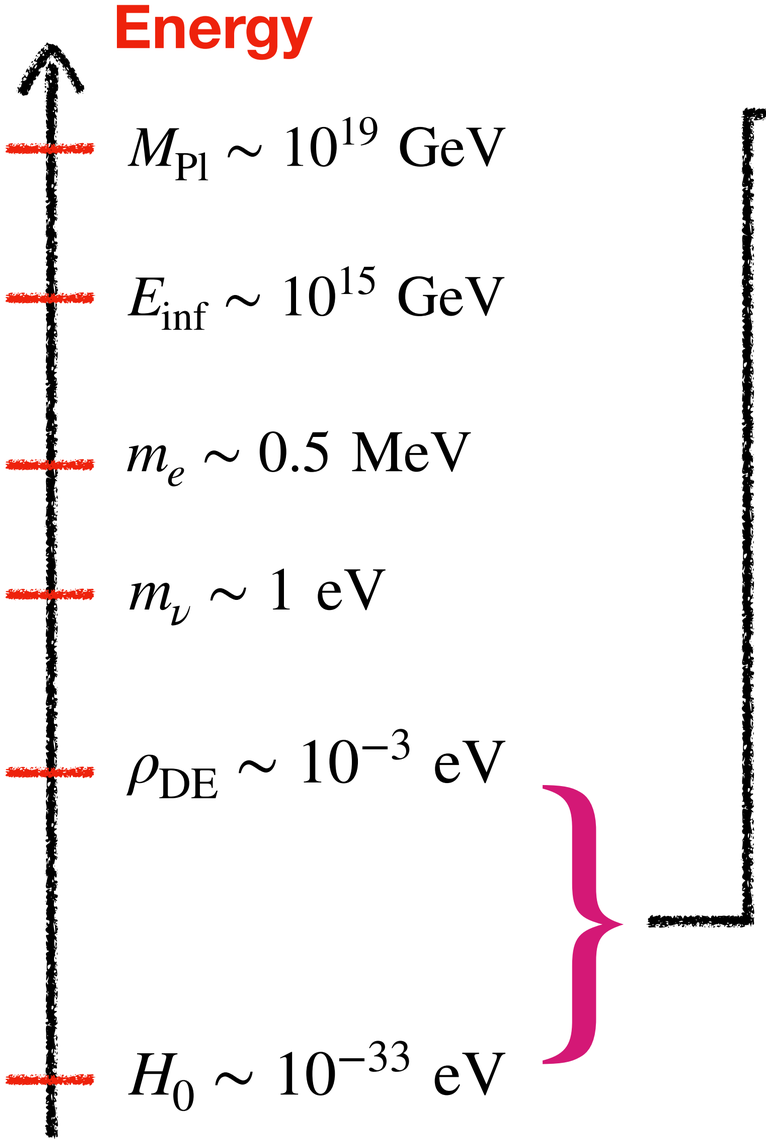
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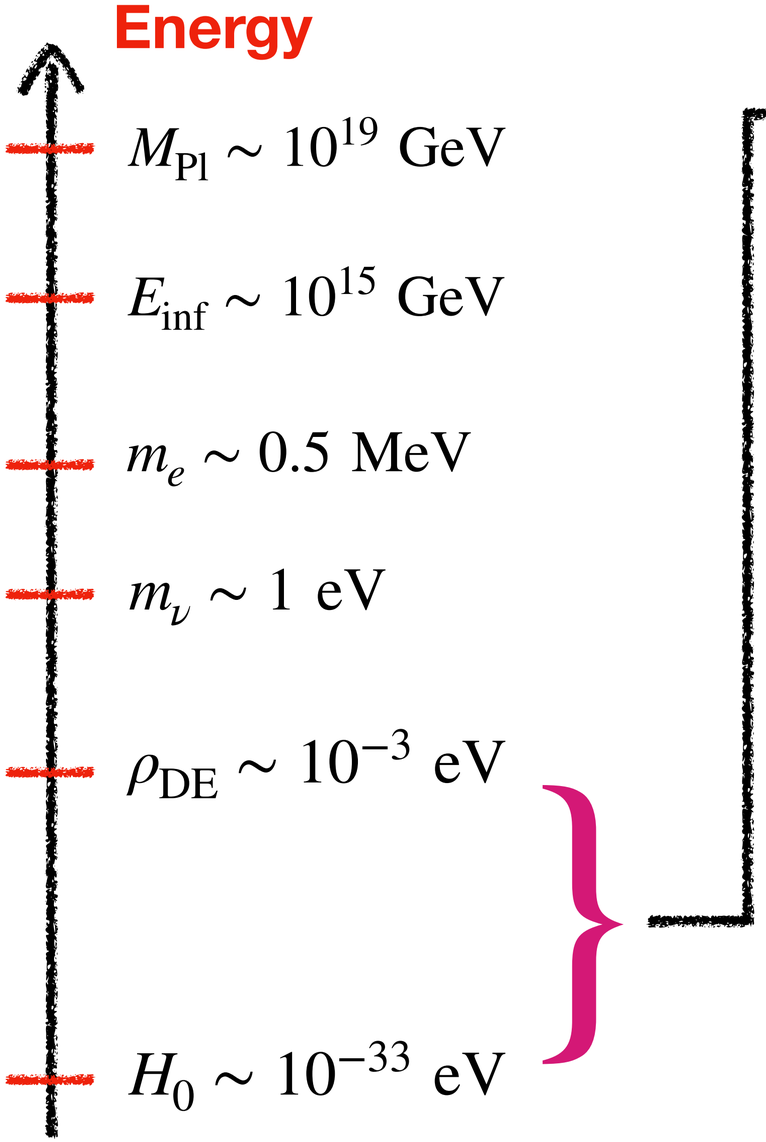
Physics of Dark Energy = Low-Energy Physics

On large scales, gravity (+ possible fifth force) matters

$$\mathscr{L} = \frac{M_{\rm Pl}^2}{2}R + \mathscr{L}_\phi(\phi, \partial\phi, \partial\partial\phi) + \cdots$$

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If Dark Energy = Cosmological Constant Λ

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Quintessence Dark Energy

- Additional scalar field: pheno. extension of $\Lambda\mathrm{CDM}$

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- Slow roll $\dot{\phi}^2/2 \ll V \quad \Rightarrow \quad H^2 \sim \dfrac{V}{M_{\mathrm{Pl}}^2}$ and $H\dot{\phi} \sim V'$

$$V \sim m_\phi^2\phi^2 \quad \Rightarrow \quad m_\phi \ll H \sim 10^{-33}\ \mathrm{eV}$$ Very light field

Quintessence Dark Energy

- Additional scalar field: pheno. extension of $\Lambda\mathrm{CDM}$

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Very light field

Displacement in one Hubble time

$$\Delta\phi \sim \frac{\dot{\phi}}{H} \sim \frac{V'}{H^2}$$

$$\frac{\Delta\phi}{\phi} \ll 1 \quad \Rightarrow \quad \frac{V'}{\phi} \ll H^2$$

Quintessence Dark Energy

Steinhardt, Wang, Zlatev 98

- Tracker solution $V(\phi) = \dfrac{M^{n+4}}{n\phi^n}$ $n > 2$

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$\rho_{\rm R} \propto a^{-4} \propto t^{-2}$

$\rho_{\rm M} \propto a^{-3} \propto t^{-2}$

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- EoS parameter $n > 2$

$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} = \frac{n}{n+2} w_i - \frac{2}{n+2}$$

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Not slow roll

w_i of component that dominates Universe

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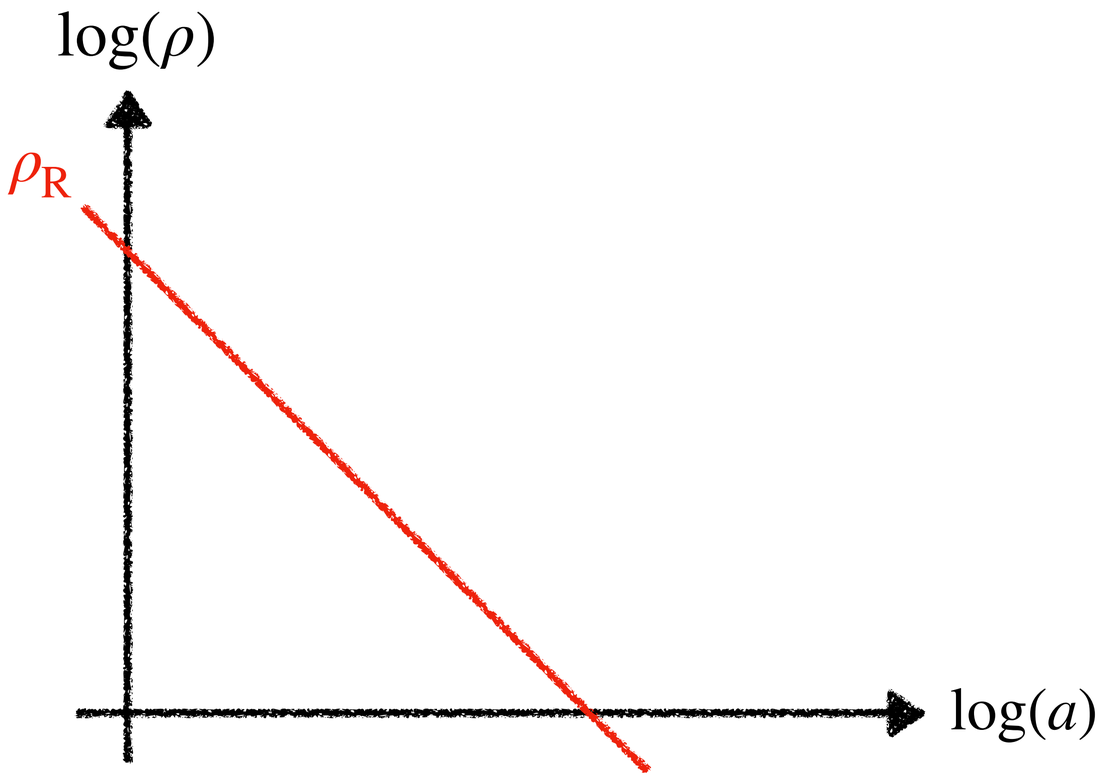
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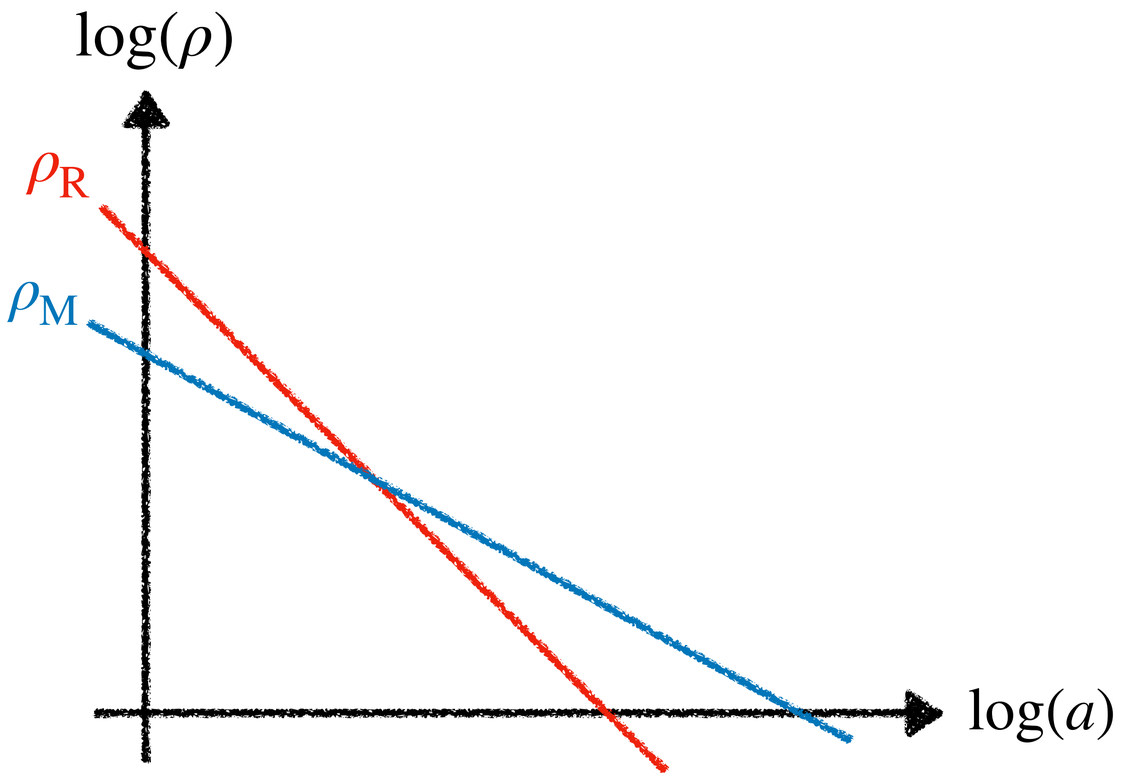
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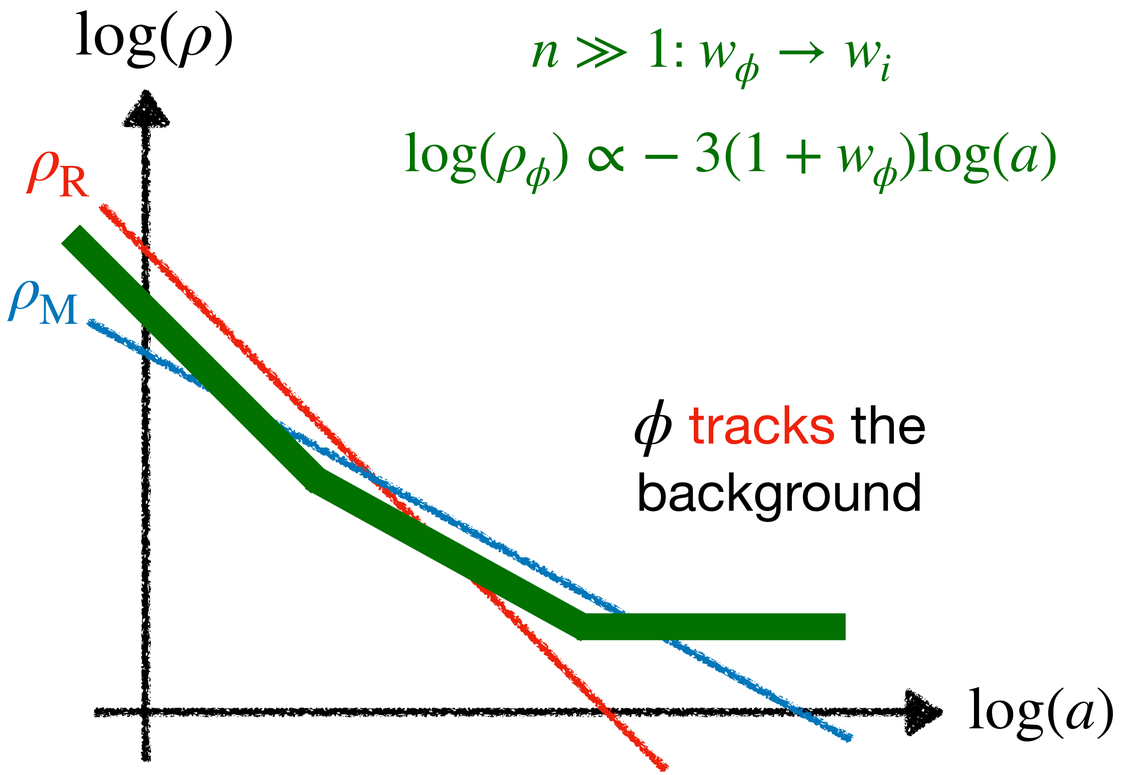
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$$\log(\rho_\phi) \propto -3(1+w_\phi)\log(a)$$



ϕ tracks the background

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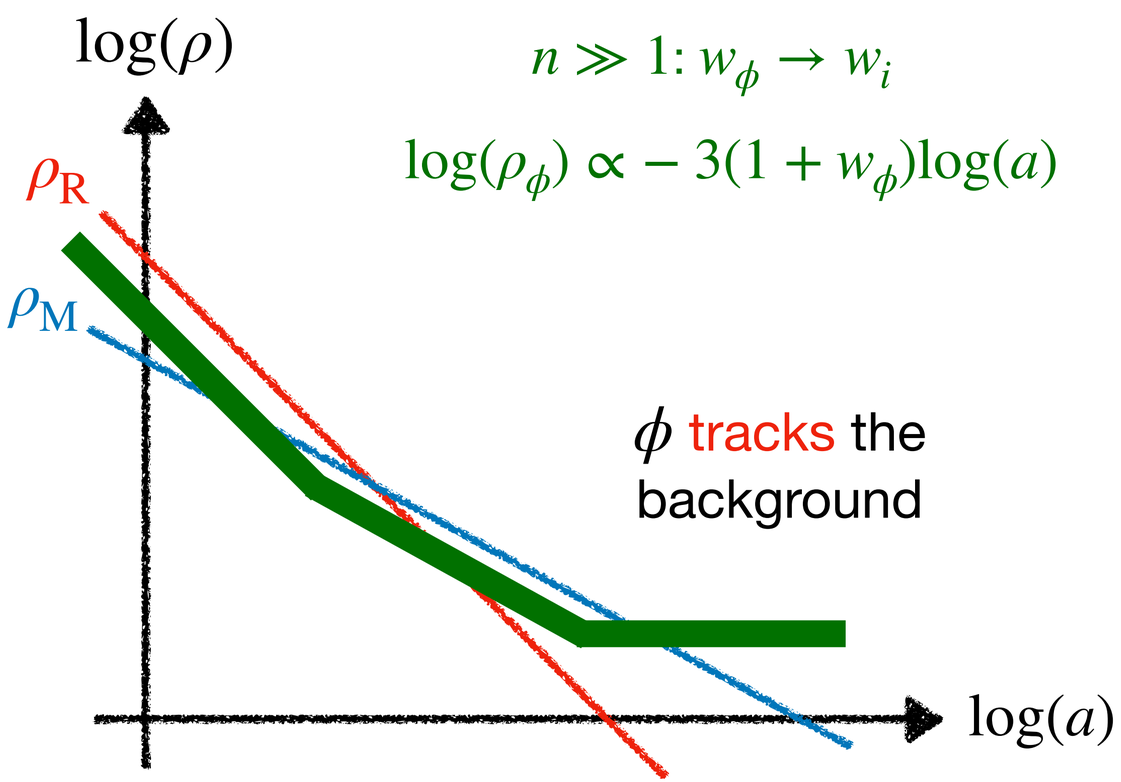
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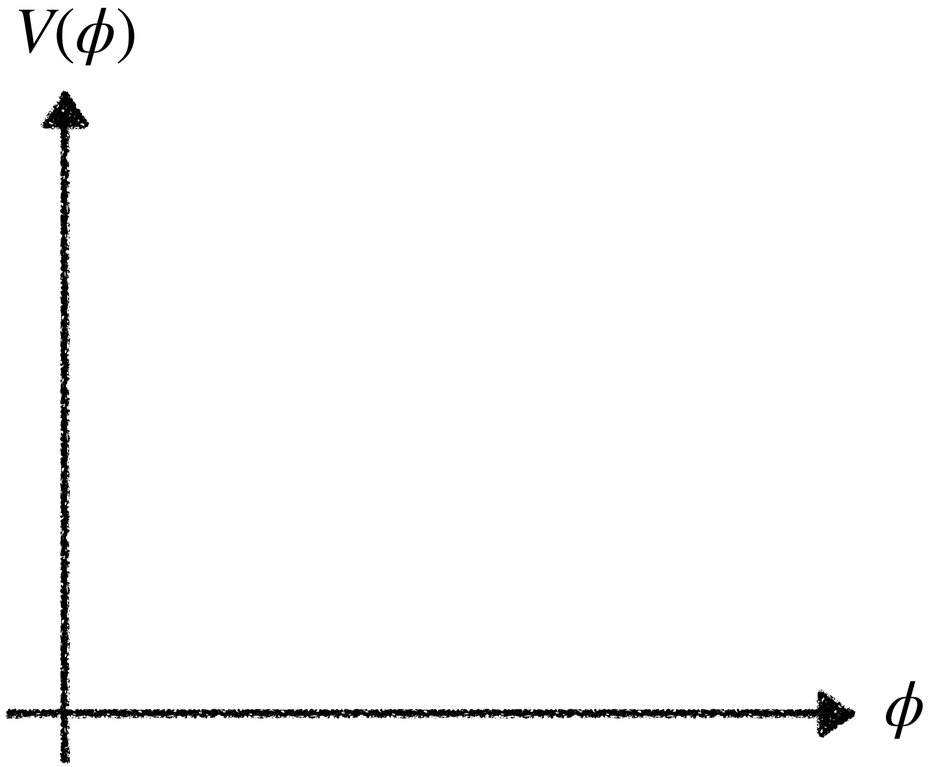
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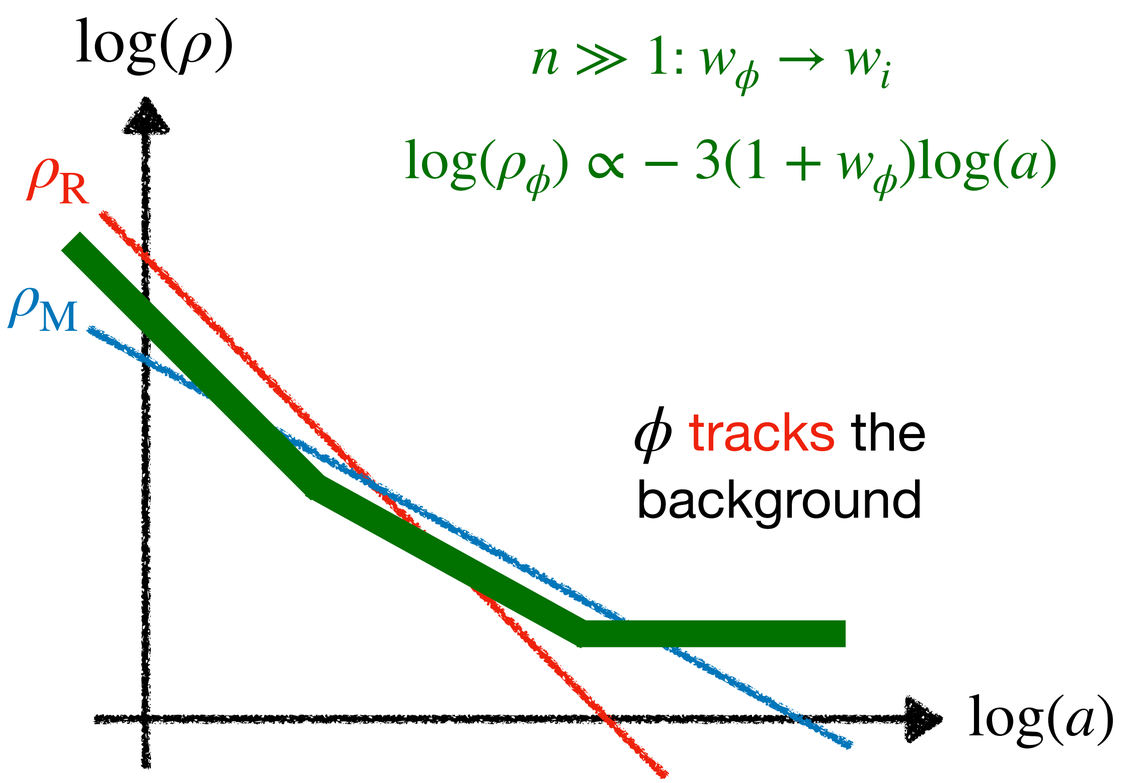
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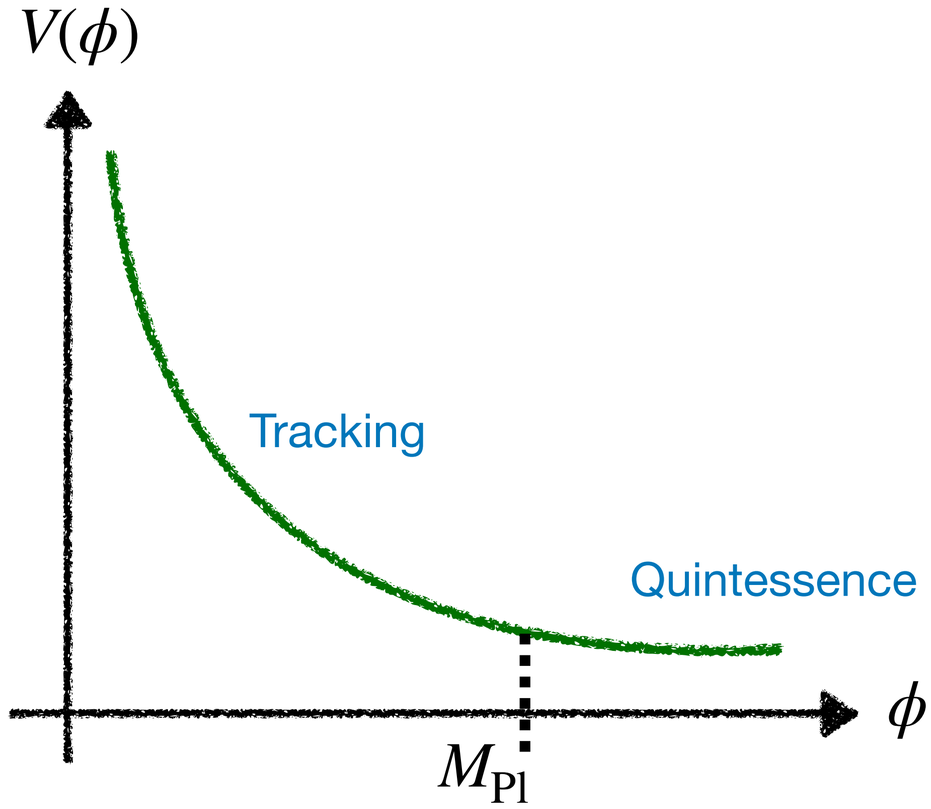
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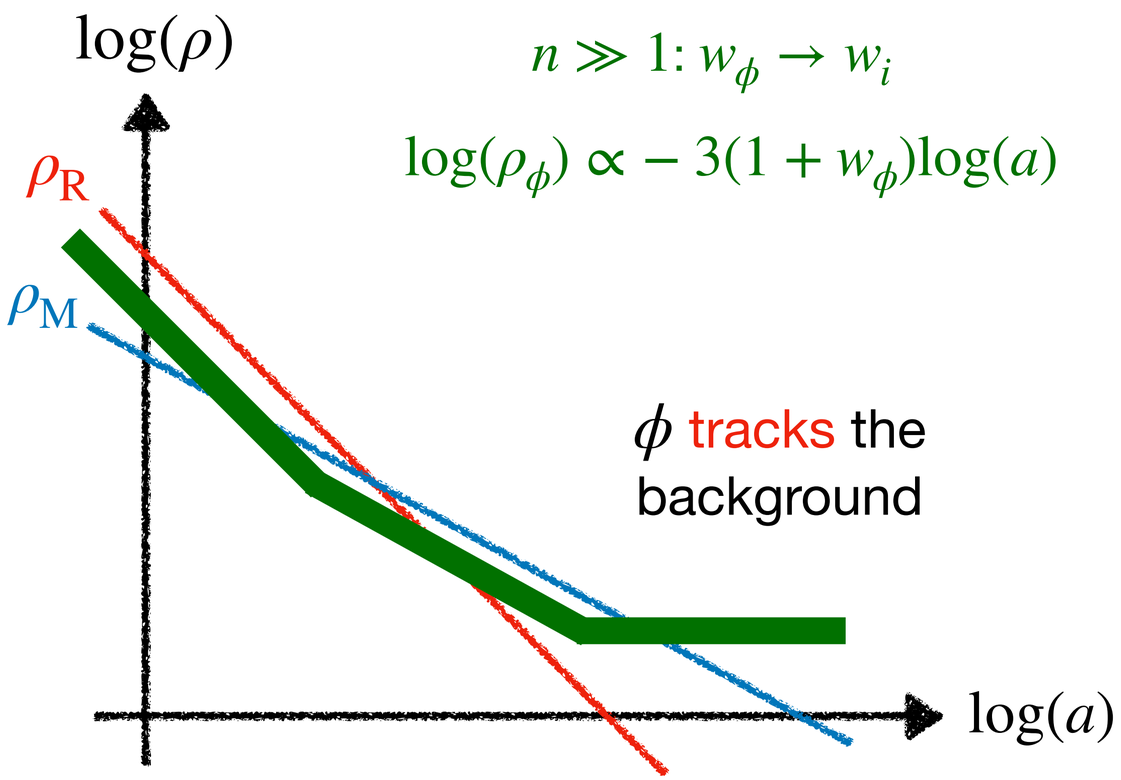
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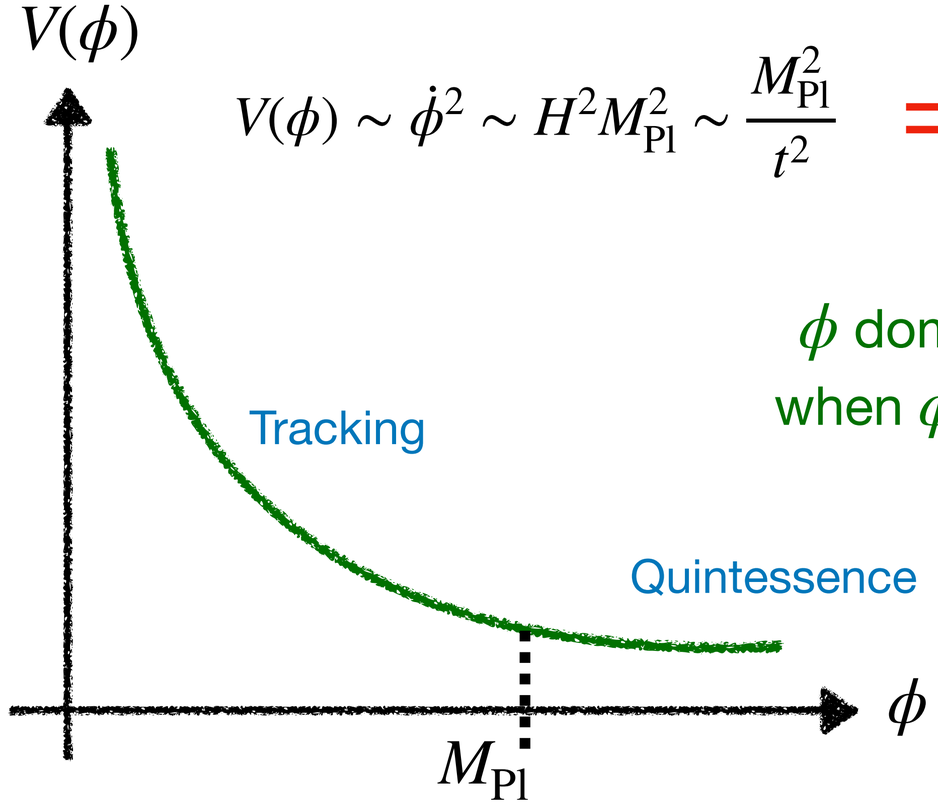
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$$V(\phi) \sim \dot{\phi}^2 \sim H^2 M_{\rm Pl}^2 \sim \frac{M_{\rm Pl}^2}{t^2} \Rightarrow \phi \sim M_{\rm Pl}$$

ϕ dominates when $\phi \sim M_{\rm Pl}$

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Quadratic action for $\varphi(t,\vec{x})$:

$$S_\varphi^{(2)} = \int \mathrm{d}^3x\mathrm{d}t\, a^3\left[\frac{1}{2}\dot{\varphi}^2 - \frac{1}{2a^2}(\partial_i\varphi)^2 + \cdots\right]$$

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$$\omega^2 = k^2$$

The speed of φ = speed of light

Subhorizon limit: $k \gg aH$

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- k-essence model = clustering of quintessence

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + P(X,\phi)\right] \qquad X \equiv (\partial_\mu\phi)^2$$

Arkani-Hamed et al. 03+04, Creminelli et al. 06, +++

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Clustering allows φ to travel with speed ≠ speed of light

$$\mathscr{L}^{(2)}_\varphi = P_X(c^2)\left[\dot\varphi^2 - (\nabla_i\varphi)^2\right] + 2P_{XX}(c^2)c^2\dot\varphi^2 \Rightarrow \boxed{c_s^2 = \frac{P_X(c^2)}{P_X(c^2) + 2XP_{XX}(c^2)}}$$

$c_s^2 \ll 1$, DE actively contributes to structure formation as DM

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For $P(X) = X^2$

$c_s^2 = \frac{1}{3}$ (Rad fluid)

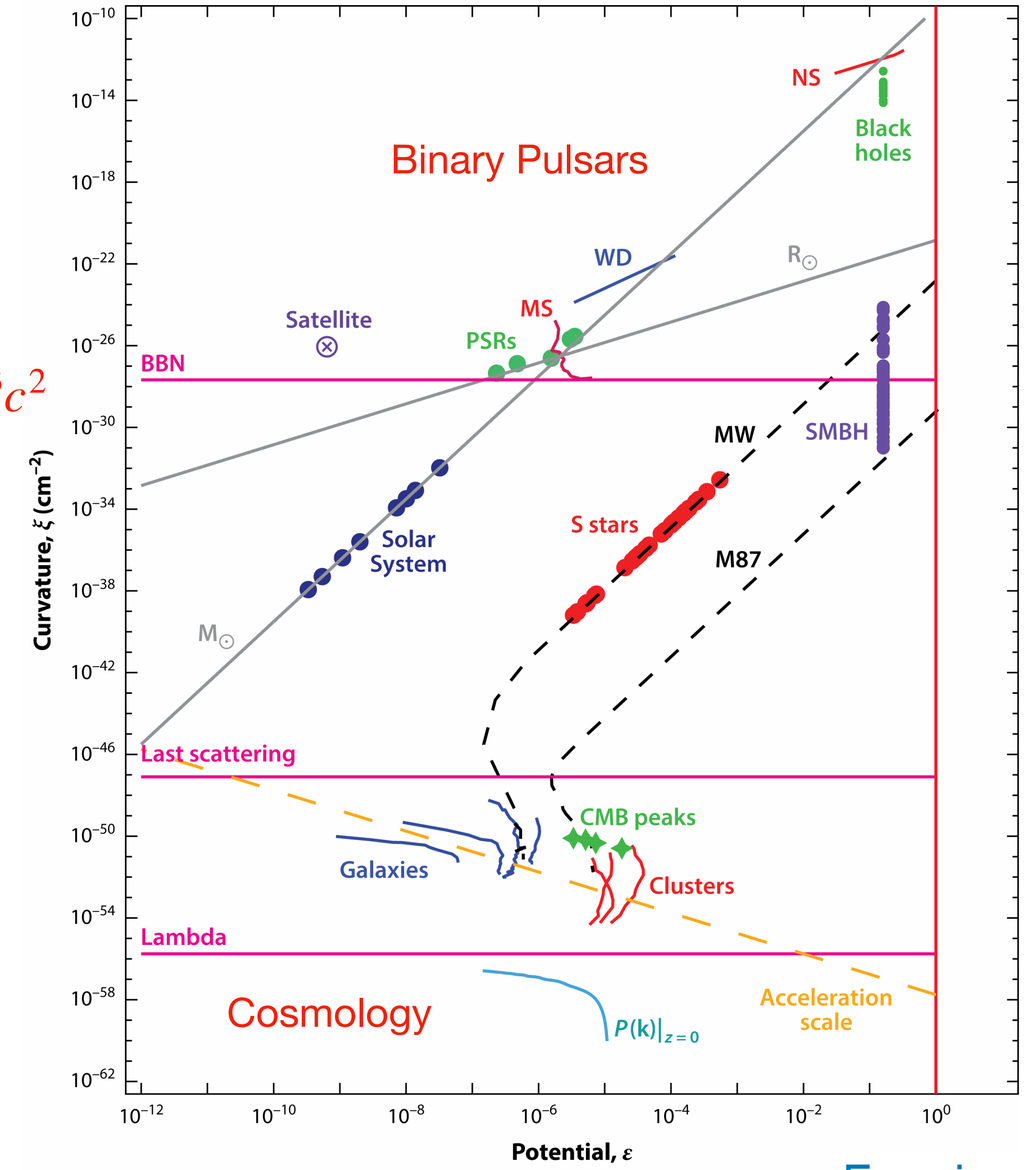
$c_s^2 \ll 1$, DE actively contributes to structure formation as DM

Arkani-Hamed et al. 03+04, Creminelli et al. 06, +++

Gravity on large scales

- Accelerated expansion of our Universe today leads to a lot of activities trying to extend gravity and cosmology beyond ΛCDM

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Grav. potential

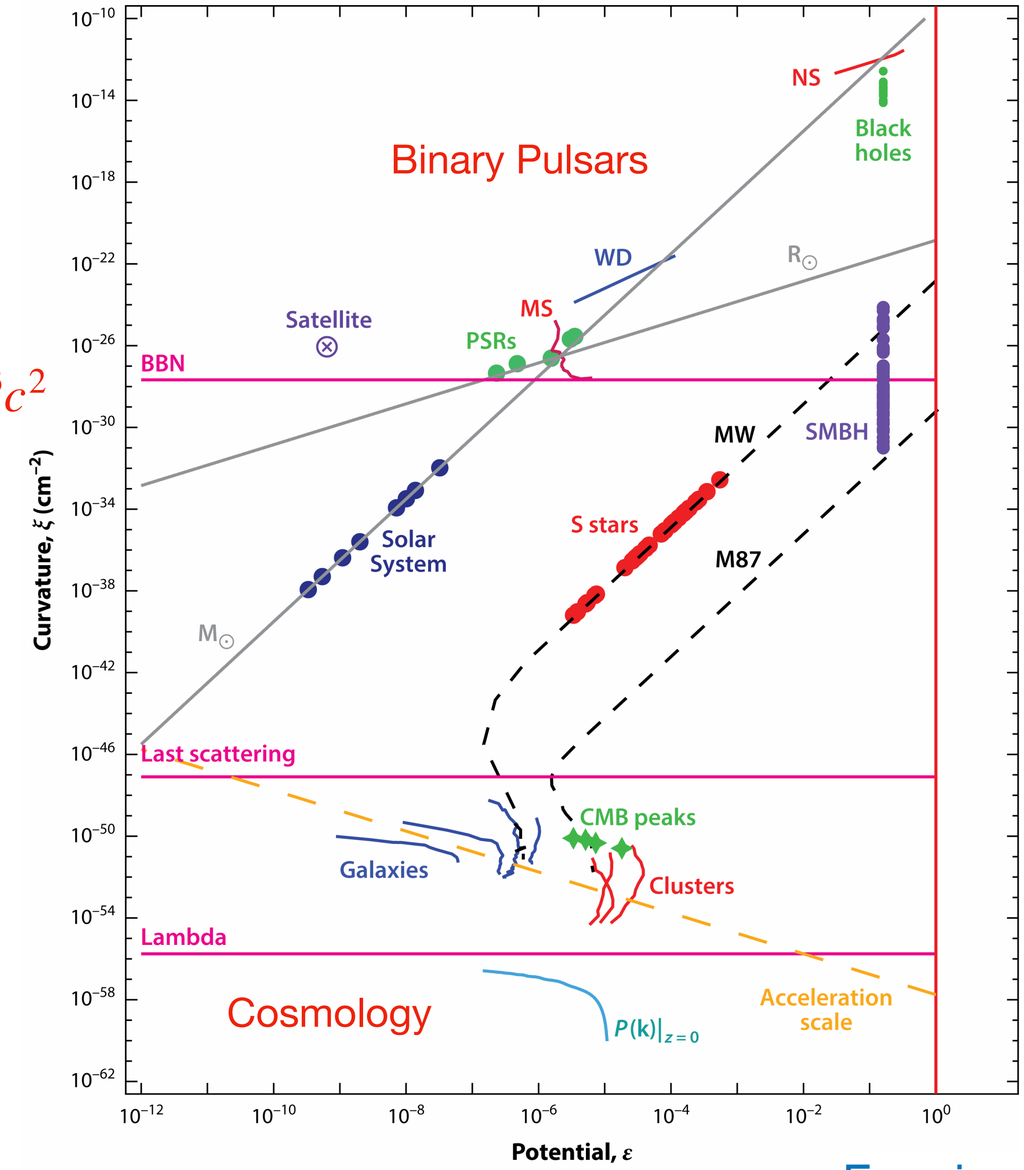
Ferreira 19

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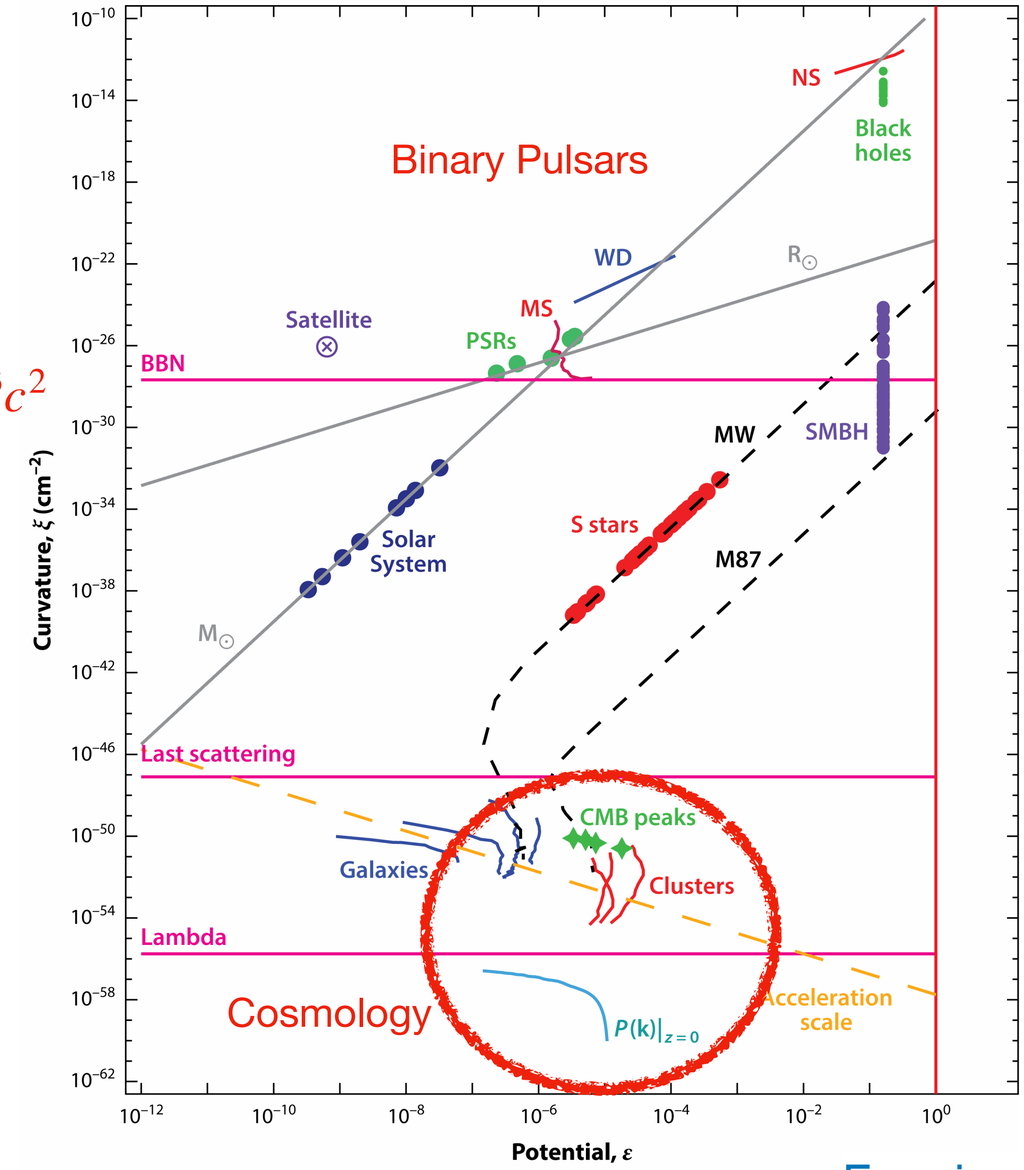
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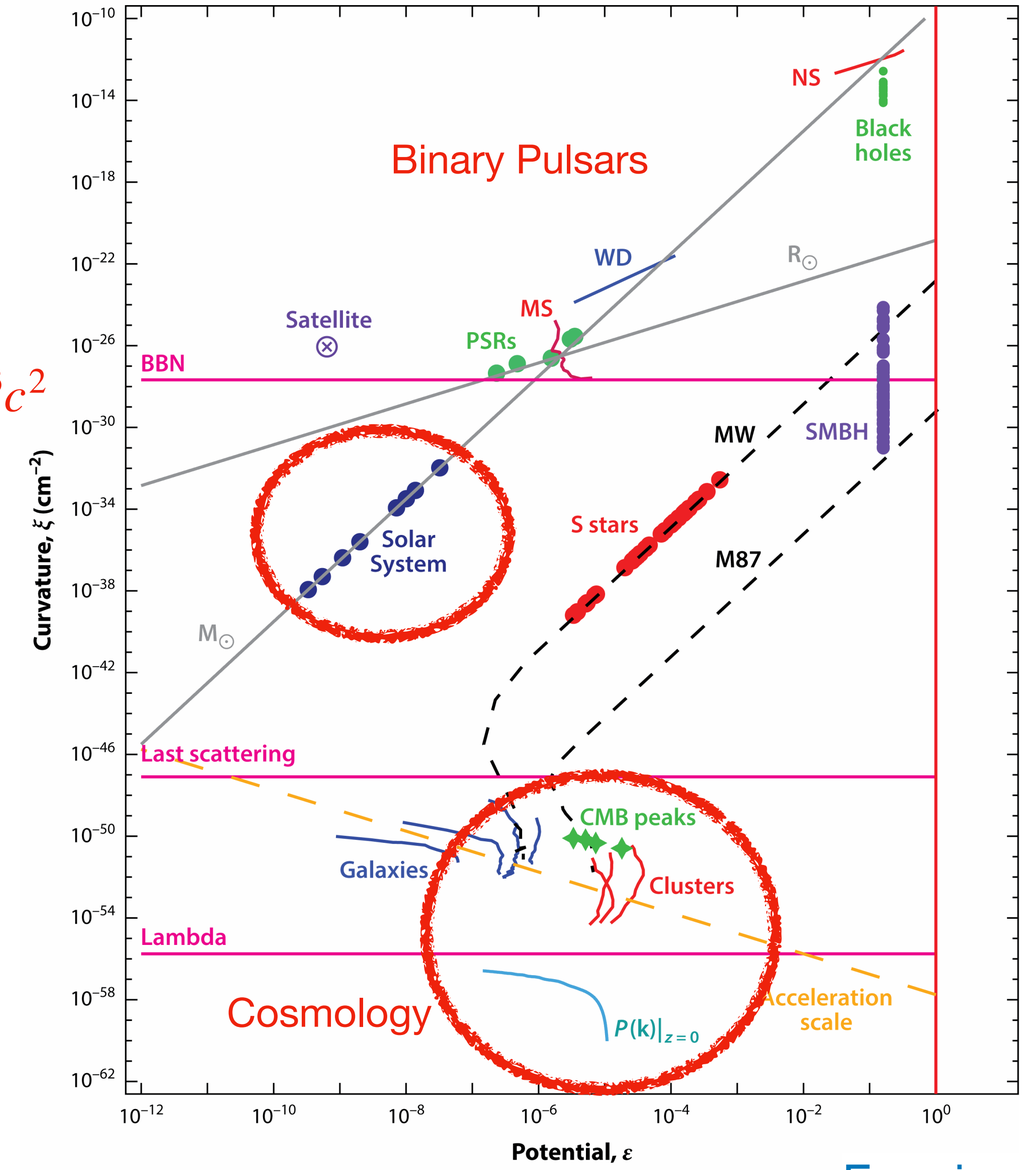
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Understand and test gravity better



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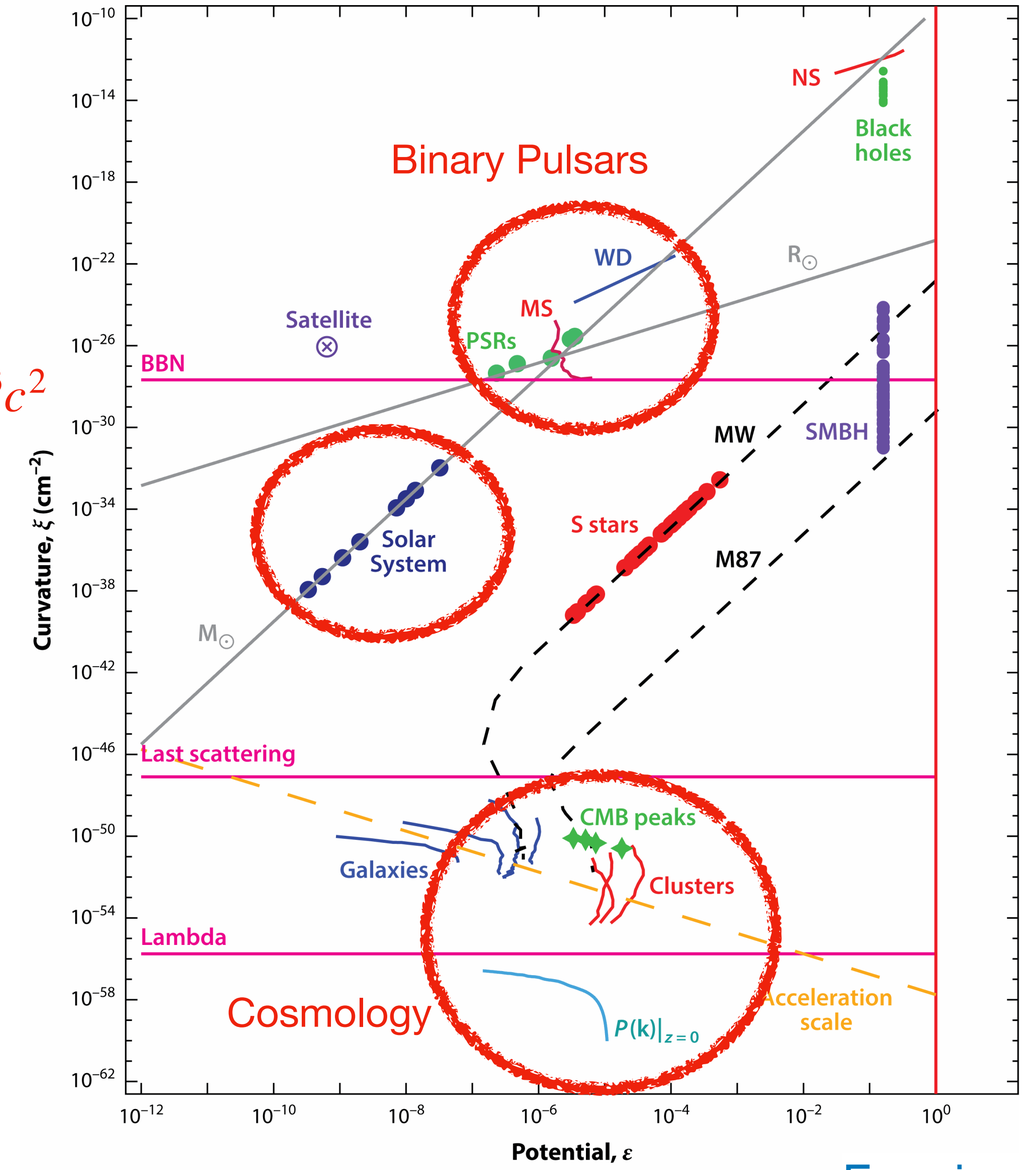
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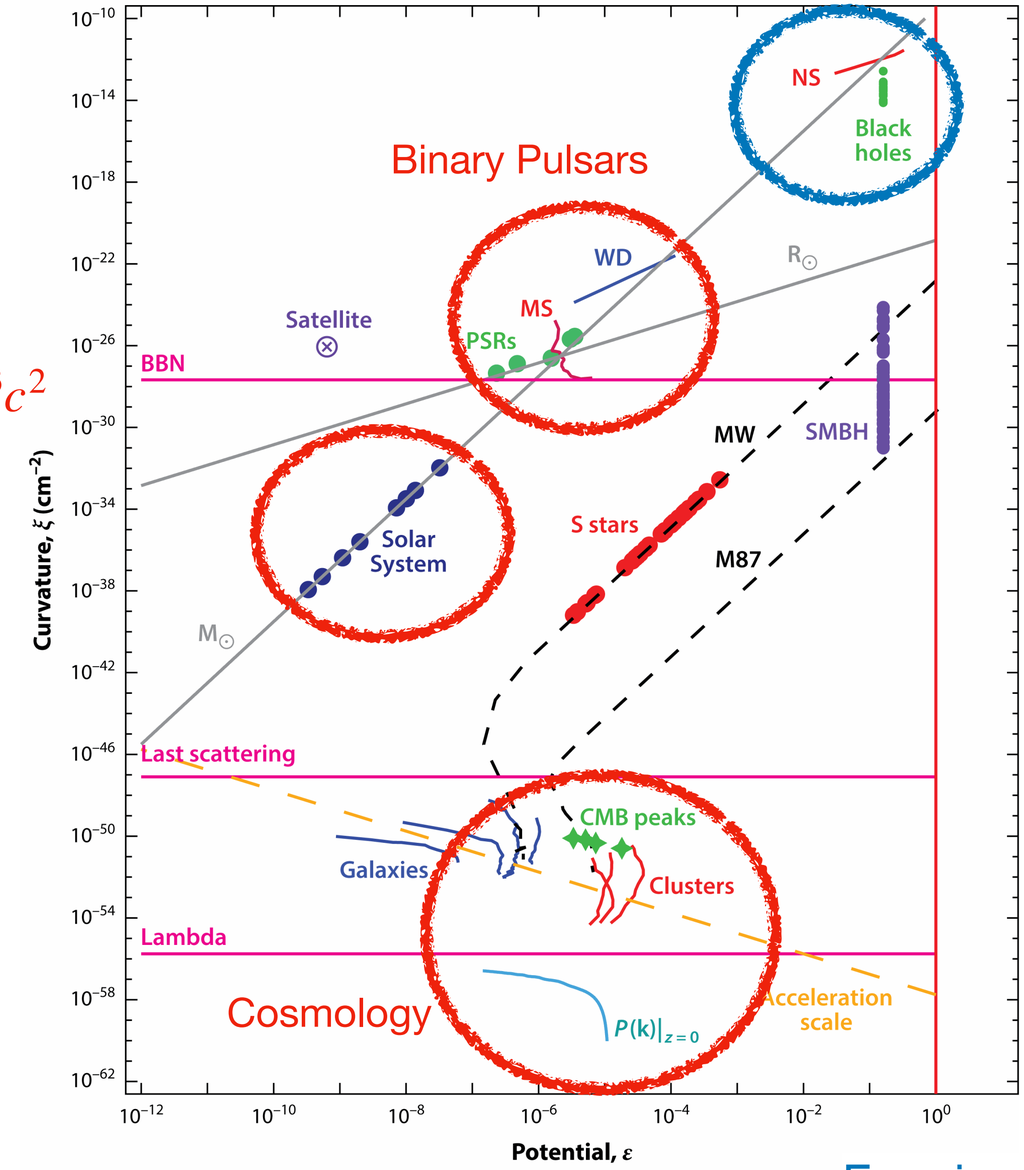
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- Focus on strong-gravity regime where deviations from GR are expected: BH/NS mergers



Grav. potential

Ferreira 19

Scalar-tensor theories

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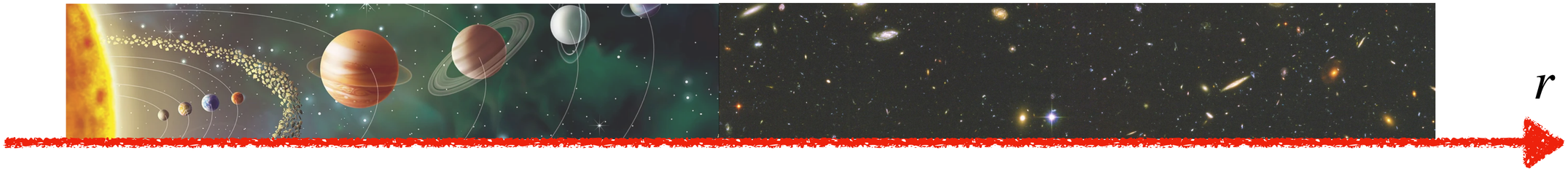
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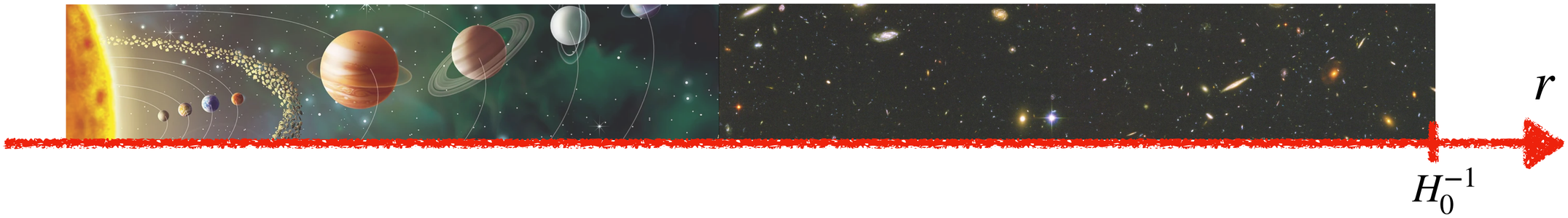


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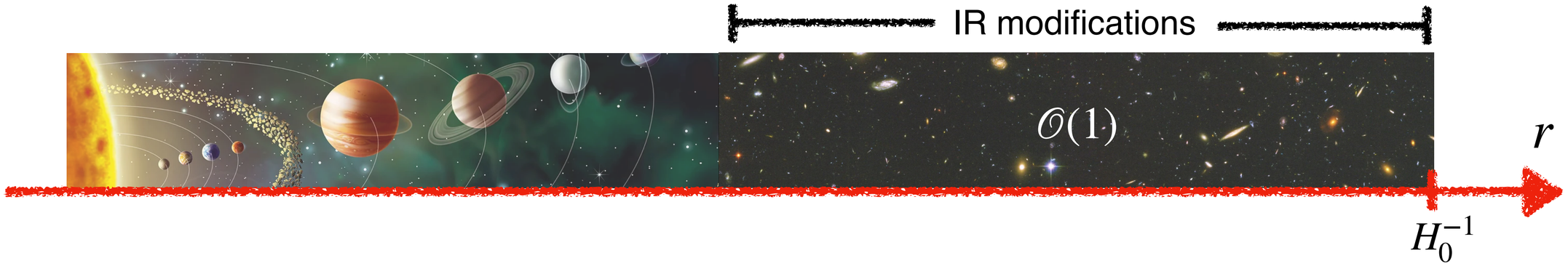


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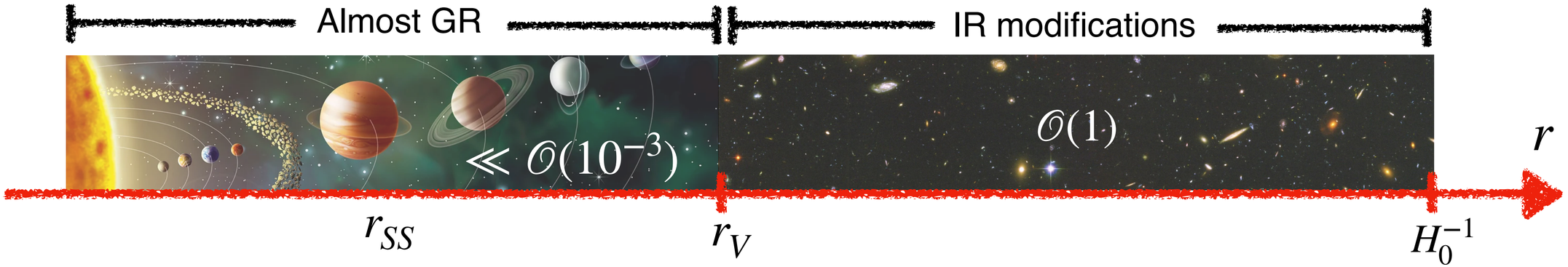


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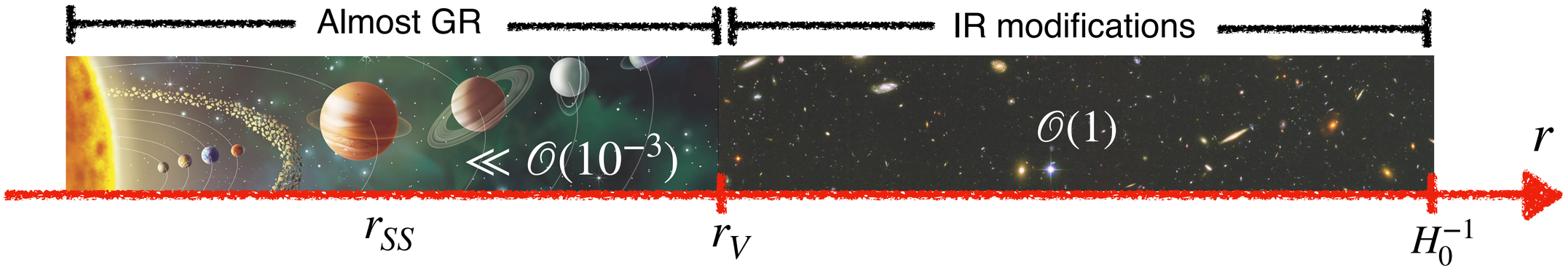
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Large classical non-linearities



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- Solar system tests impose severe bounds on modifications of gravity
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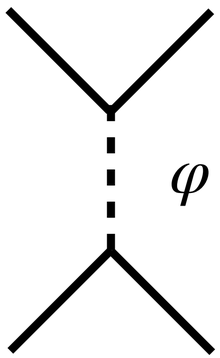
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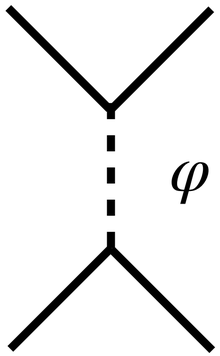
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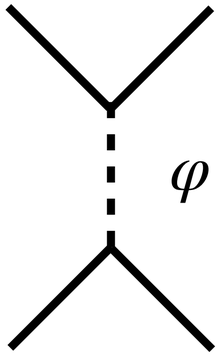


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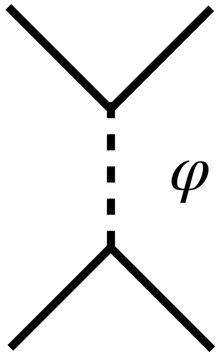
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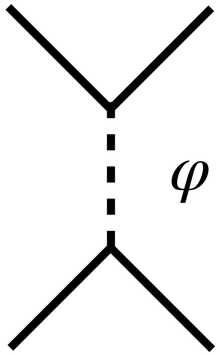
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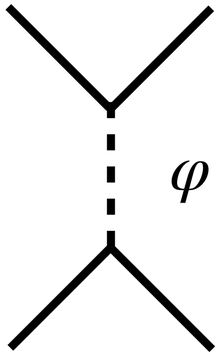
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Chameleon mechanism

3. $Z(\bar{\phi}) \gg 1$: large nonlinearities

Kinetic screening

e.g. Vainshtein & k-mouflage

Vainshtein screening

- Consider the cubic Galileon theory

Vainshtein 72, Babichev & Deffayet 13, +++

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Use divergence thm.

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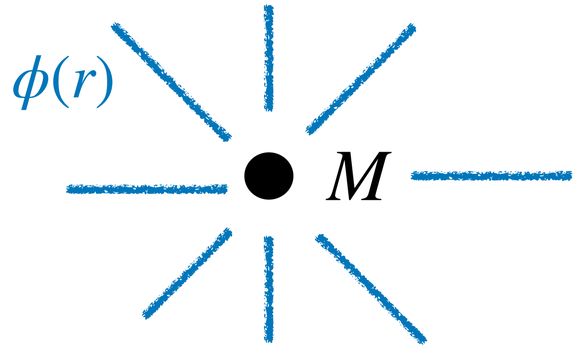
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$+\Phi$

Newton potential

Vainshtein screening

- Solution for $\phi'(r)$

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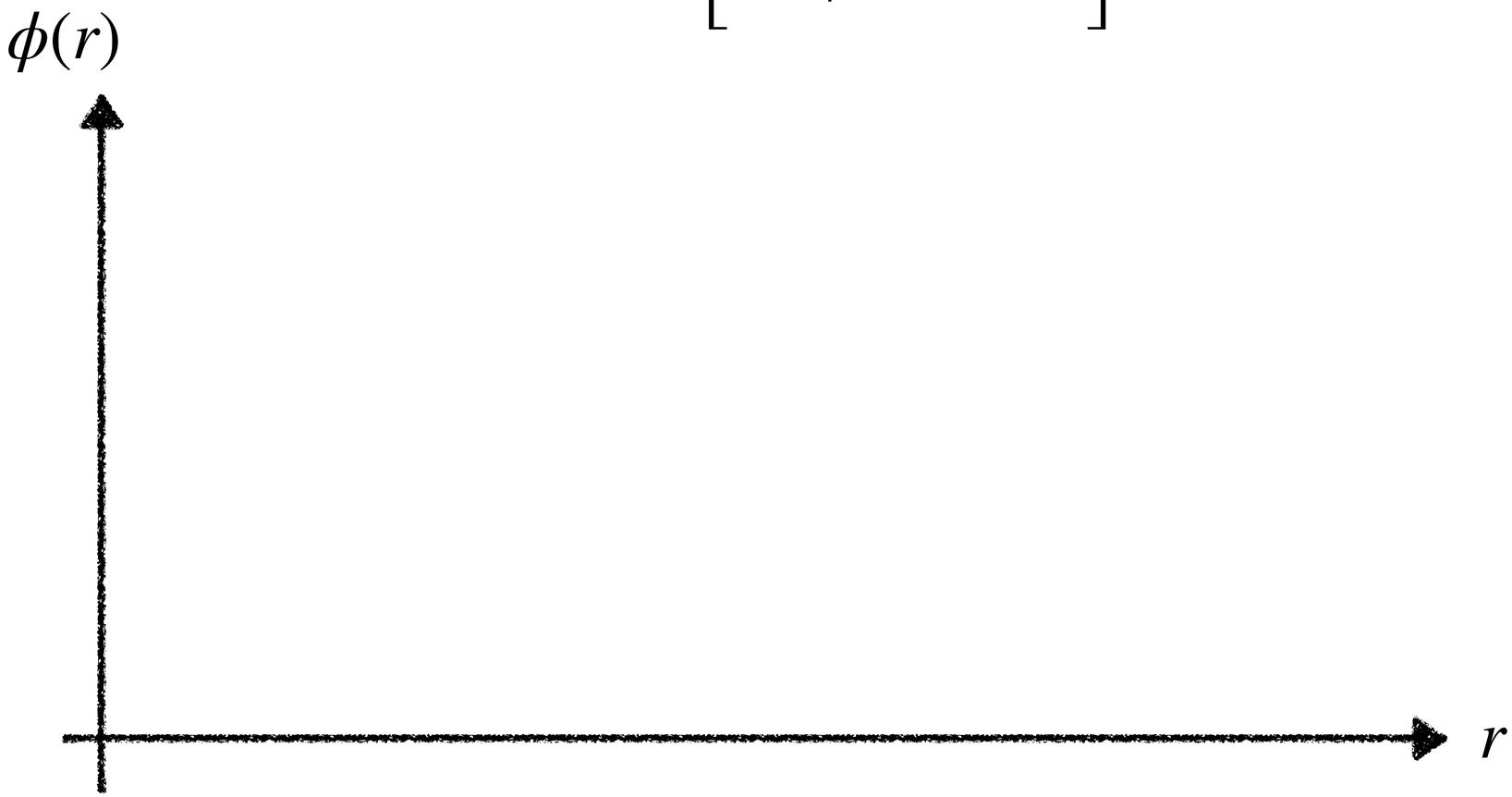
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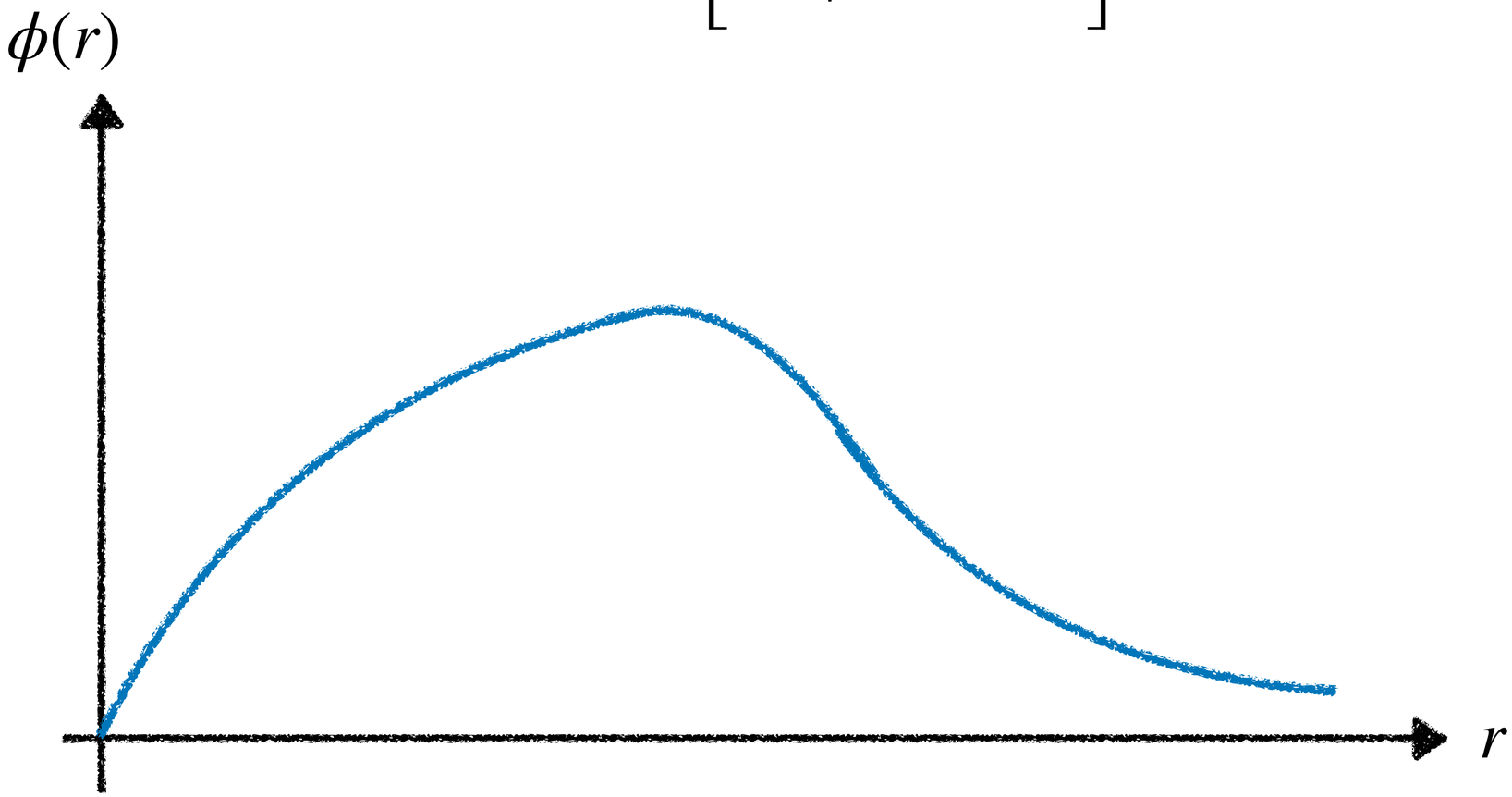
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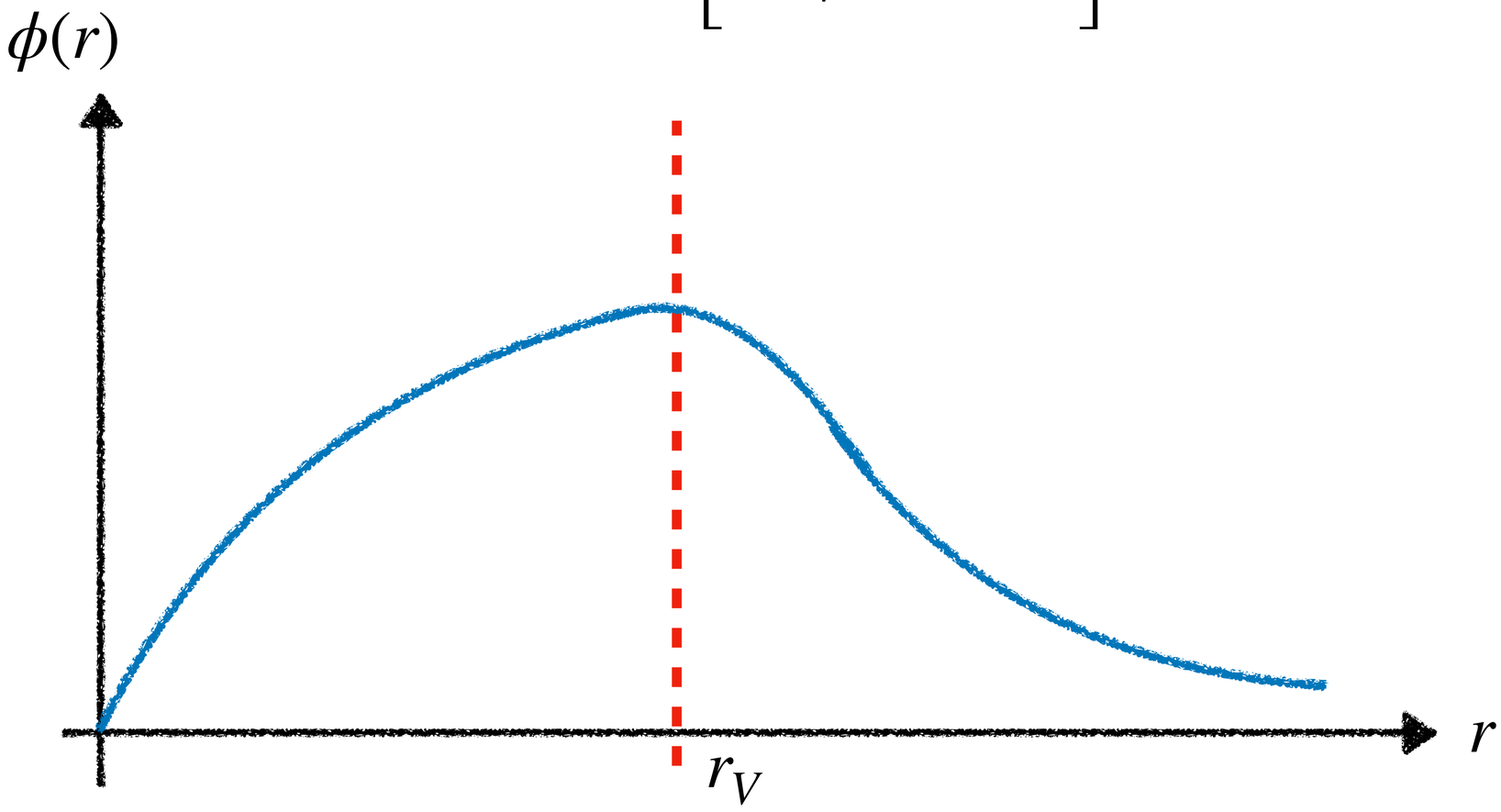
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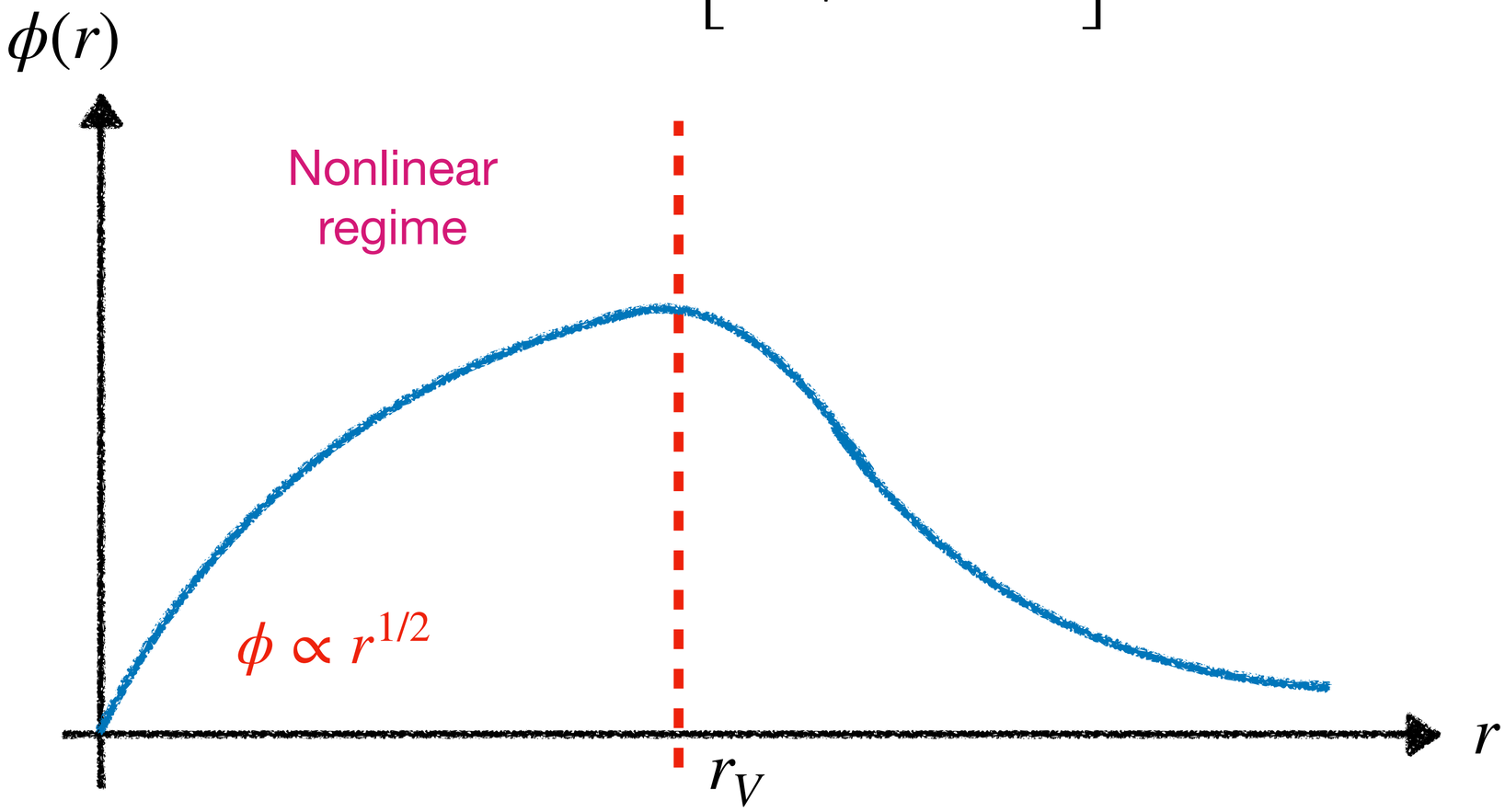
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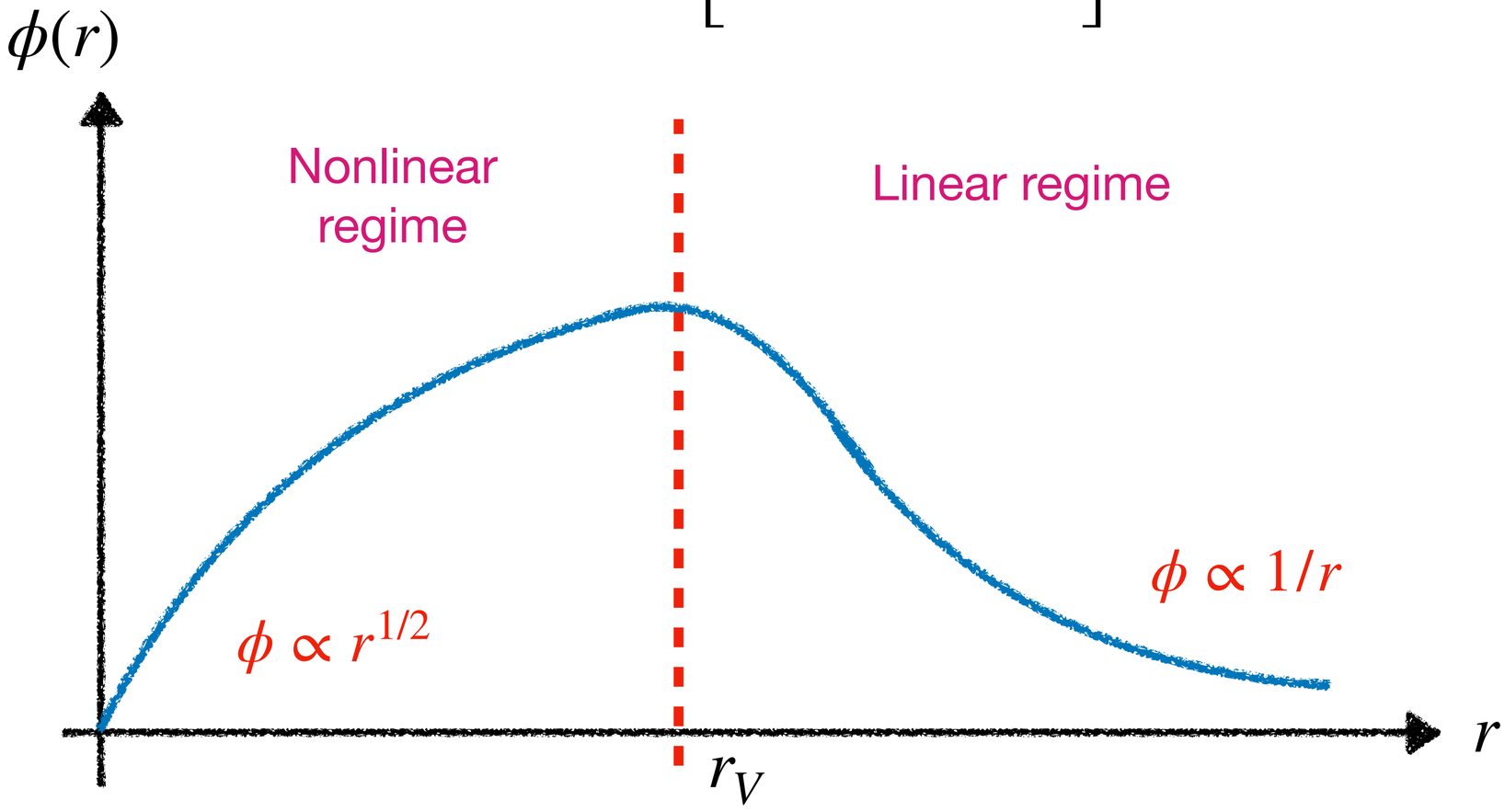
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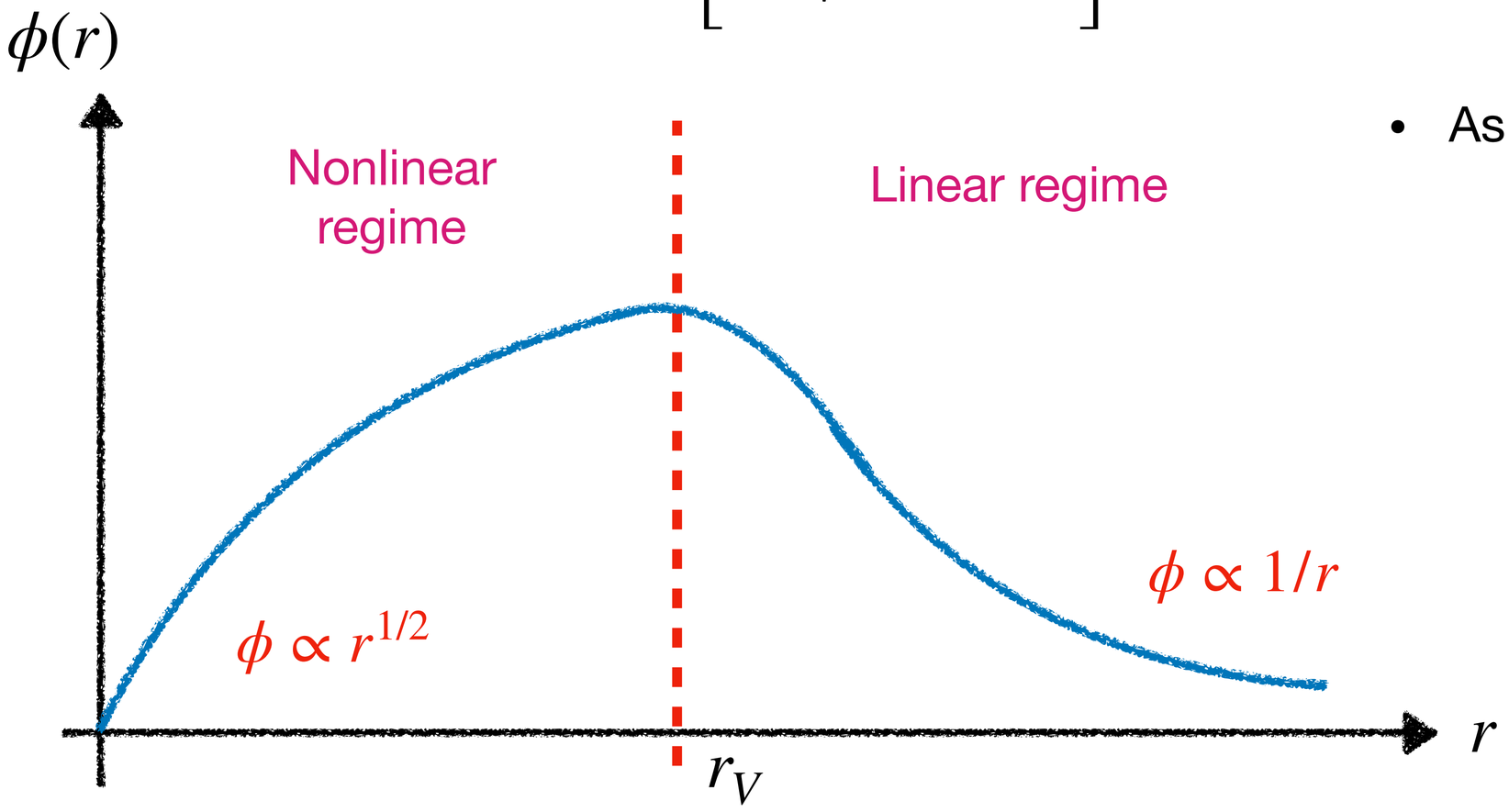
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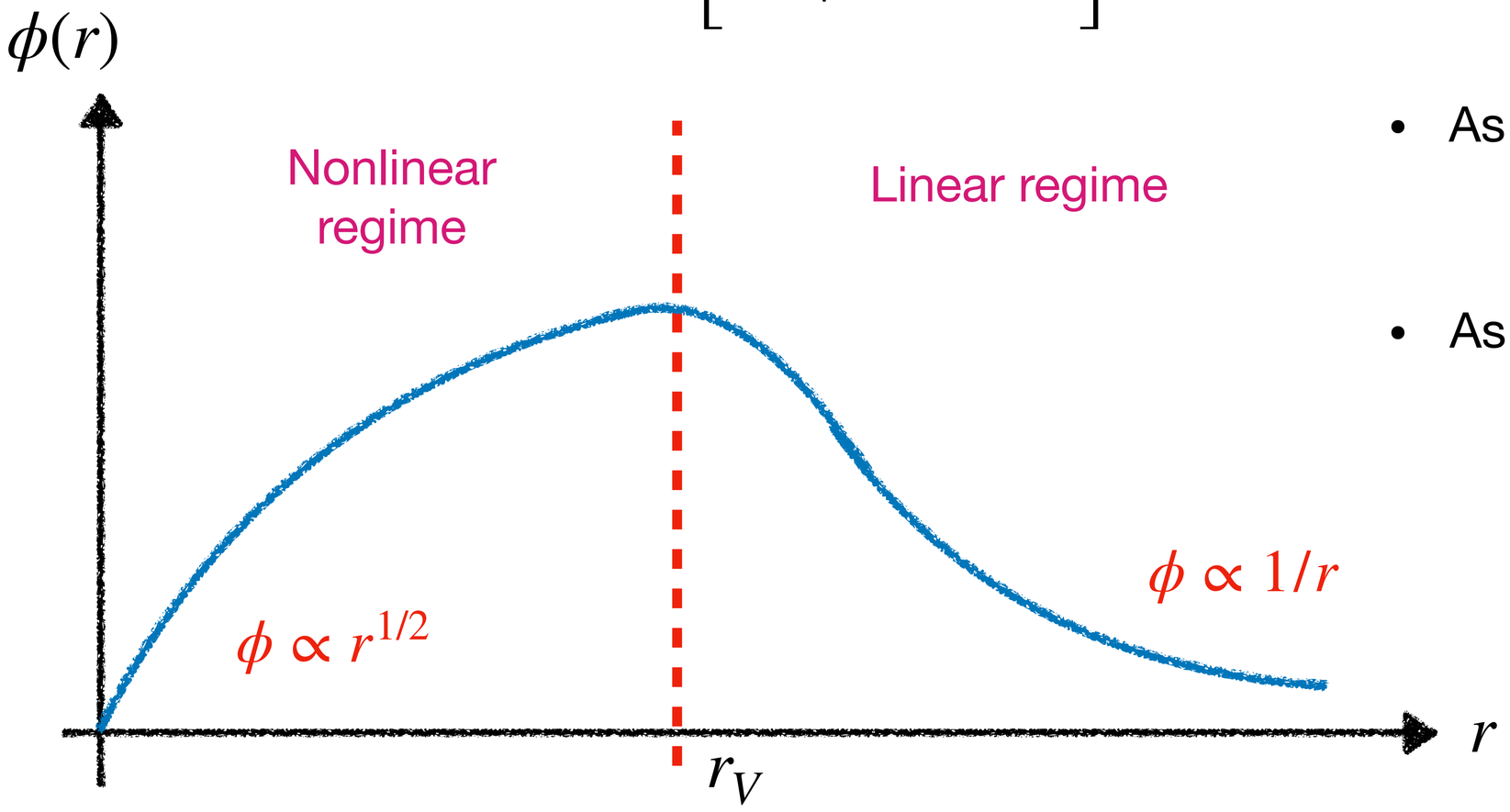
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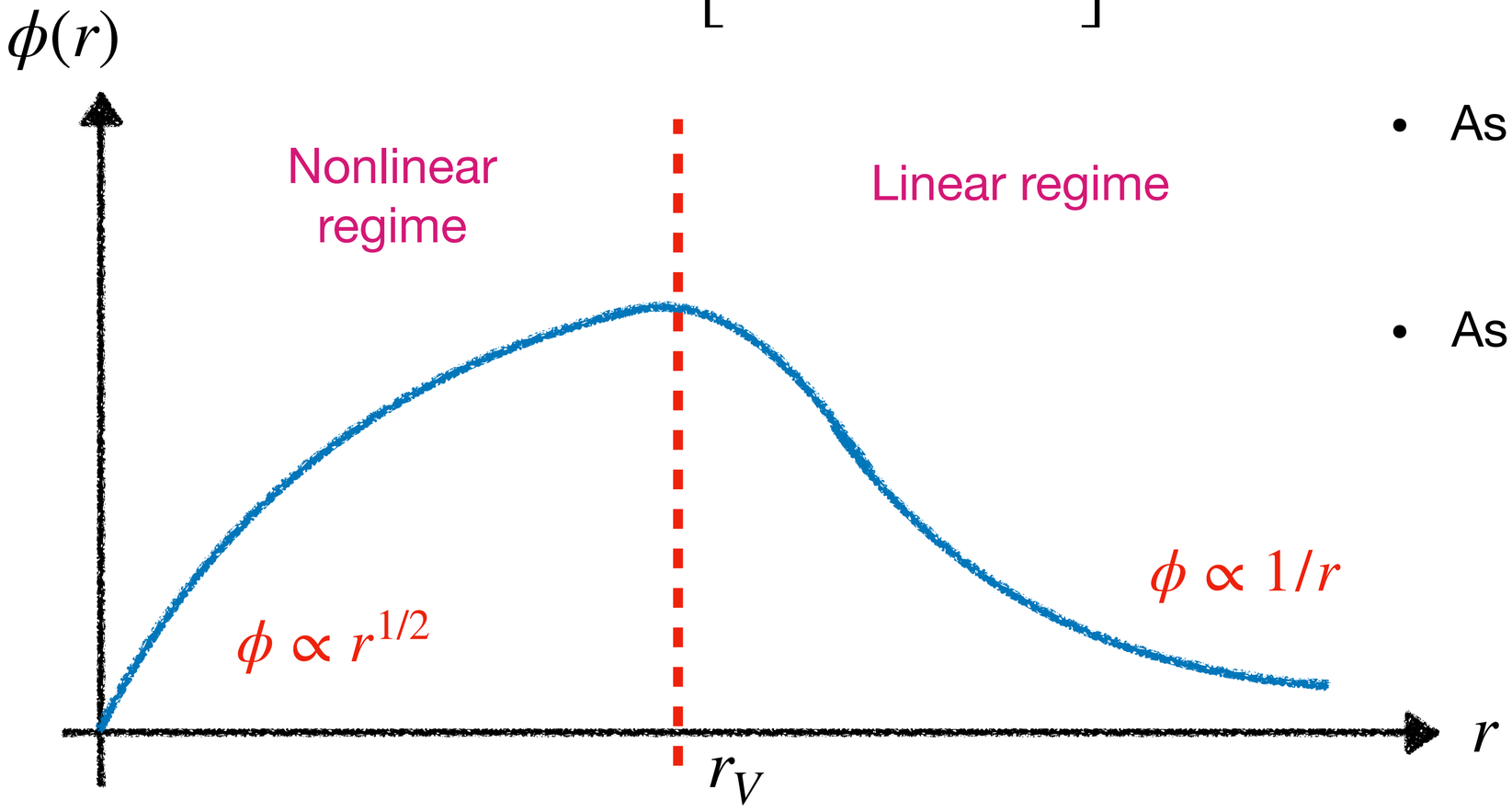
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$$\frac{F_\phi}{F_{\rm gravity}} \sim \left(\frac{r}{r_V}\right)^{3/2} \ll 1$$

Vainshtein screening

General scalar-tensor theories

- The most general scalar-tensor theories without the Ostrogradski ghost
  : (Beyond) Horndeski Theories

$$L_2 = G_2(\Phi, X) \qquad L_3 = G_3(\Phi, X)\Box\Phi$$

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Horndeski 74, Deffayet et al. 11, Zumalacárregui and García-Bellido 14, Gleyzes et al. 14 +++

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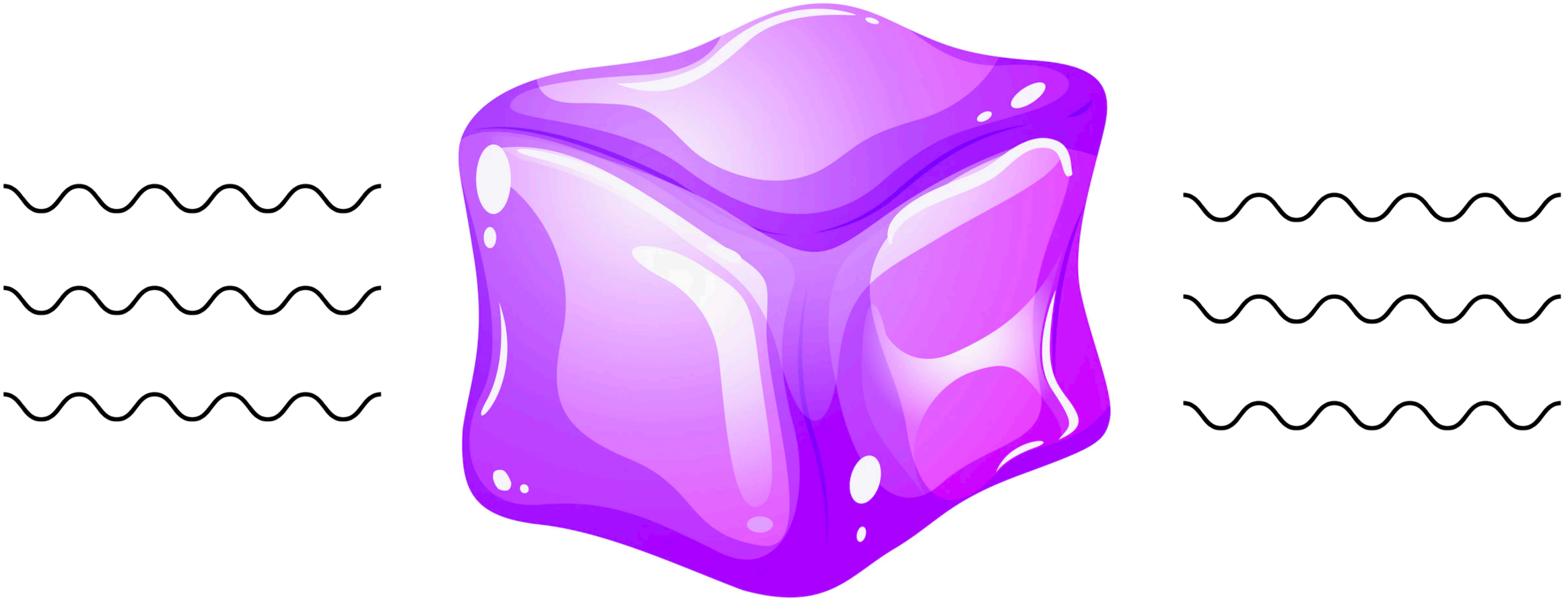
Horndeski 74, Deffayet et al. 11, Zumalacárregui and García-Bellido 14, Gleyzes et al. 14 +++

- (D)HOST Theories: Lagrangian contains second-order derivatives of a scalar field

Langlois and Noui 15, Langlois 17 +++

GW is traveling through a Medium of Dark Energy $\Phi(t)$

(Lorentz violating medium)



In general $c^2_{\mathrm{GW}} \neq c^2_{\mathrm{Light}}$

GWs in Modified Gravity

GR: $$\ddot{h}_{ij} + 3H(t)\dot{h}_{ij} - \frac{1}{a^2(t)}\nabla_k^2 h_{ij} = 0$$

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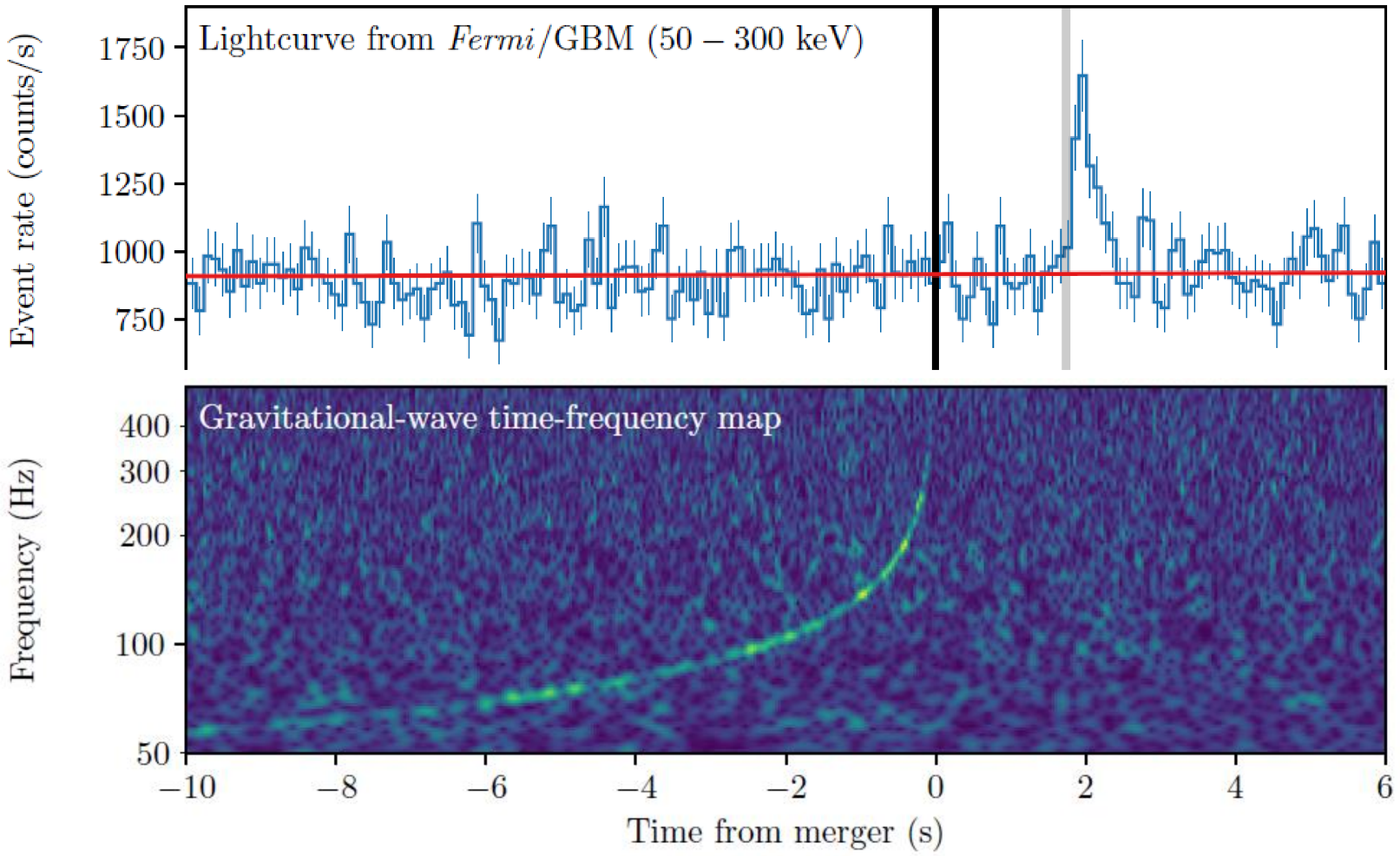
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Effective friction term — Speed of GW — Effective mass term

A Test of GR and Modified Gravity

GW170817 @ LIGO = GRB170817A @ Fermi

LIGO/Virgo + Fermi/GBM + INTEGRAL 17



$$|c_{\rm GW}^2 - 1| \lesssim 10^{-15}$$

$f_{\rm LIGO} \sim 100\ {\rm Hz} \sim 10^{-13}\ {\rm eV}$

$r \sim 40\ {\rm Mpc}$

Chirp mass

$M_c \simeq 1.18\ M_\odot$

Consequence of $c^2_{\rm GW} = 1$

- Expand the (beyond) Horndeski action on $\Phi(t)$: $S[h_{ij}] = \frac{M^2}{8}\int d^4x\ a^3(t)\left[\dot{h}^2_{ij} - \frac{c^2_{\rm GW}}{a^2(t)}(\partial_k h_{ij})^2\right]$

Creminelli and Vernizzi 17, Ezquiaga and Zumalacárregui 17, Baker et al. 17, Sakstein and Jain 17

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Example of using the constraint $c_{\rm GW}^2 = 1$

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Caveats: Validity of the theories — the cut-off scale of the theories

de Rham and Melville 18

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de Rham and Melville 18

- Scalar-tensor Theories with $c_{\rm GW}^2 = 1$, no GW decay, and no instabilities of dark energy

$$L_{c_{\rm GW}^2=1,{\rm no-decay}} = G_2(\Phi, X) + C(\Phi, X)R + \frac{6C_X(\Phi, X)^2}{C(\Phi, X)}\Phi^\mu\Phi_{\mu\nu}\Phi_\lambda\Phi^{\lambda\nu}$$

After performing $g_{\mu\nu} \to C(\Phi, X)g_{\mu\nu}$

Creminelli et al. 18, Creminelli, Tambalo, Vernizzi and VY 19 + 20

EFT of Dark Energy

Effective Field Theory approach

- Efficient way to study a perturbation around fixed background
- Effective Field Theory of Inflation/Dark Energy

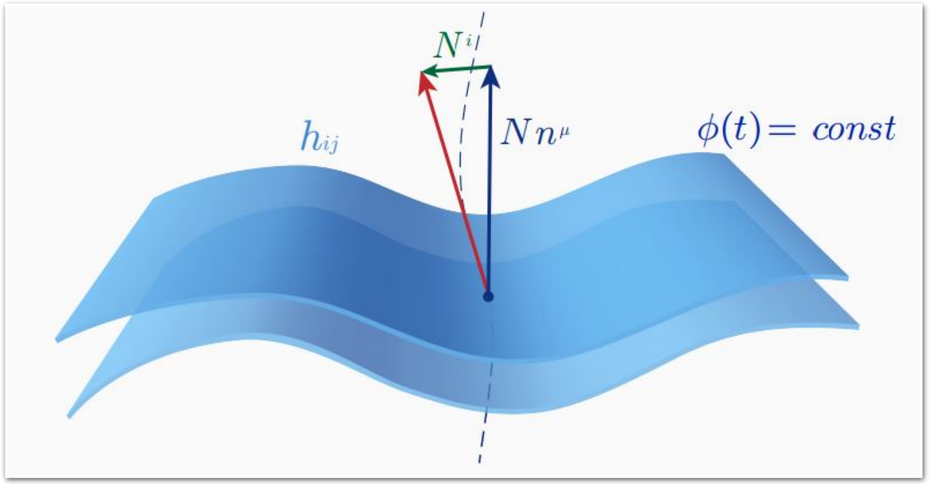
Cheung et al. 08, Gubitosi et al. 12, +++

$$ds^2 = -dt^2 + a(t)^2 dx_i dx^i$$

$\Phi(t)$ spontaneously breaks time diffeomorphism

$$S = \int d^4x \sqrt{-g}\ L[t; N, K^i_j, {}^{(3)}R, \ldots]$$

The action is invariant under 3d diff.



$$ds^2 = -N^2 dt^2 + h_{ij}(N^i dt + dx^i)(N^j dt + dx^j)$$

Simple example

Quintessence:

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The EFT action: $$S_{\rm EFT} = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \Lambda(t) - c(t)g^{00} + \cdots\right]$$

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$$c_s^2 = \frac{c}{c + 2m_2^4}$$

For $\bar{g}^{00} = -1$

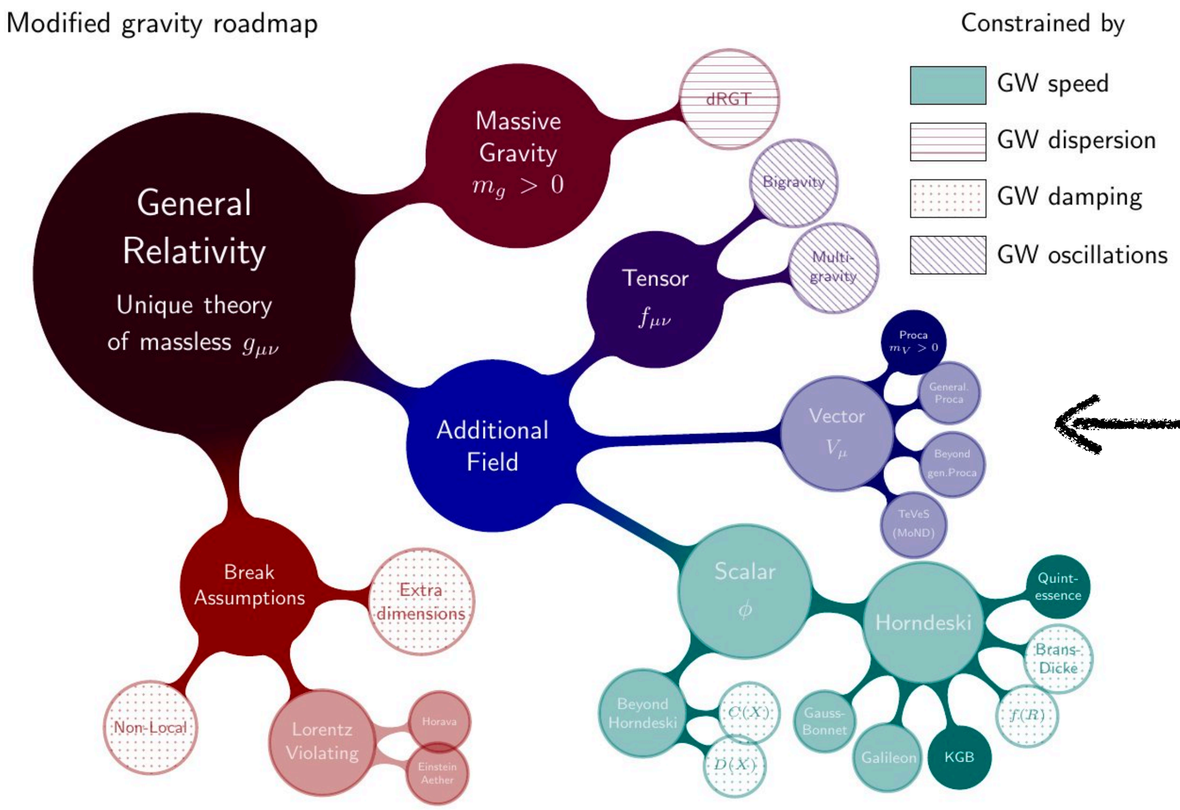
$$S^{(2)} = \int d^4x \sqrt{h} \frac{M^2}{2} \left[\delta K^i_j \delta K^j_i - \delta K^2 + {}^{(3)}\!R \delta N {}^{(3)}\!R + \sum_i \alpha_i(t) \mathcal{O}^{(2)}_i(\delta N, \delta K, \cdots) \right]$$

$$\delta K^i_j = K^i_j - H \delta^i_j$$

Models

Deviations from GR

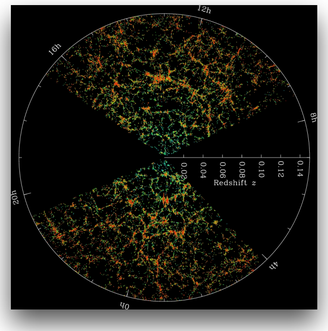
Observations



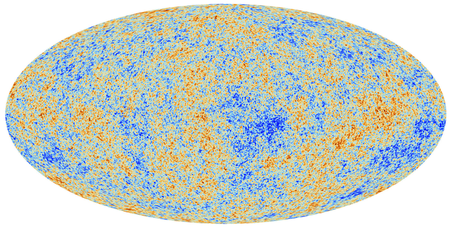
α_K, α_B, α_H,
α_M, α_T, etc.

EFT of DE

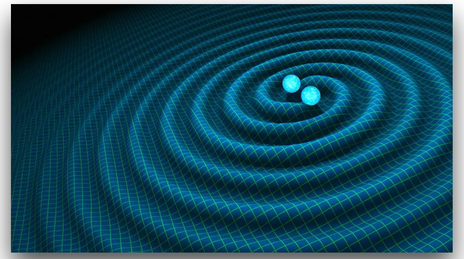
Bridges the models and observations in a minimal and systematic way



Galaxy clusters — SDSS



CMB — Planck



Grav. Waves

Ezquiaga and Zumalacárregui 18

EFT of DE after GW170817

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$$S^{\rm EFT} = \int d^4x\sqrt{-g}\left[\frac{M_\star^2}{2}f(t)R - \Lambda(t) - c(t)g^{00} + \frac{m_2^4(t)}{2}(\delta g^{00})^2 - \frac{m_3^3(t)}{2}\delta g^{00}\delta K - m_4^2(t)\delta\mathcal{K}_2\right.$$

$$\left. + \frac{\tilde{m}_4^2(t)}{2}\delta g^{00(3)}R - \frac{m_5^2(t)}{2}\delta g^{00}\delta\mathcal{K}_2 - \frac{m_6(t)}{3}\delta\mathcal{K}_3 - \tilde{m}_6(t)\delta g^{00}\delta\mathcal{G}_2 - \frac{m_7(t)}{3}\delta g^{00}\delta\mathcal{K}_3\right]$$

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$c_{\rm T}^2 = 1$ Creminelli and Vernizzi 17, Ezquiaga and Zumalacárregui 17, Baker et al. 17, Sakstein and Jain 17

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$$m_4^2\delta\mathcal{K}_2 \supset m_4^2\dot{\gamma}_{ij}^2 \quad \Rightarrow \quad 1 - c_{\rm GW}^2 = \frac{2m_4^2}{M_\star^2 f}$$

GW decay $\gamma \to \pi\pi$

- EFT of DE after GW170817

Creminelli et al. 18

$$S^{\rm EFT}_{c_T^2=1} = \int d^4x\sqrt{-g}\left[\frac{M_*^2}{2}f(t)R - \Lambda(t) - c(t)g^{00} + \frac{m_2^4(t)}{2}(\delta g^{00})^2 - \frac{m_3^3(t)}{2}\delta g^{00}\delta K + \frac{\tilde{m}_4^2(t)}{2}\left(\delta g^{00(3)}R - \delta g^{00}\delta\mathcal{K}_2\right)\right]$$

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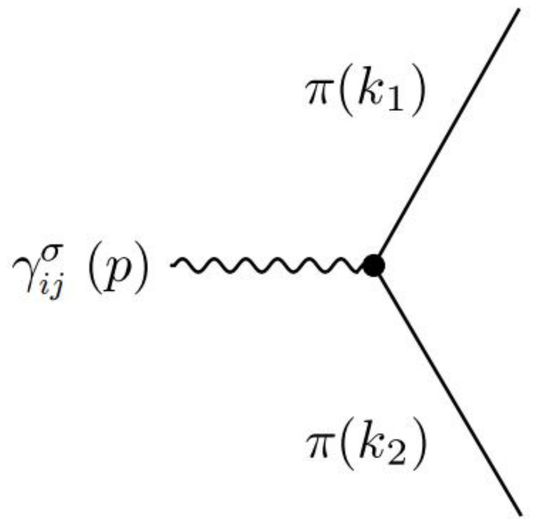
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$\Lambda_3 = 10^{-13}$ eV



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Creminelli et al. 18

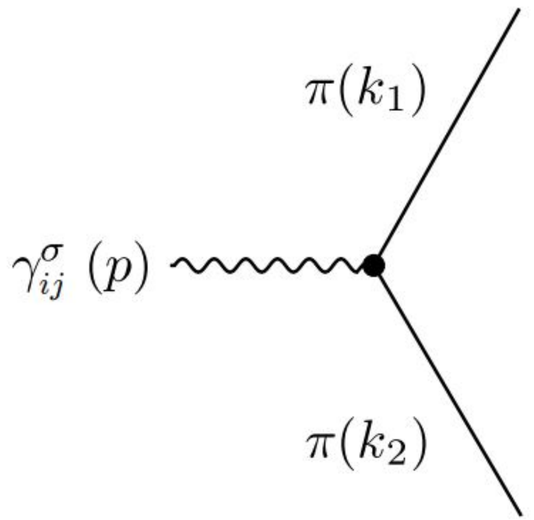
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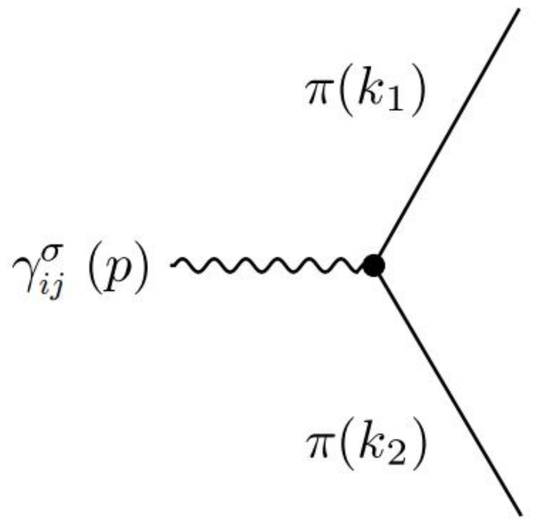
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Beyond Horndeski is highly constrained as DE models

Resonant GW decay $\gamma \to \pi\pi$

- Take into account large occupation number of gravitons

Creminelli, Tambalo, Vernizzi and VY 19

- EoM of $\pi(t, \vec{x})$ in the background of GW $\gamma_{ij} = M_{\rm Pl} h_0^+ \sin(\omega(t-z))\epsilon_{ij}^+$

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Due to the coupling $\ddot{\gamma}_{ij}\partial_i\pi\partial_j\pi$

$$\beta \equiv \frac{2\alpha_{\rm H}\omega^2 M_{\rm Pl} h_0^+}{\Lambda_3^3}$$

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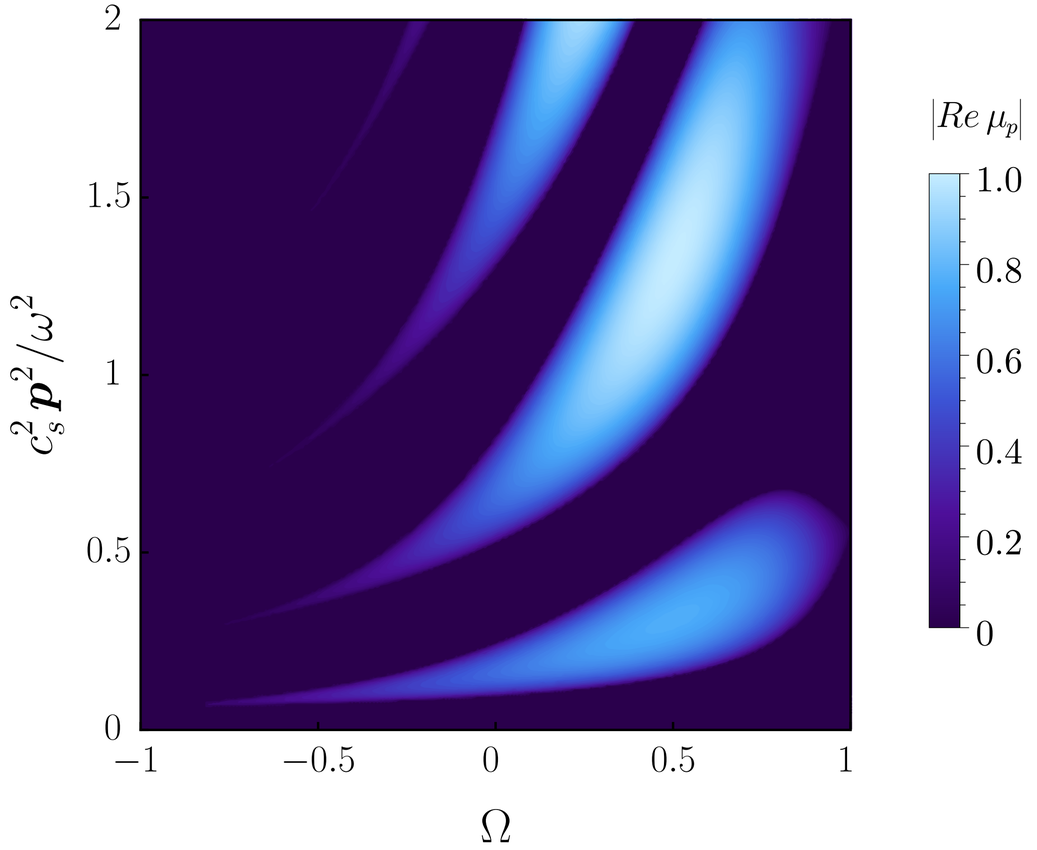
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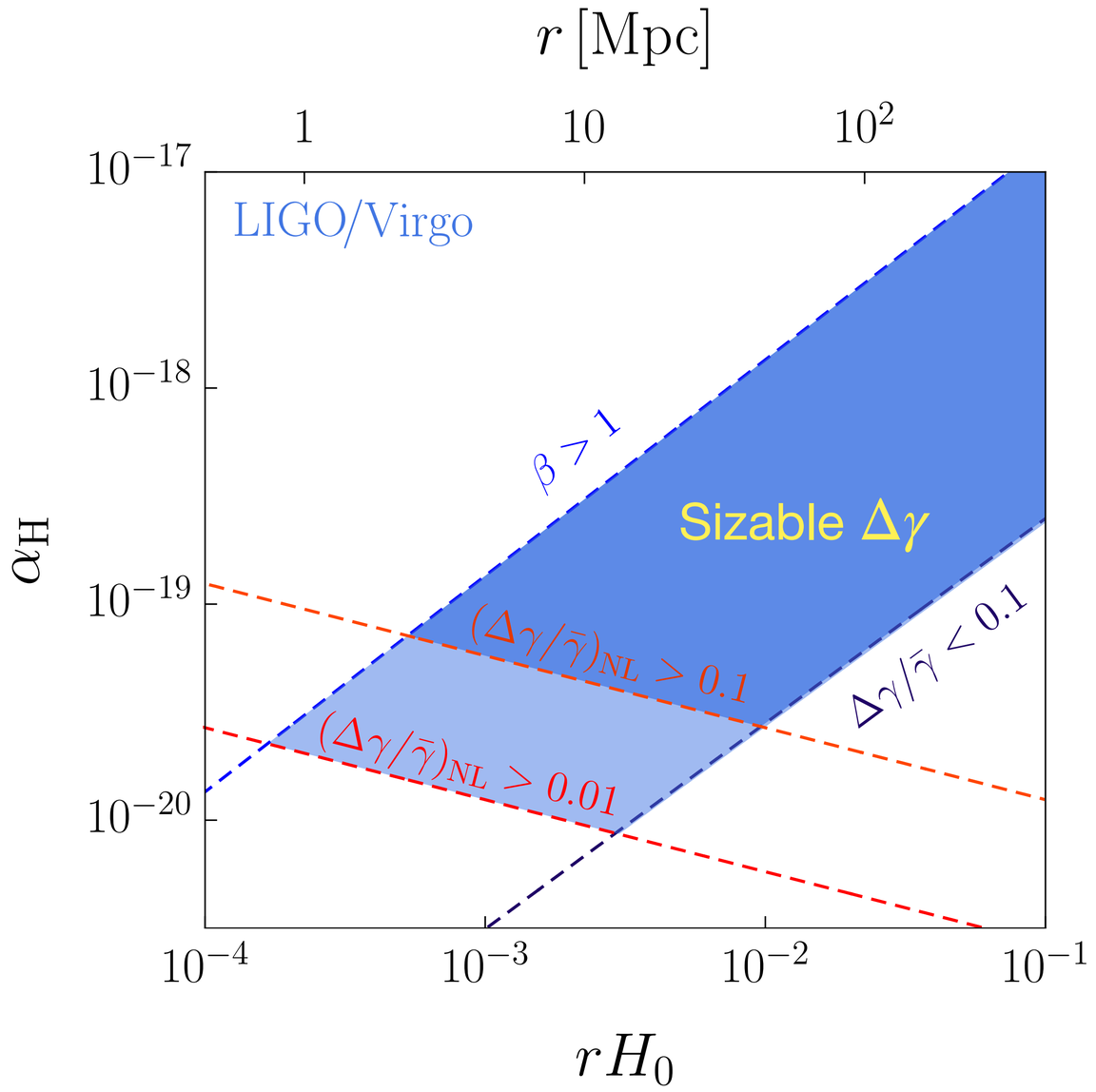
- $\beta < 1$: Resonant instability

$$\frac{{\rm d}^2\pi_{\vec{p}}}{{\rm d}\tau^2} + [A - 2q\cos(2\tau)]\pi_{\vec{p}} = 0 \quad \Rightarrow \quad \pi_{\vec{p}} \sim e^{\mu\tau} \qquad \mu \sim \beta$$

Floquet exponent



Resonant GW decay $\gamma \to \pi\pi$



- Modification to GWs: $\gamma_{ij} \equiv \bar{\gamma}_{ij} + \Delta\gamma_{ij}$

$$\Delta\gamma_{ij} \sim \frac{1}{\Lambda_\star^3}\partial_t^2\left(\partial_i\pi\partial_j\pi\right)$$

- Neglect nonlinearities of π's and
- Absence of sizable $\Delta\gamma_{ij}$ rules out

$$\boxed{10^{-20} \lesssim |\alpha_{\rm H}| \lesssim 10^{-17}}$$

Perturbative bound: $|\alpha_{\rm H}| \lesssim 10^{-10}$

Creminelli, Tambalo, Vernizzi and VY 19

DE Instabilities by GWs

- Large GW amplitude: $\beta > 1$

Creminelli, Tambalo, Vernizzi and VY 20

$$\ddot{\pi} - c_s^2 \nabla^2 \pi - c_s^2 \beta \sin[\omega(t-z)](\partial_x^2 - \partial_y^2)\pi = 0$$

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- Too naive: one needs to consider nonlinearities of π

$$\Lambda_2 = 10^{-3}\ \text{eV}$$

$$\Lambda_3 = 10^{-13}\ \text{eV}$$

$$\mathscr{L}_\pi = -\frac{1}{2}\bar{\eta}^{\mu\nu}\partial_\mu\pi\partial_\nu\pi - \frac{1}{\Lambda_3^3}\Box\pi(\partial\pi)^2 + \frac{\alpha_{\rm B}}{\Lambda_2^2}\dot{\gamma}_{ij}\partial_i\pi\partial_j\pi + \frac{\alpha_{\rm B}}{M_{\rm Pl}}\pi\dot{\gamma}_{ij}^2 \qquad \alpha_{\rm B} = -\frac{m_3^3}{2M_{\rm Pl}^2 H}$$

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Creminelli, Tambalo, Vernizzi and VY 20

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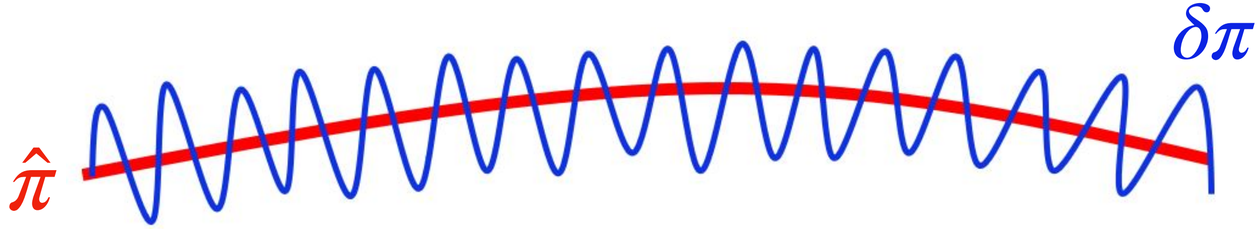
- Too naive: one needs to consider nonlinearities of π

$\Lambda_2 = 10^{-3}$ eV

$\Lambda_3 = 10^{-13}$ eV

$$\mathscr{L}_\pi = -\frac{1}{2}\bar{\eta}^{\mu\nu}\partial_\mu\pi\partial_\nu\pi - \frac{1}{\Lambda_3^3}\Box\pi(\partial\pi)^2 + \frac{\alpha_{\rm B}}{\Lambda_2^2}\dot{\gamma}_{ij}\partial_i\pi\partial_j\pi + \frac{\alpha_{\rm B}}{M_{\rm Pl}}\pi\dot{\gamma}_{ij}^2 \qquad \alpha_{\rm B} = -\frac{m_3^3}{2M_{\rm Pl}^2 H}$$

- Expand around nonlinear solution: $\pi = \hat{\pi} + \delta\pi \quad \Rightarrow \quad \mathscr{L}_{\delta\pi} = Z^{\mu\nu}(x)\,\partial_\mu\delta\pi\partial_\nu\delta\pi$



DE Instabilities by GWs

- Large GW amplitude: $\beta > 1$

Creminelli, Tambalo, Vernizzi and VY 20

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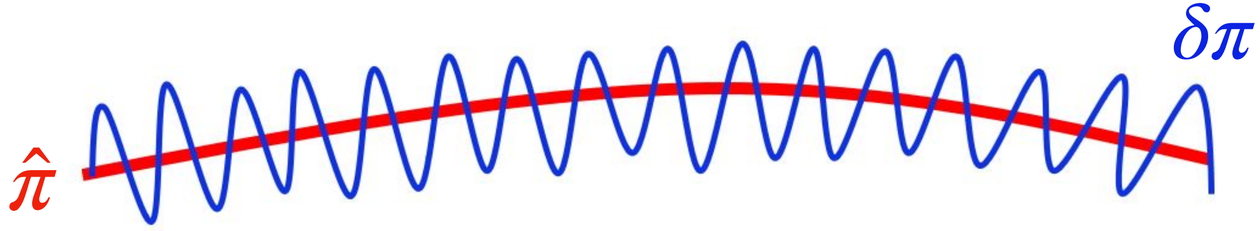
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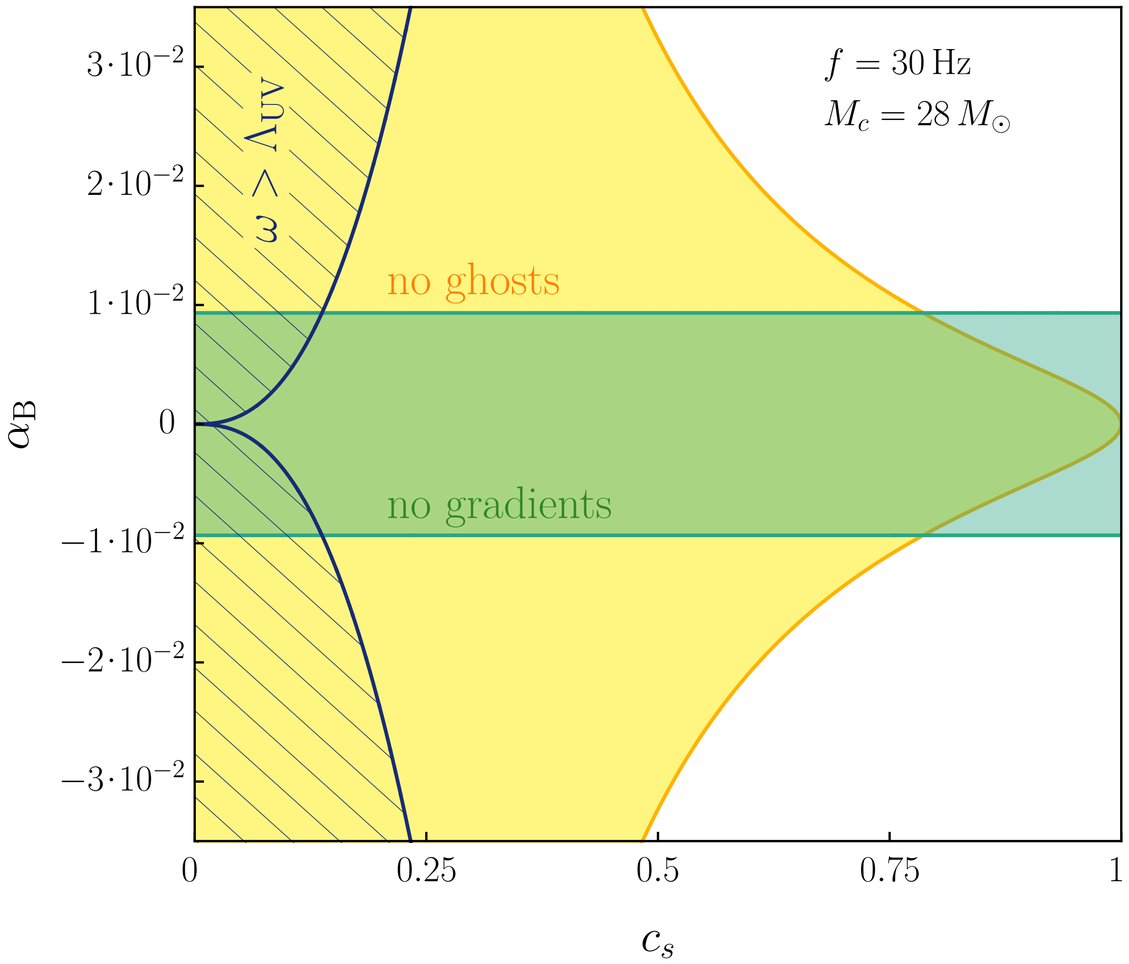
$Z^{00} > 0$: No ghost

$Z^{0i}Z^{0j} - Z^{ij}Z^{00} > 0$: No gradient



DE Instabilities by GWs

- Bound on $\alpha_{\rm B}$: cubic Horndeski



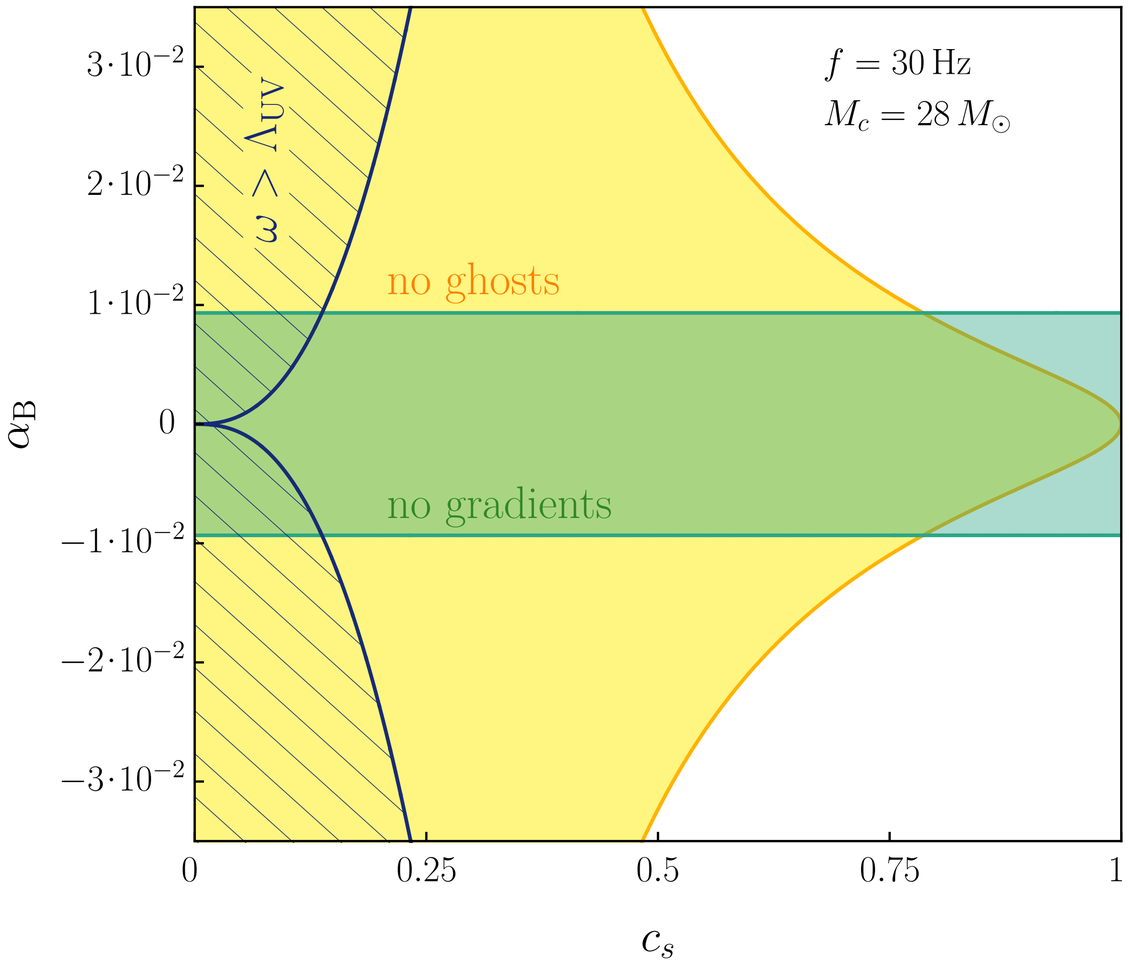
Creminelli, Tambalo, Vernizzi and VY 20

- Absence of ghost/gradient instabilities

$$|\alpha_{\rm B}| \lesssim 10^{-2}$$

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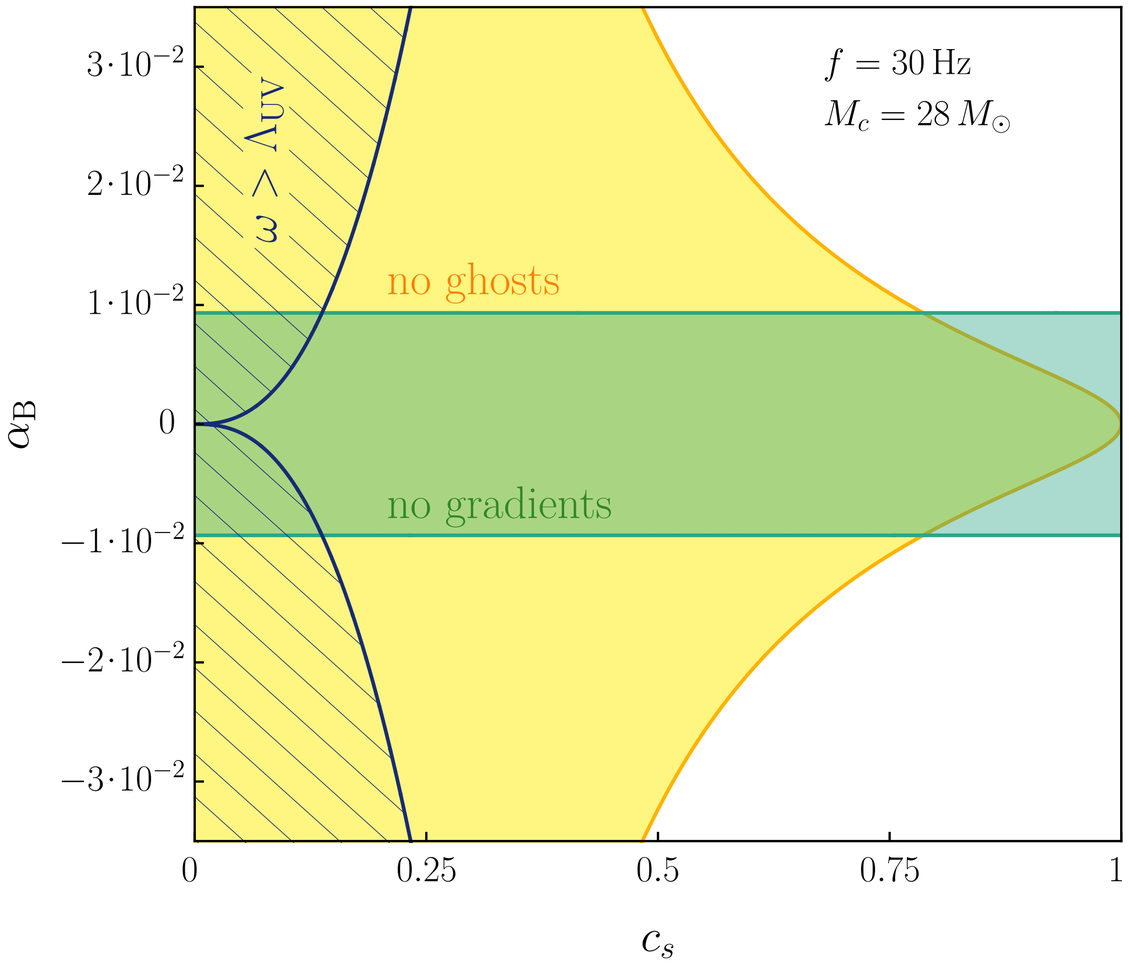
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- Surviving scalar-tensor theories as DE models

$$S^{\rm EFT}_{c_T^2=1} = \int d^4x\sqrt{-g}\left[\frac{M_*^2}{2}f(t)R - \Lambda(t) - c(t)g^{00} + \frac{m_2^4(t)}{2}(\delta g^{00})^2\right]$$

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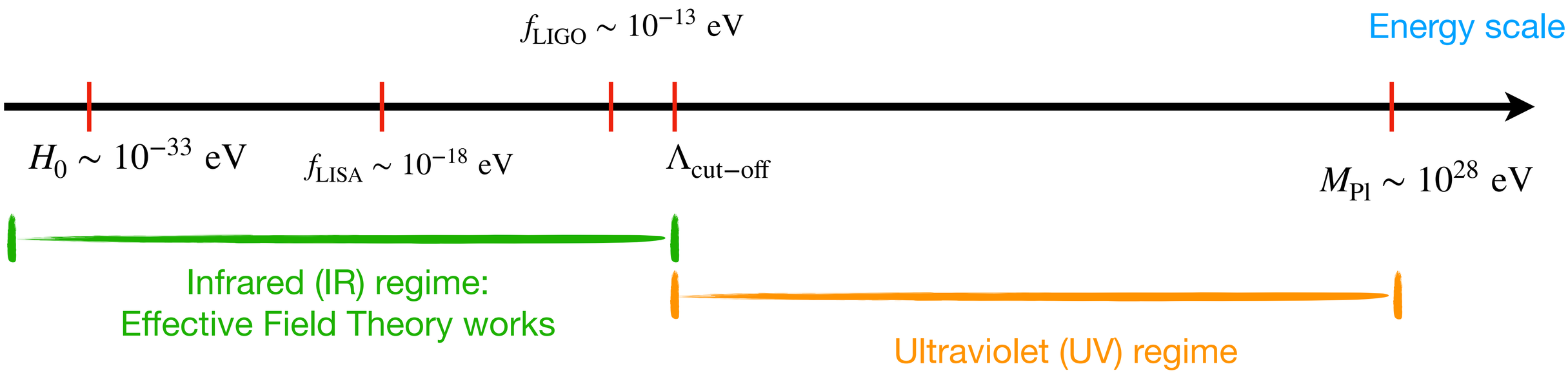
$$\Updownarrow$$

$$\mathscr{L} = G_2(\phi, X) + C(\phi, X)R + \frac{6C_{,X}(\phi, X)^2}{C(\phi, X)}\phi^{;\mu}\phi_{;\mu\nu}\phi_{;\lambda}\phi^{;\nu\lambda}$$

K-essence + non-minimally coupled with gravity

Modified Gravity as an EFT

- On larges scale gravity is modified



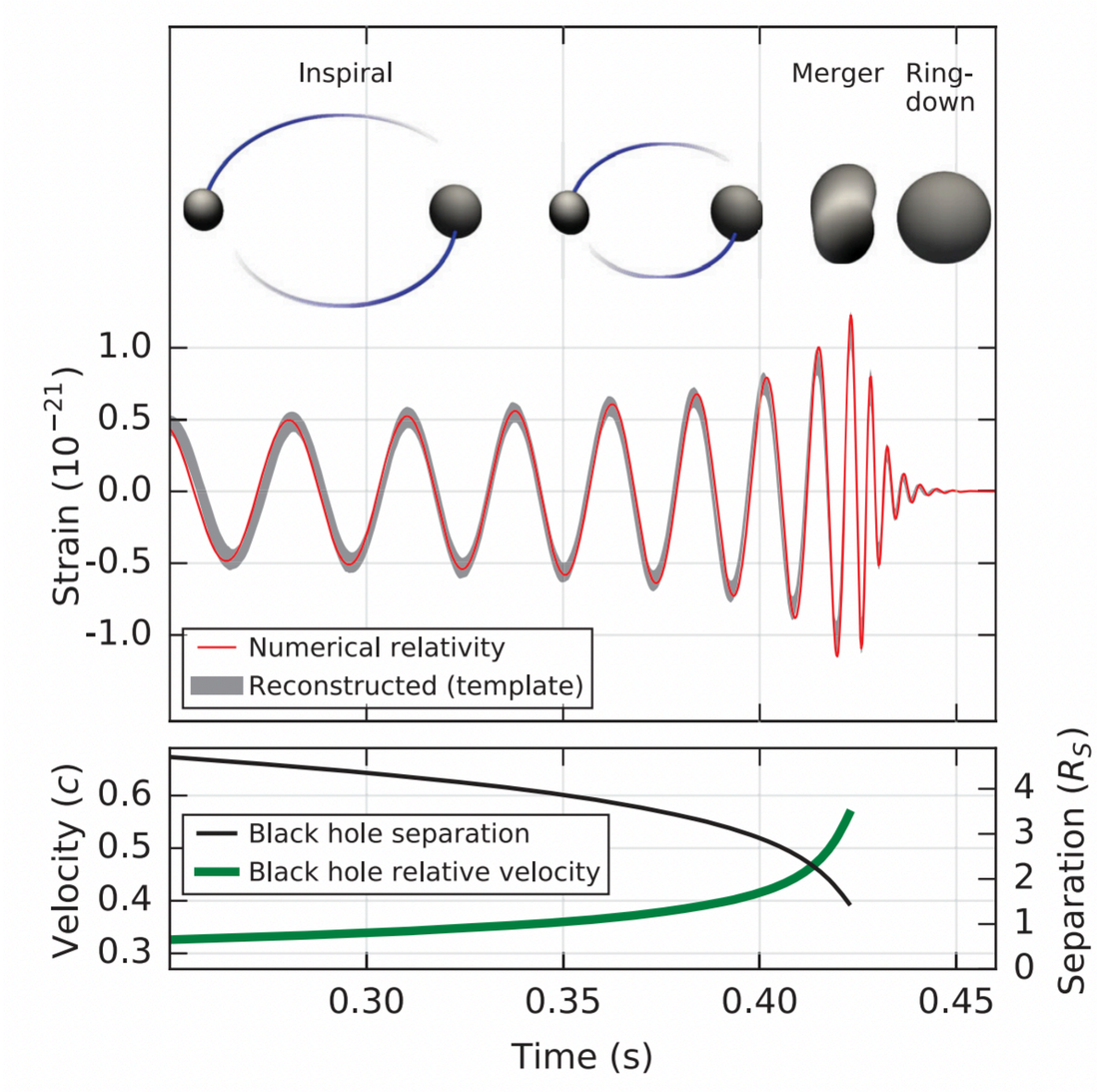
- On small scales (solar system) gravity behaves the same as in GR

The presence of the fifth force (on small scales) can be typically suppressed by screening mechanism (but not necessary)

$$r_V \sim 100 \left(\frac{M}{M_\odot} \right)^{1/3} \text{pc}$$

EFT of BH Perturbations

Black hole regime



LIGO/Virgo 2016

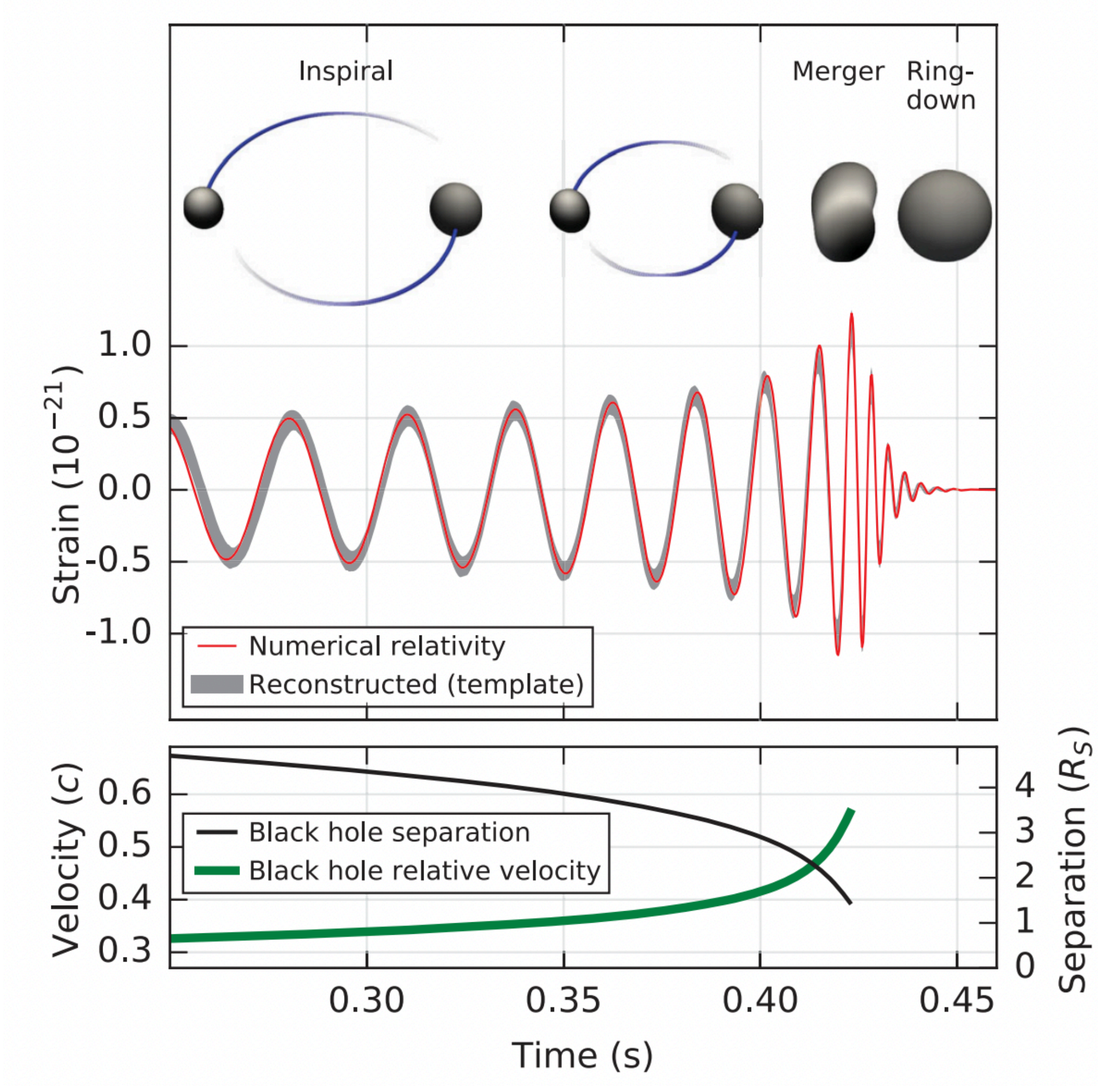
- We have detected many mergers e.g. GW150914, GW170817, GW190424 etc.
- Each phase can be described by:

  Inspiral: Post-Newtonian method

  Merger: Numerical relativity

  Ring-down: Black hole perturbation theory

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LIGO/Virgo 2016

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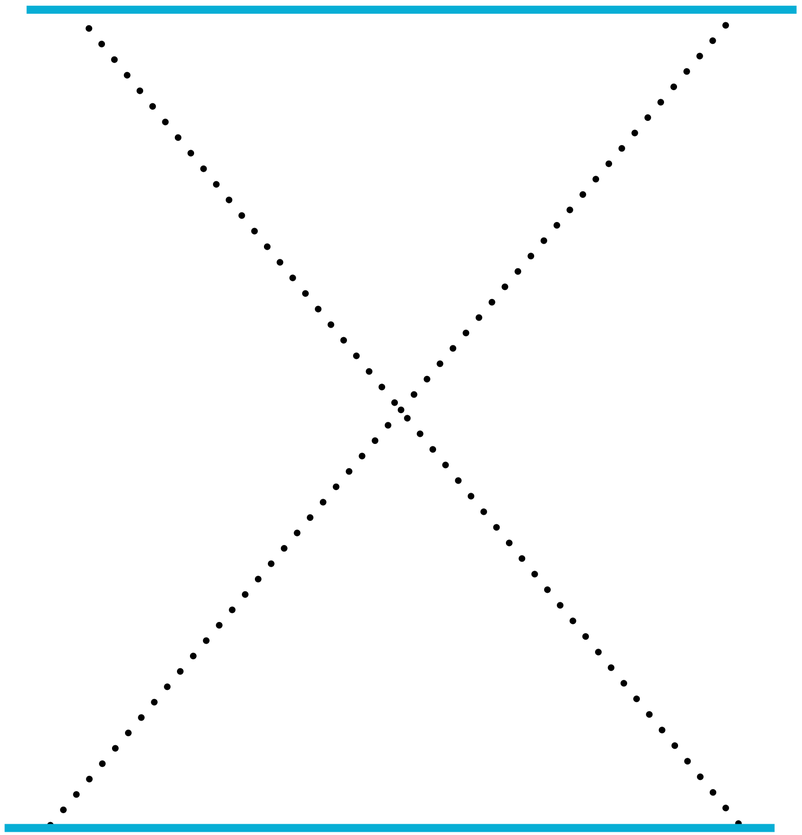
- Formulate the Effective Field Theory

  ⇒Test of GR and modified gravity (model-indep.)

BH Phenomenology from EFT of BH

e.g. QNM, Love number

Construction of the EFT



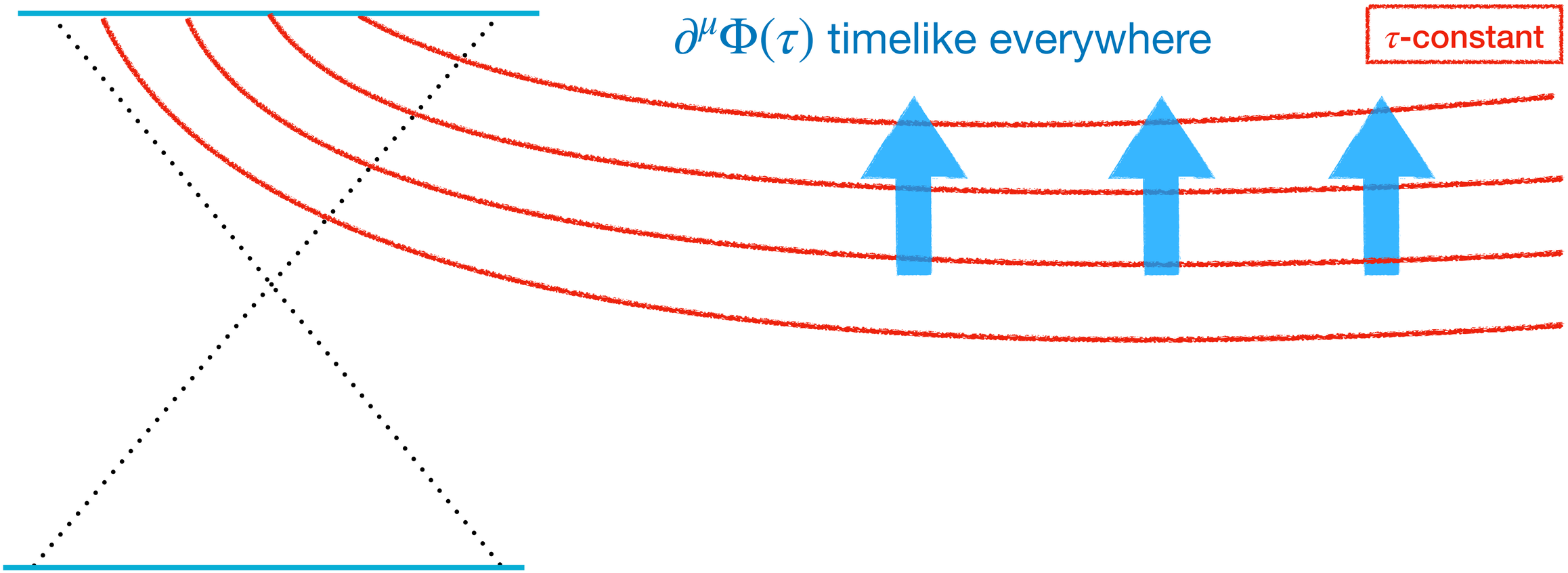
Black Hole regime

Cosmological regime

(EFT of Inflation/DE)

Cheung et al. 08, Gubitosi et al. 12, +++

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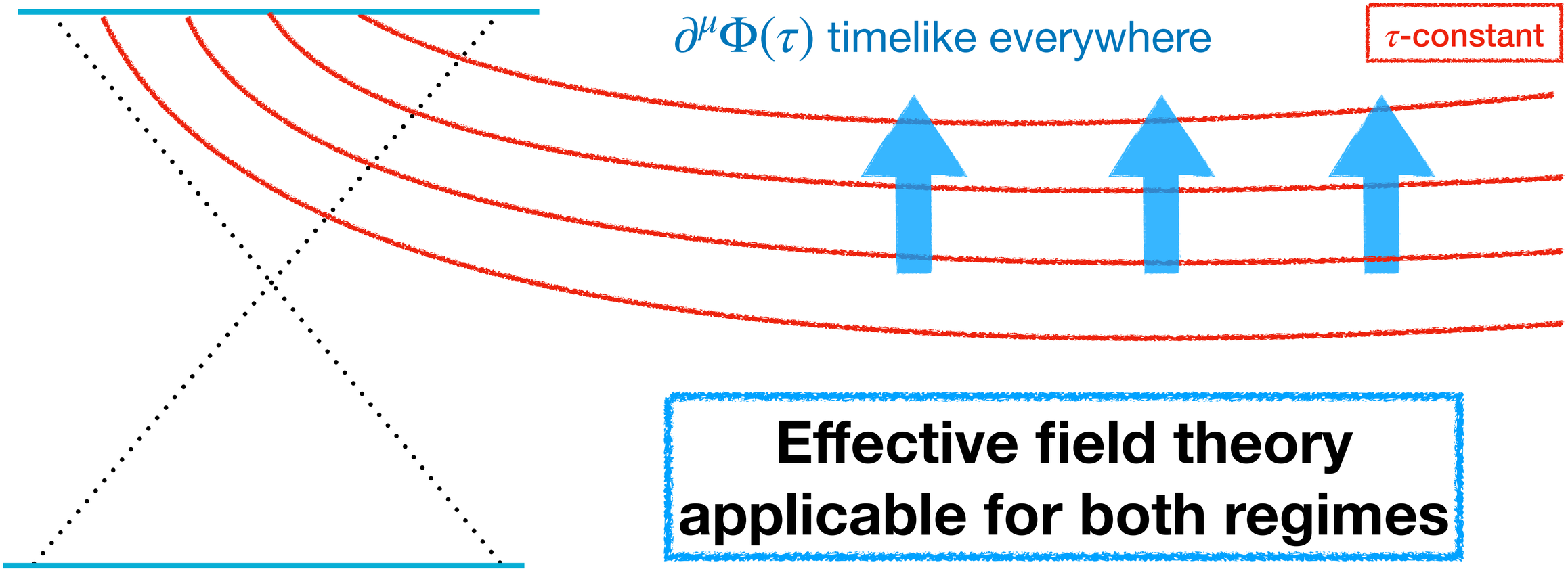
Black Hole regime

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Construction of the EFT



Effective field theory applicable for both regimes

Black Hole regime

Cosmological regime

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Cheung et al. 08, Gubitosi et al. 12, +++

Construction of the EFT

- Similar to EFT of Inflation/DE on FLRW metric Cheung et al. 08, Gubitosi et al. 12, +++

  ⇔ Scalar-tensor theories e.g. Horndeski and beyond

  Horndeski 74, Deffayet et al. 11, Zumalacárregui and García-Bellido 14, Gleyzes et al. 14, Langlois and Noui 15, Langlois 17, +++

- $\bar{\Phi}(\tau)$ breaks τ-diffeo. spontaneously

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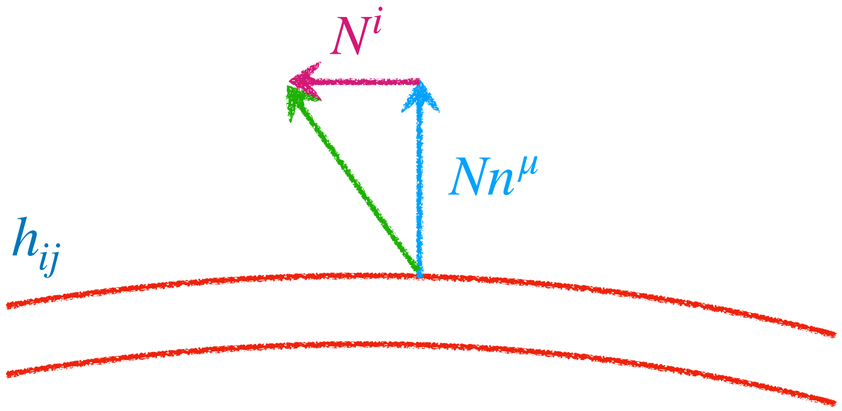
- $\bar{\Phi}(\tau)$ breaks τ-diffeo. spontaneously

- EFT action in unitary gauge: $\delta\Phi = 0$

$$S = \int d^4x\sqrt{-g}\ F(R_{\mu\nu\alpha\beta}, g^{\tau\tau}, K_{\mu\nu}, \nabla_\nu, \tau)$$

$$ds^2 = -N^2 d\tau^2 + h_{ij}(dx^i + N^i d\tau)(dx^j + N^j d\tau)$$

$$g^{\tau\tau} = -1/N^2$$



This action is invariant under 3d diffeo and valid for generic background geometries

Dynamics in odd-parity sector

- Background metric: $ds^2 = -d\tau^2 + [1 - A(r)]d\rho^2 + r^2 d\Omega^2 \quad \Leftrightarrow \quad ds^2 = -A(r)dt^2 + \dfrac{dr^2}{B(r)} + r^2 d\Omega^2$

- The odd EFT action up to second order:

$$S_{\rm odd} = \int d^4x\sqrt{-g}\left[\frac{M_\star^2}{2}R - \Lambda(y) - c(y)g^{\tau\tau} - \tilde{\beta}(y)K - \alpha(\tau)\bar{K}^\mu_\nu K^\nu_\mu + \frac{1}{2}M_3^2(y)\delta K^\mu_\nu \delta K^\nu_\mu\right]$$

Modifies speed of GW

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$$\mathscr{L}_2 = a_1(\partial_t\chi)^2 - a_2(\partial_r\chi)^2 - a_4\chi^2$$

Function of
$M_\star^2, M_3^2, \alpha, A(r)$ and $B(r)$

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Function of
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$$\Psi = (a_1 a_2)^{1/4}\,\chi$$

$$r_* \equiv \int\sqrt{\frac{a_1}{a_2}}\,dr$$

$$\frac{\partial^2\Psi}{\partial r_*^2} - \frac{\partial^2\Psi}{\partial t^2} - V_{\rm eff}(r)\Psi = 0$$

Generalized Regge-Wheeler

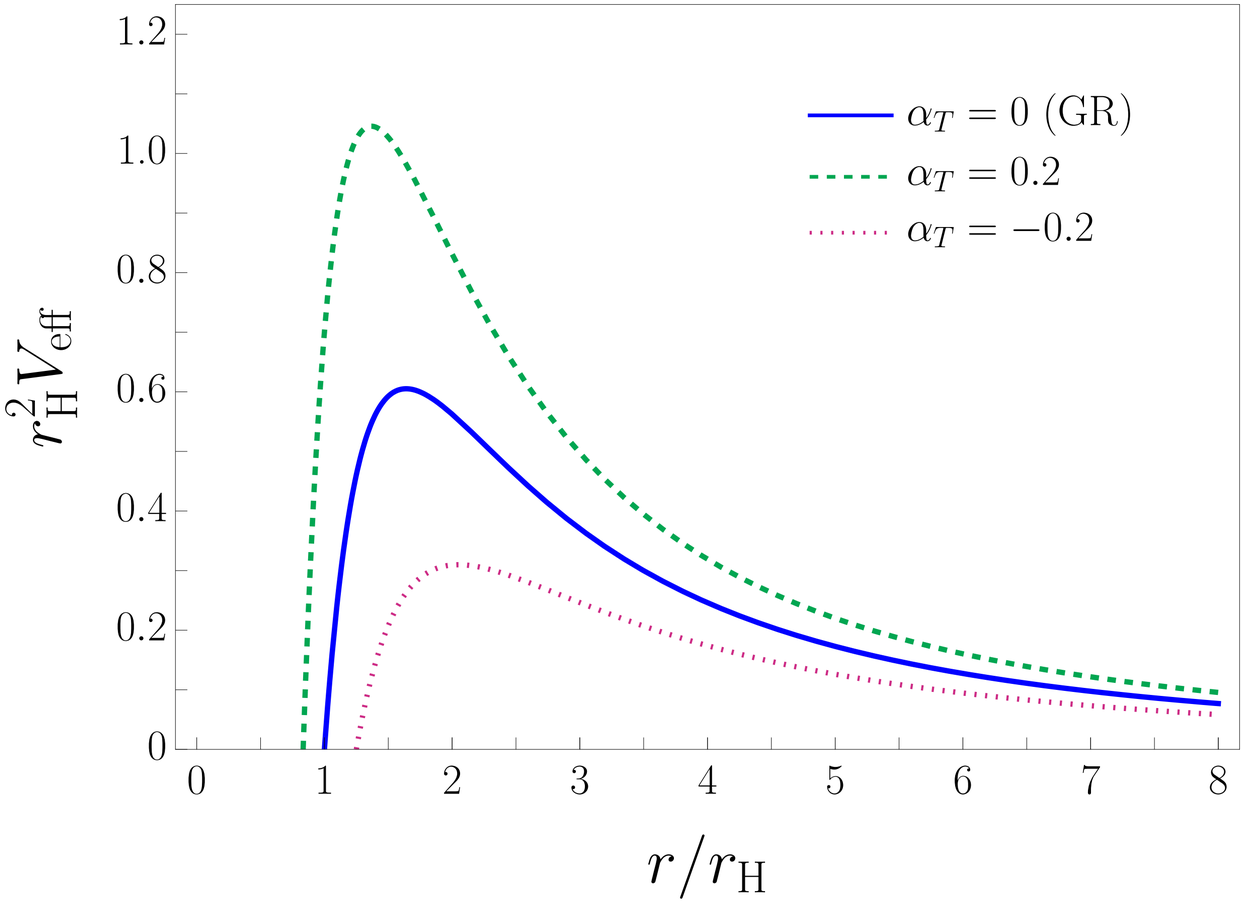
$$V_{\rm eff}(r) \equiv \frac{a_4}{a_1} + \frac{1}{2\sqrt{a_1 a_2}}\frac{d^2\sqrt{a_1 a_2}}{dr_*^2} - \frac{1}{4a_1 a_2}\left(\frac{d\sqrt{a_1 a_2}}{dr_*}\right)^2$$

We want to solve this diff. Eq.!!

Stealth Schwarzschild solution

We choose: $A(r) = B(r) = 1 - \dfrac{r_{\rm H}}{r}$ $\qquad \alpha_T(r) \equiv c_T^2 - 1 = -\dfrac{M_3(r)^2}{M_\star^2 + M_3(r)^2}$

$\alpha_T = const\,.$



$$V_{\rm eff}(r) = (1+\alpha_T) f(r) \left[\frac{\ell(\ell+1)}{r^2} - \frac{3 r_g}{r^3} \right]$$

$f(r) \equiv 1 - r_g/r$

Graviton horizon: $r_g \equiv r_{\rm H}/(1+\alpha_T)$

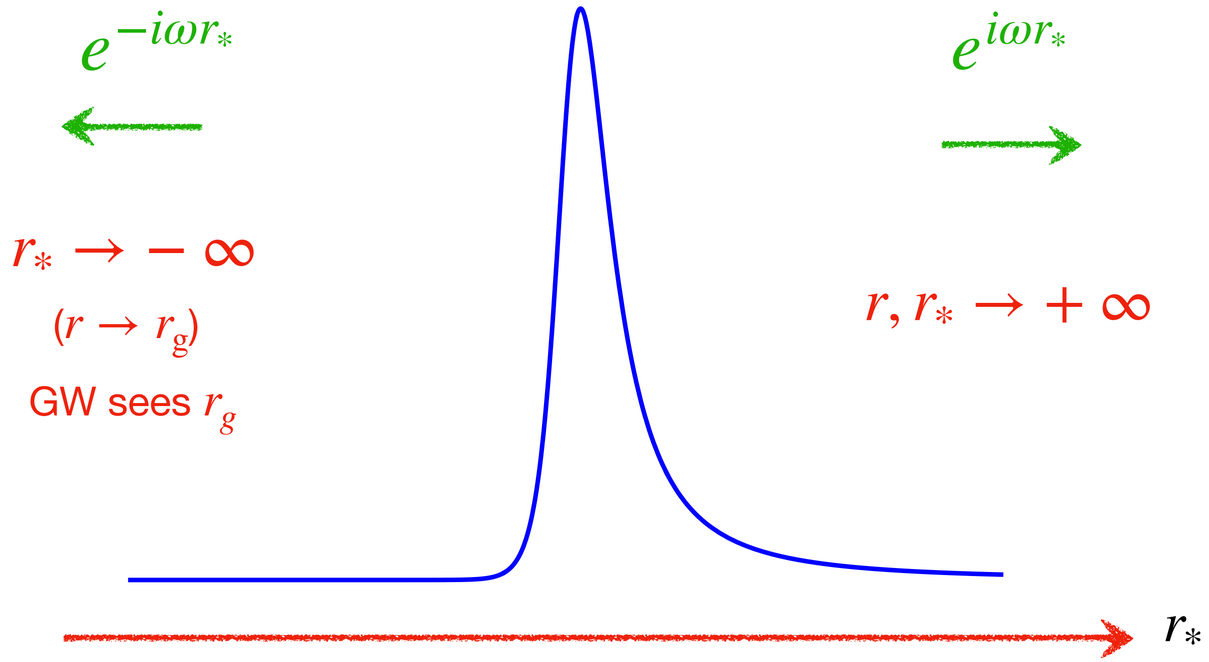
- $\alpha_T > 0 \Rightarrow r_g < r_{\rm H}$
- $\alpha_T < 0 \Rightarrow r_g > r_{\rm H}$

The presence of α_T changes the height of $V_{\rm eff}$

$$r_*(r) = (1+\alpha_T)^{-1/2} \left[r + r_g \log \left| \frac{r}{r_g} - 1 \right| \right]$$

QNM calculation

$$\Psi(t, r_*) = Q(r_*)e^{-i\omega t}$$

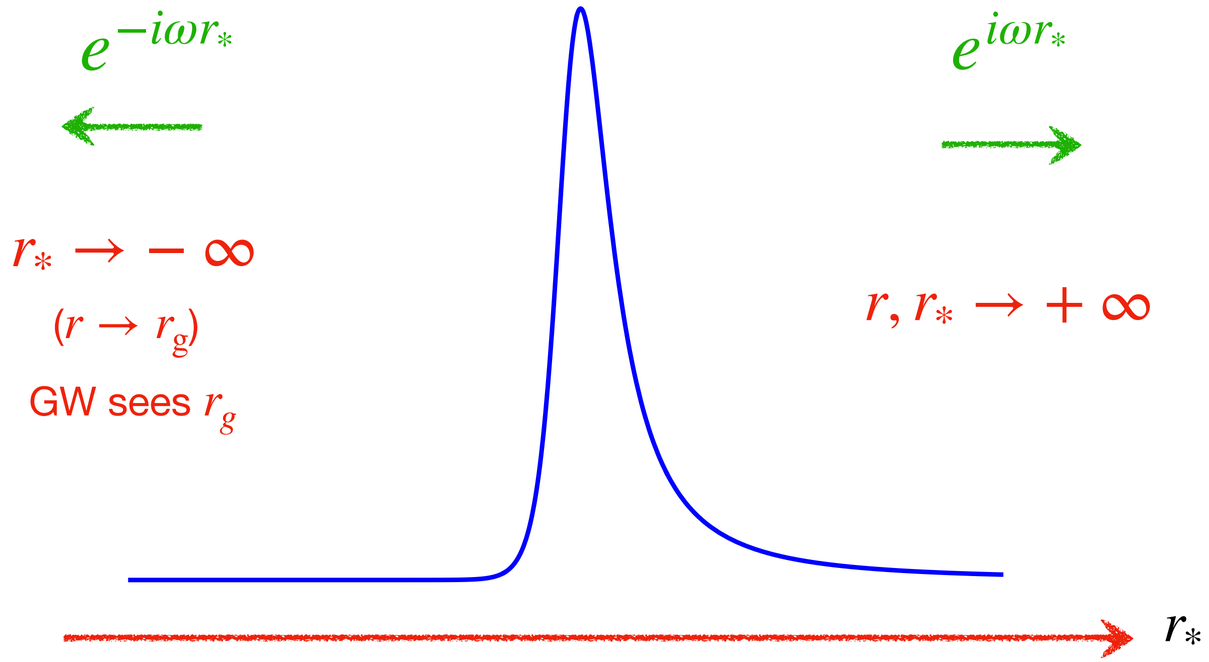


$$\frac{d^2}{dr_*^2}Q(r_*) + (\omega^2 - V_{\rm eff})Q(r_*) = 0$$

1d time-indep. Schrödinger eq.

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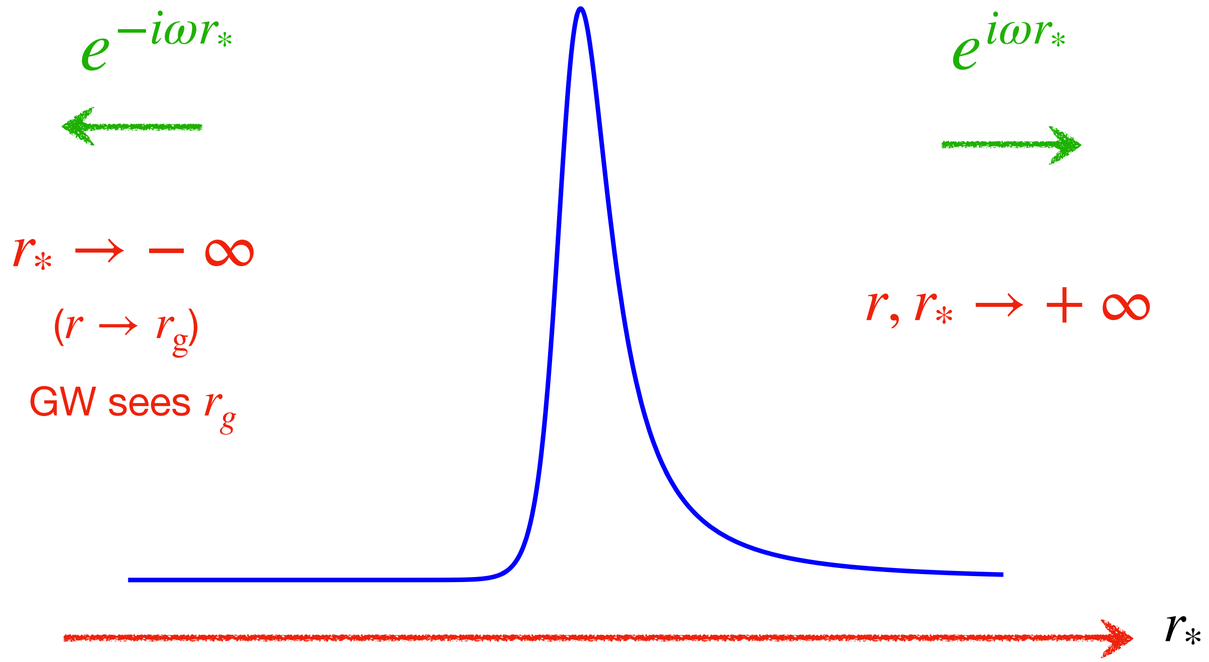
1d time-indep. Schrödinger eq.

A specific frequency that satisfies the two BCs is the quasinormal frequency

Review: Hatsuda and Kimura 21

QNM calculation

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1d time-indep. Schrödinger eq.

A specific frequency that satisfies the two BCs is the quasinormal frequency

Review: Hatsuda and Kimura 21

- ω is complex number $\Rightarrow$ the Im part describes the damping of the GW signal
- Other methods: WKB method, Leaver's method, parametrized QNM method

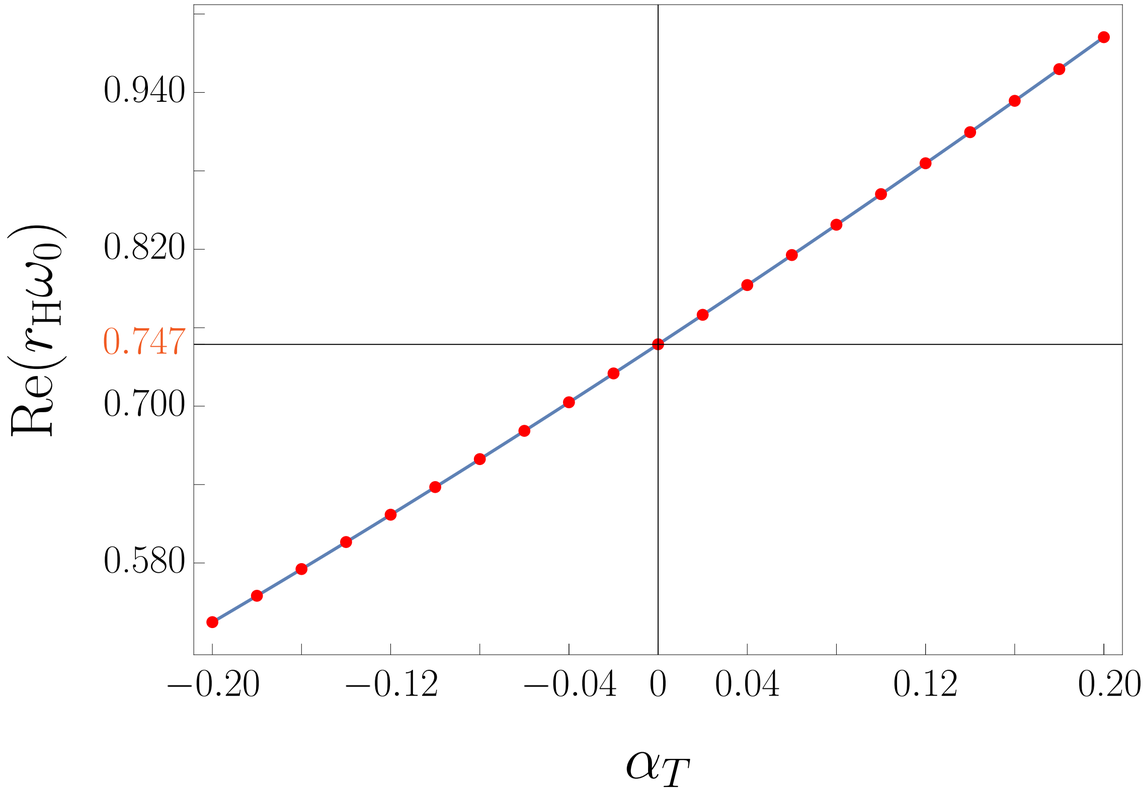
Chandrasekhar and Detweiler 75, Iyer and Will 87, Konoplya 03
Leaver 85, Schutz and Will 85, Nollert 93, McManus et al. 19, Kimura 20, +++

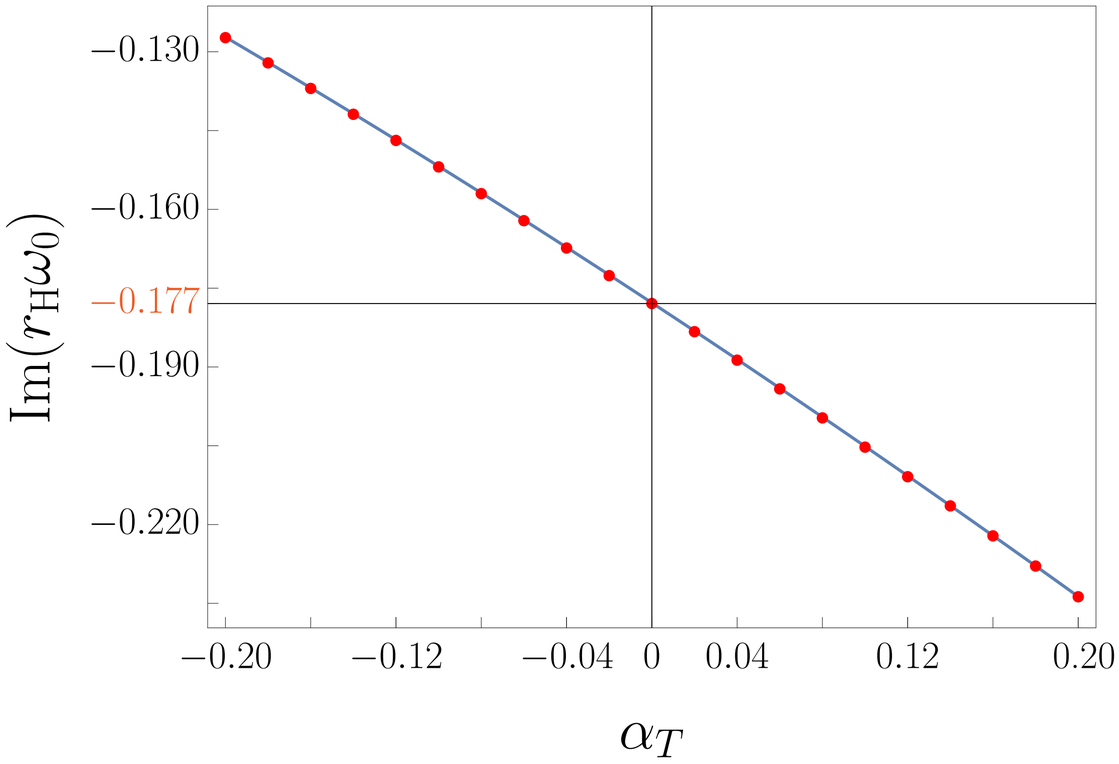
QNM in Stealth Schwarzschild case

- The fundamental QNM frequencies can be obtained by a simple rescaling from $\omega_{\rm GR}$

$$A(r) = B(r) = 1 - \frac{r_{\rm H}}{r}$$

$$r_{\rm H}\omega = r_{\rm H}\omega_{\rm GR}(1+\alpha_T)^{3/2}$$



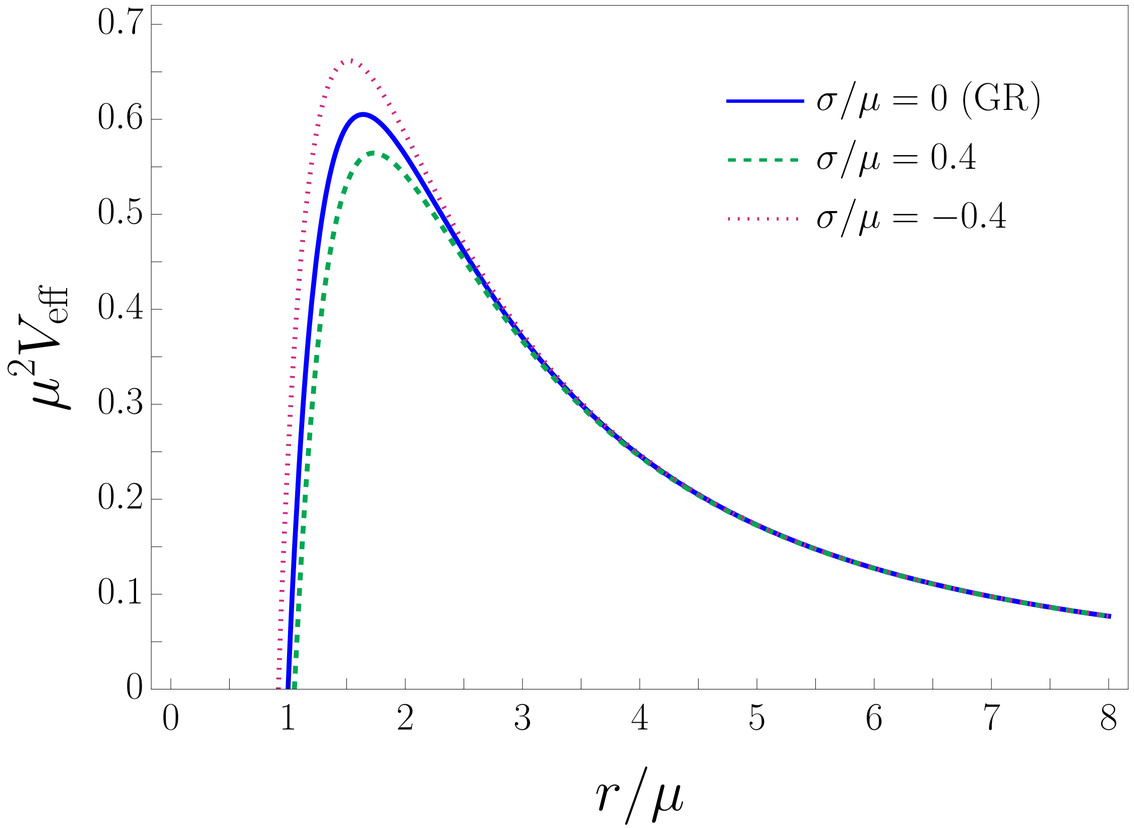


Red data points are QNFs from direct integration method

The Hayward potential

- We choose: $A(r) = B(r) = 1 - \dfrac{\mu r^2}{r^3 + \sigma^3}$ dS core as $r \to 0$ & Schwarzschild limit as $r \to \infty$

$\alpha_T(r) = -\dfrac{\sigma^3(2r^3 + \sigma^3)}{(r^3 + \sigma^3)^2}$ $\alpha_T(r) \sim 1/r^3$ as $r \to \infty$ (LIGO bound)



$$\mu^2 V_{\rm eff}(\tilde{r}) = \left[1 - \frac{\tilde{r}^3 + \tilde{\sigma}^3}{\tilde{r}^4}\right] \times \times \left\{ \frac{\ell(\ell+1)\tilde{r}^4}{(\tilde{r}^3 + \tilde{\sigma}^3)^2} - \frac{3\left[4\tilde{r}^9 + 2\tilde{\sigma}^3\tilde{r}^6(8\tilde{r} - 1) + \tilde{\sigma}^6 r^3(\tilde{r} - 7) - \tilde{\sigma}^9\right]}{4(\tilde{r}^3 + \tilde{\sigma}^3)^4} \right\}$$

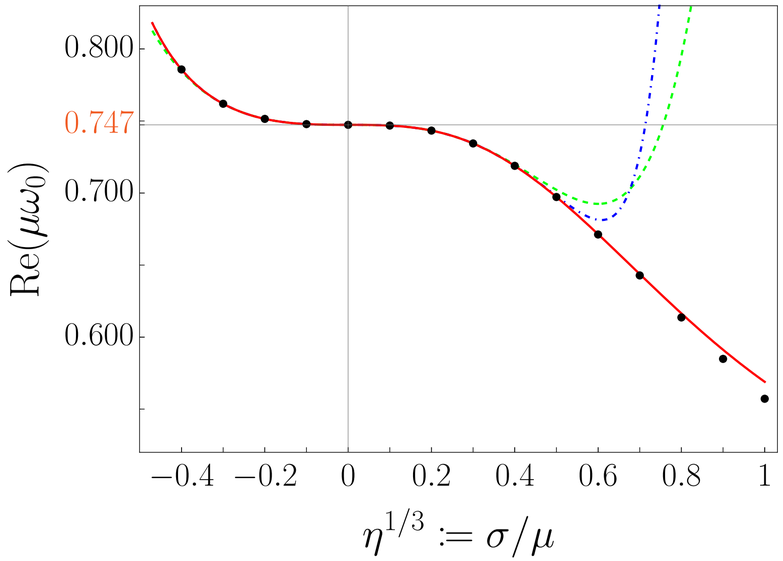
$\tilde{r} \equiv r/\mu$ $\tilde{\sigma} \equiv \sigma/\mu$

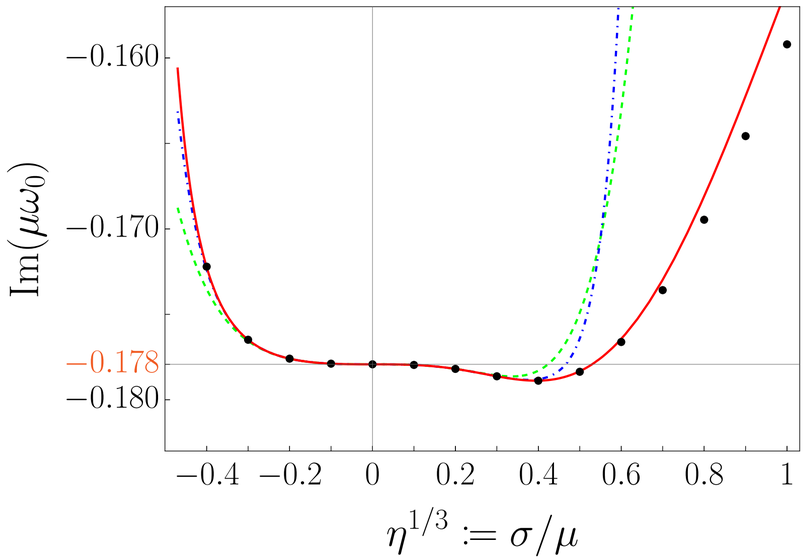
The only free parameter is σ/μ

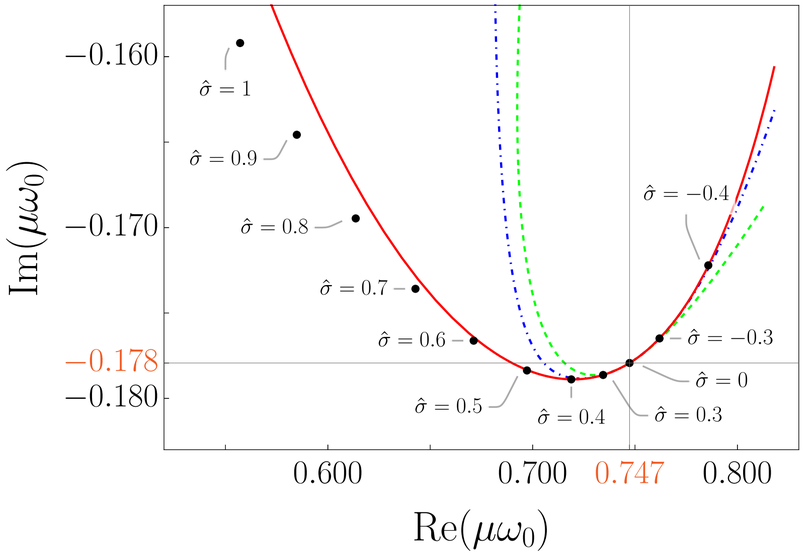
- The graviton horizon satisfies

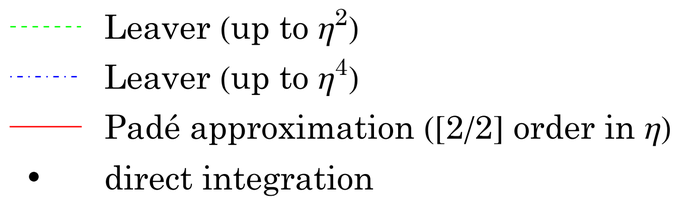
$$r_g^4 - \mu(r_g^3 + \sigma^3) = 0$$

QNM in Hayward case









- The fundamental QNM frequencies are fully numerics, using direct integration — black dot
- Other methods e.g. Leaver method + Padé approx. are a useful cross-check.
- Higher overtones give a possible probe of geometry around the horizon (Konoplya 23)

Tidal Love number

- Describes how compact objects response to the external field

Barura, Kobayashi, Mukohyama,
Oshita, Takahashi, VY 24

Tidal Love number

- Describes how compact objects response to the external field
- In our EFT, the master equation in the odd sector:

$$\left[\frac{d^2}{dr_*^2}+\omega^2-F(r)V_{\rm eff}(r)\right]\Psi(r_*,\omega)=0$$

- Impose regularity condition at the (graviton) horizon

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- Large-r limit:

$$\Psi(r)\sim\left(\frac{r}{r_g}\right)^{\ell+1}\left[1+\mathcal{O}\left(\frac{r_g}{r}\right)\right]+K_\ell\left(\frac{r}{r_g}\right)^{-\ell}\left[1+\mathcal{O}\left(\frac{r_g}{r}\right)\right]$$

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$K_\ell = 0$ in GR for BH in 4d

Damour & Nagar 09, Binnington & Poisson 09, +++

- Impose regularity condition at the (graviton) horizon

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Tidal Love number

$$\Psi(r)\sim\left(\frac{r}{r_g}\right)^{\ell+1}\left[1+\mathcal{O}\left(\frac{r_g}{r}\right)\right]+K_\ell\left(\frac{r}{r_g}\right)^{-\ell}\left[1+\mathcal{O}\left(\frac{r_g}{r}\right)\right]$$

Tidal Love number

- Describes how compact objects response to the external field

Barura, Kobayashi, Mukohyama, Oshita, Takahashi, VY 24

- In our EFT, the master equation in the odd sector:

$$\left[\frac{d^2}{dr_*^2}+\omega^2-F(r)V_{\rm eff}(r)\right]\Psi(r_*,\omega)=0$$

$K_\ell = 0$ in GR for BH in 4d

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GR deviations

affects the waveform during inspiral phase

Dynamics of even sector

- There are two propagating DoFs (1 scalar + 1 tensor) $\quad \delta g_{\mu\nu} \equiv g_{\mu\nu} - \bar{g}_{\mu\nu}$

$$\delta g_{\tau\tau} = A(r)\, H_0(\tau, r) \qquad \delta g_{\tau r} = \sqrt{1 - A}\, H_1(\tau, r) \qquad \delta\Phi = \Phi - \bar{\Phi}(\tau)$$

$$\delta g_{rr} = H_2(\tau, r) \qquad \delta g_{\tau a} = \delta g_{ra} = 0 \qquad \delta g_{ab} = r^2 K(\tau, r)\gamma_{ab}$$

The background $\simeq$ approx. stealth Schwarchild solution (De Felice et al. 23)

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Background inhomogeneity

- The EFT operator contains bg. inhomogeneities

Mukohyama, Seraille, Takahashi, VY 24

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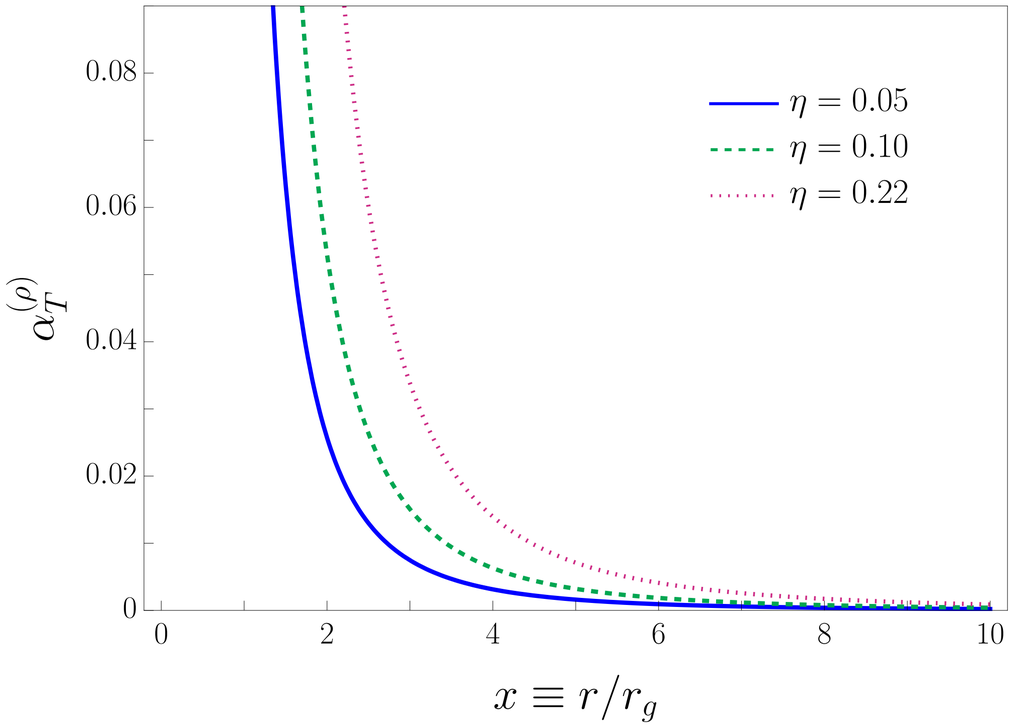
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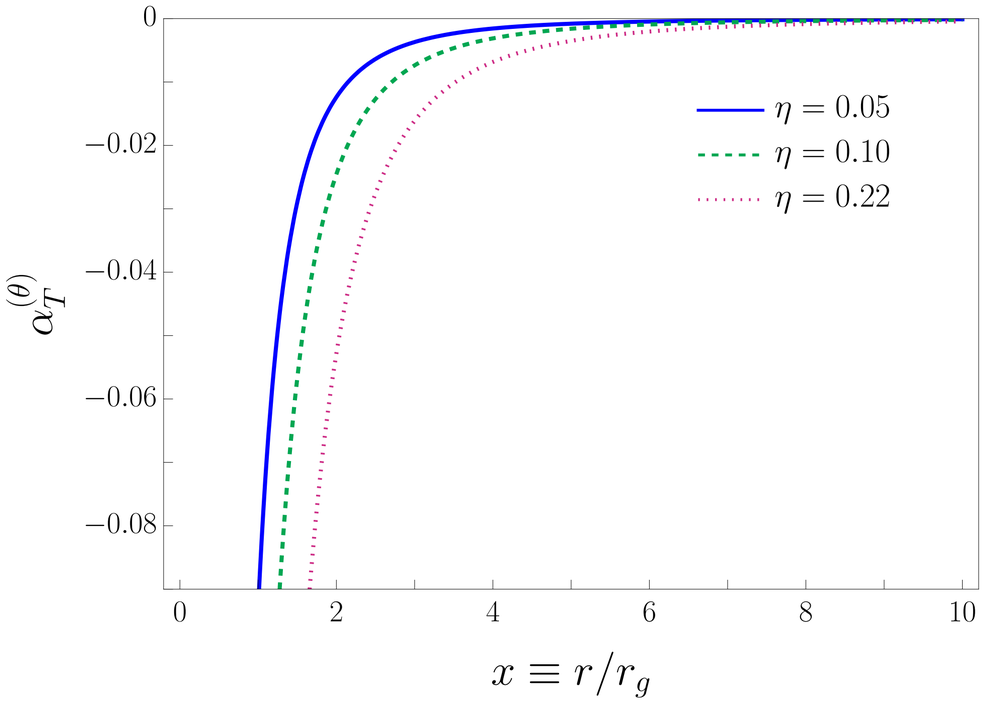
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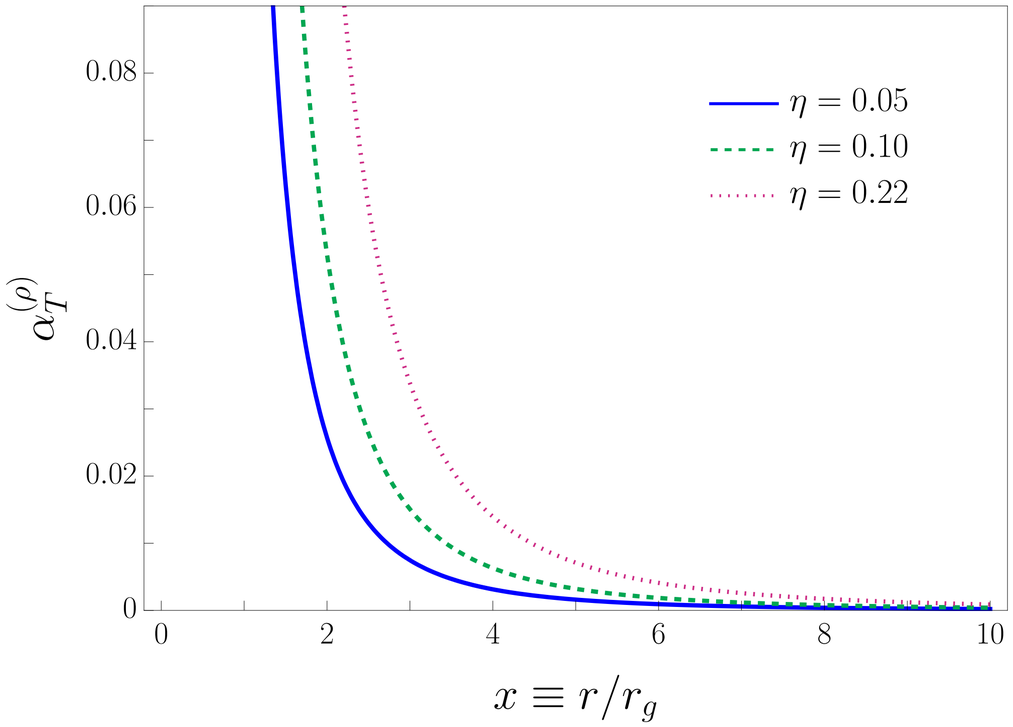
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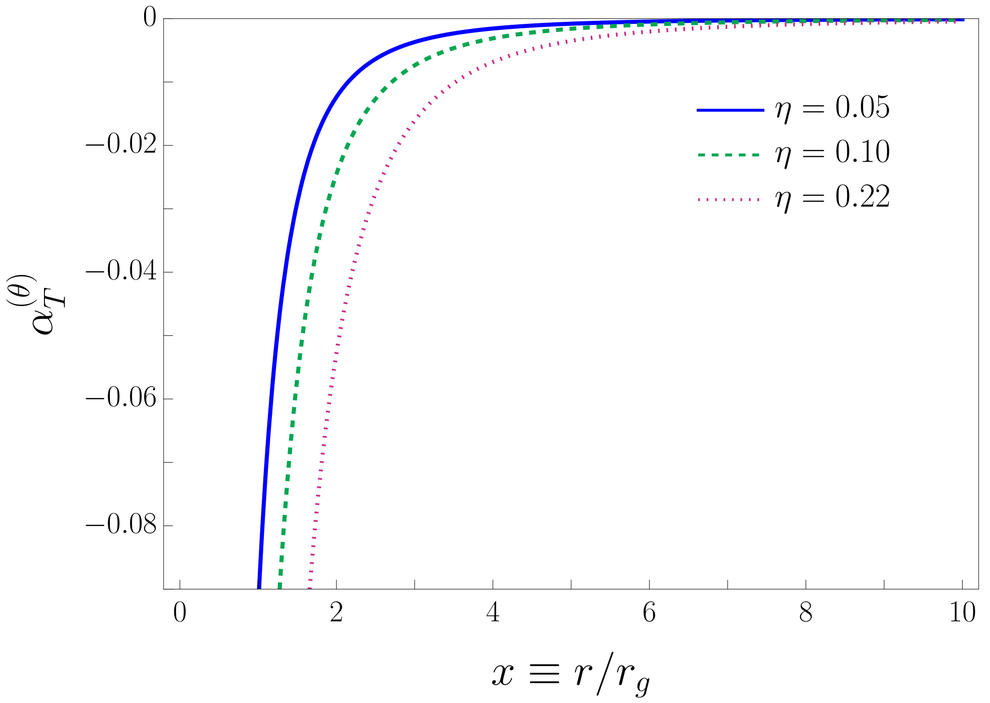


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$$c^2_\rho \neq c^2_\theta$$

around BH

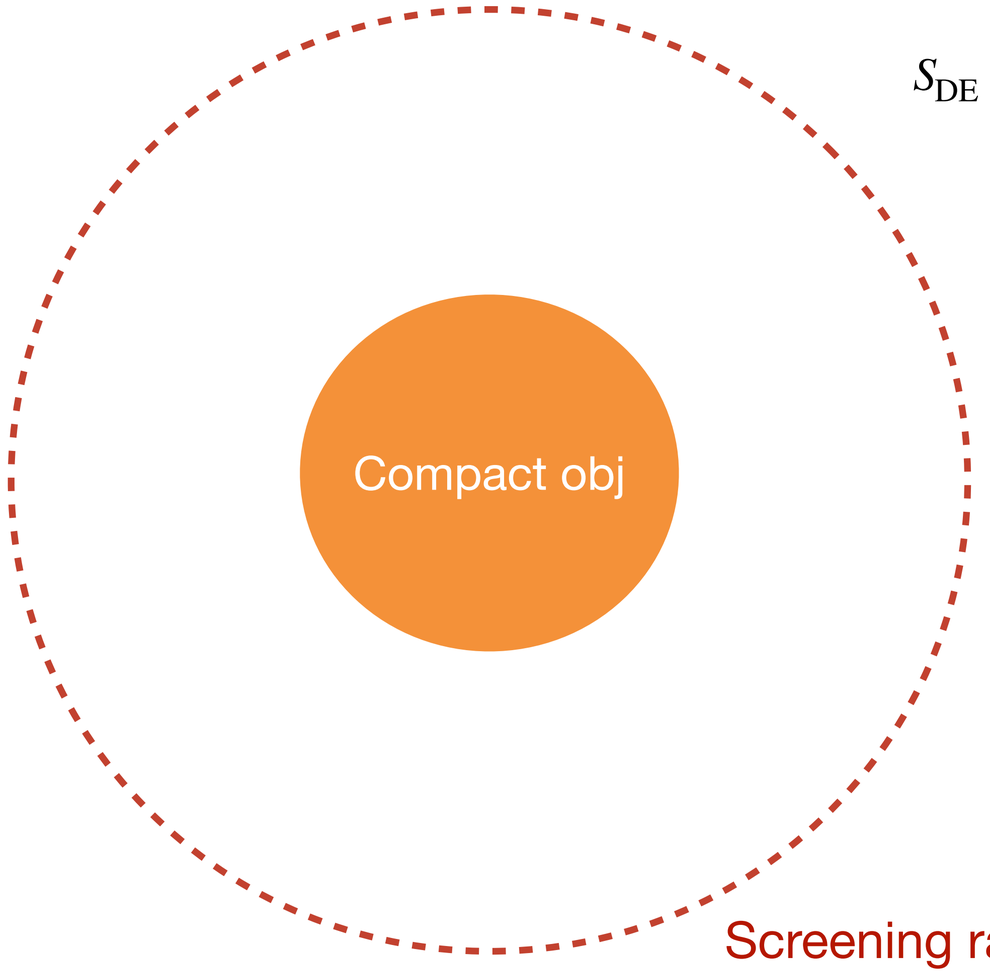


EFT with Screening

Mukohyama, Vernizzi, VY, in prep.

- Accommodate the screening mechanism in the EFT of Compact Objects

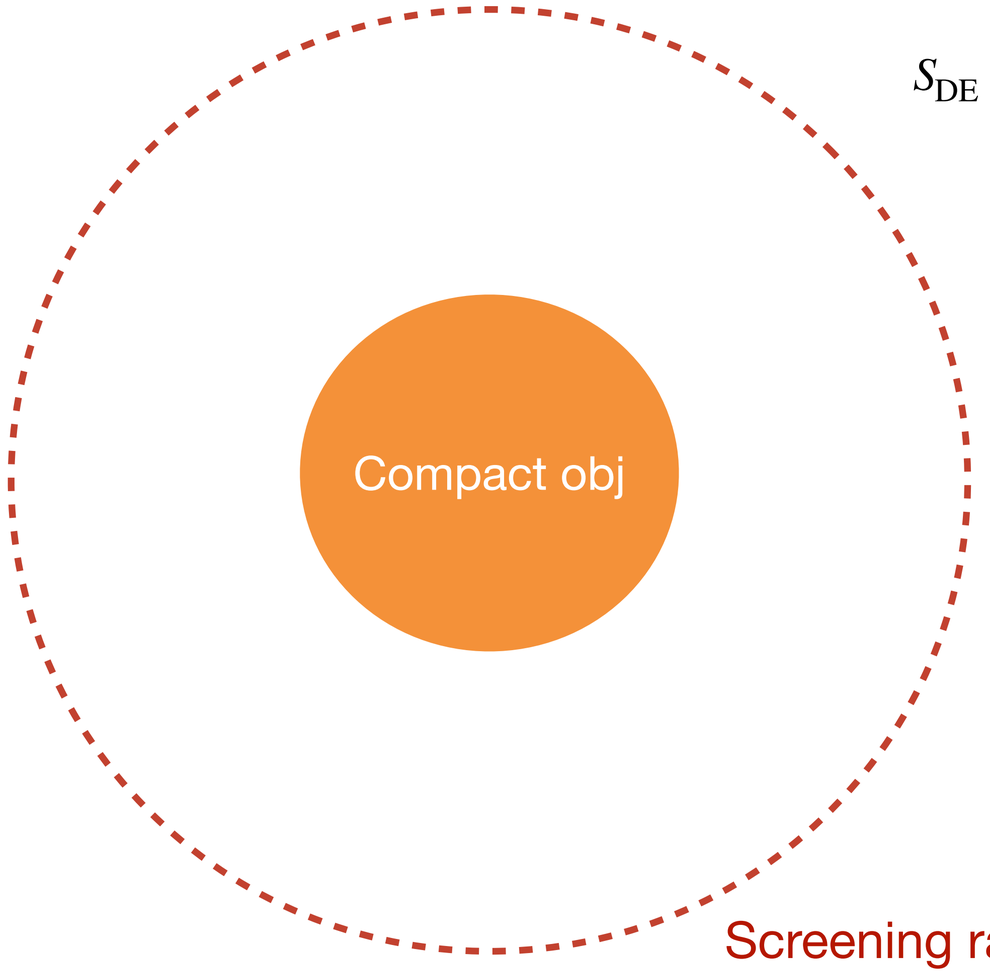
$$S_{\rm DE} = \int d^4x\sqrt{-g}\left[\frac{M_\star^2}{2}fR - \Lambda - c(y)g^{\tau\tau} + \frac{1}{2}m_2^4(\delta g^{\tau\tau}) - \frac{1}{2}m_3^3\delta g^{\tau\tau}\delta K + \cdots\right]_{\rm DE} + S_{\rm m}$$



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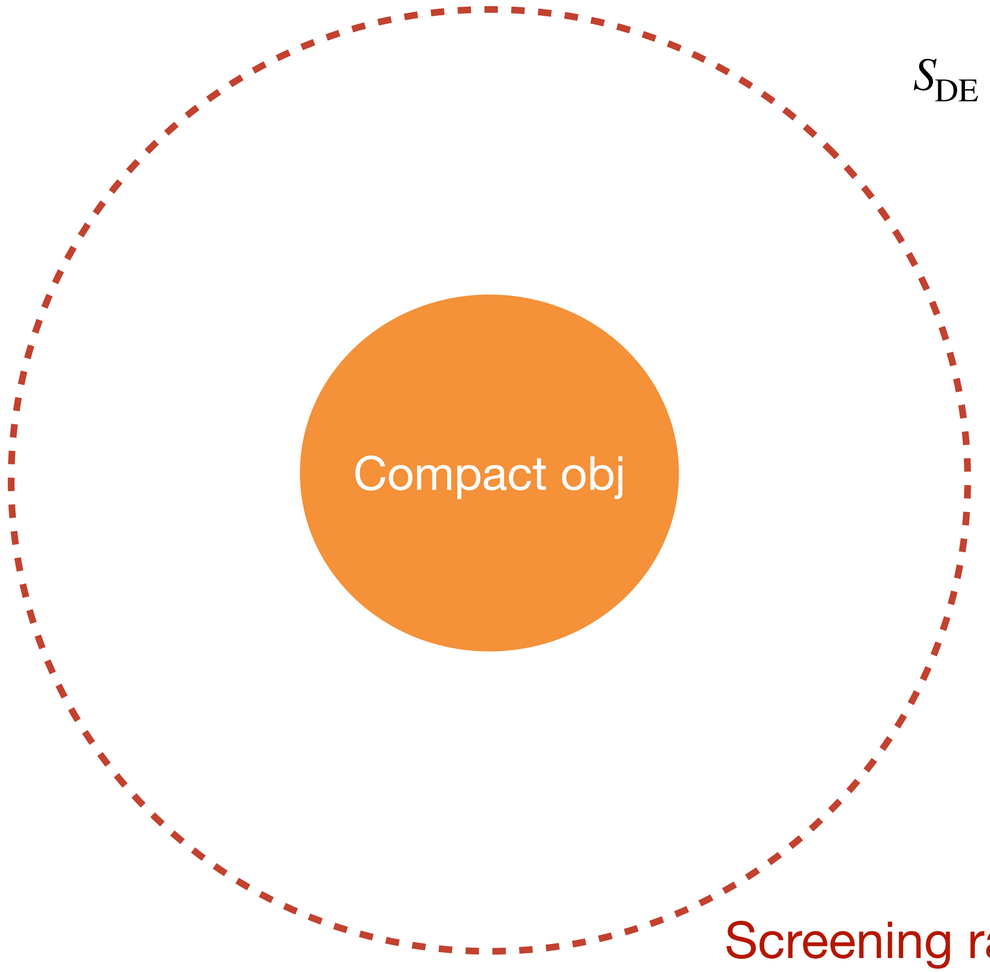
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Tadpole parameters are fixed by screening profiles

Nonlinearities of perturbations in EFTofDE are encoded up to tadpole terms in EFTofCO

Extension to vector-tensor gravity

- Formulate the EFT of BH perturbations in vector-tensor (A_μ, $g_{\mu\nu}$): 3+2 dofs

Aoki, Gorji, Mukohyama, Takahashi and VY 23

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$(A_\mu, g_{\mu\nu})$: 3+2 dofs

$$n_\mu \propto \delta^\tau_\mu + g_M A_\mu \equiv \delta^\tau_\mu \qquad n_\mu n^\mu = -1$$

preferred time direction not preferred time slicing

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Unitary gauge

4d diffeo. $\Rightarrow$ 3d diffeo. +

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NEED two sets of consistency relations

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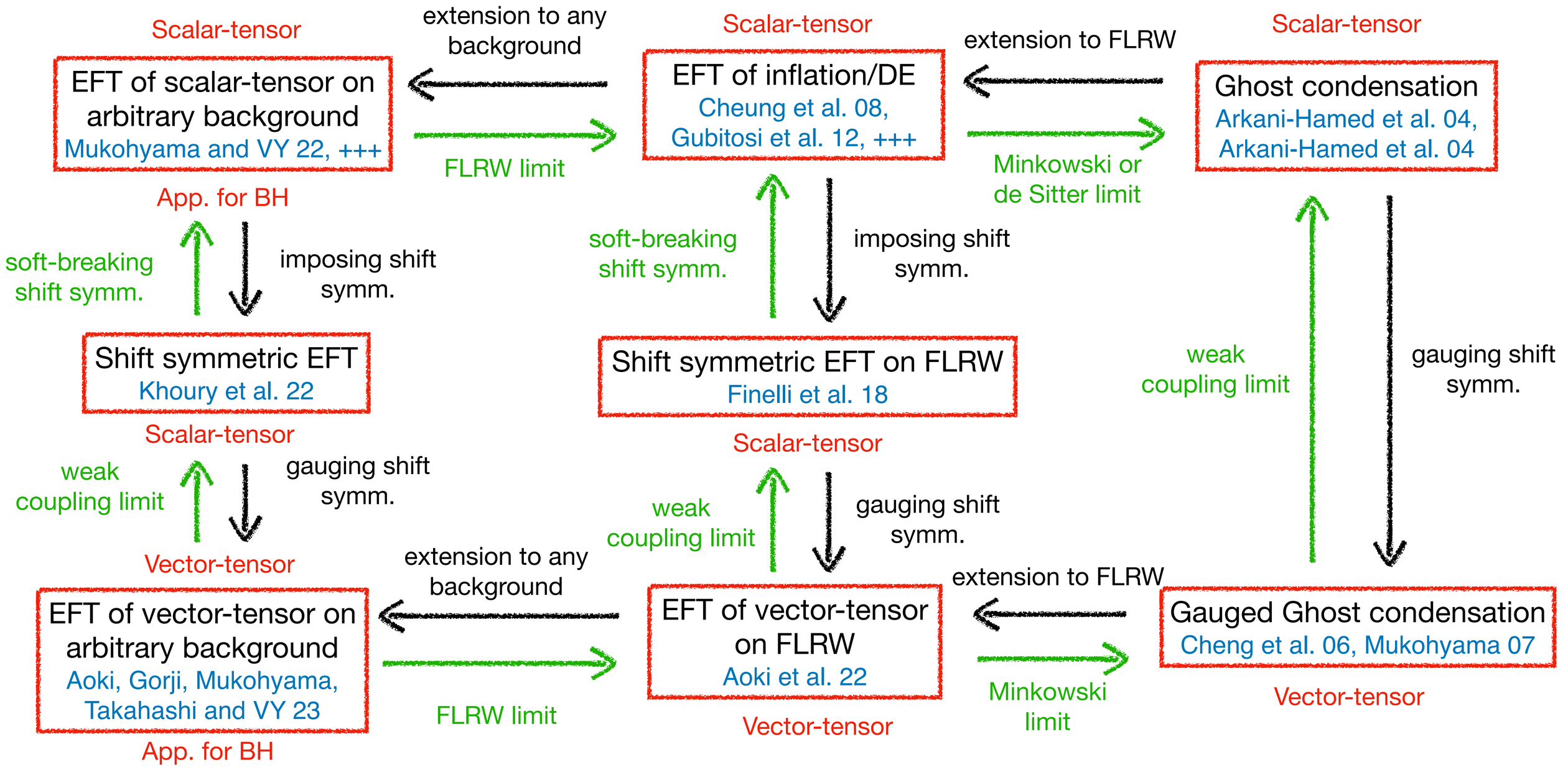
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NEED two sets of consistency relations

- EFT of vector-tensor theories on cosmological background

Aoki et. al. 22

The Web of EFTs



The Abdus Salam International Centre for Theoretical Physics

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Bangkok School on Cosmology

DESCRIPTION:

This school provides an introduction to the current state of research in cosmology and astroparticle physics. It is intended for graduate students, as well as more senior non- expert researchers interested in the field.

LECTURERS AND TOPICS:

Dark Matter
A. Arvanitaki - Perimeter Institute, Canada
Inflation
P. Creminelli - ICTP, Italy
Large Scale Structure
A. Moradinezhad - LAPTh Annecy, France
Introduction to Cosmology
F. Schmidt - Max Planck Institute, Germany
Cosmic Microwave Background
B. Sherwin - University of Cambridge, UK
Cosmology and Strings
C. Vafa - Harvard University, USA (TBC)

7 - 12 December 2026

Bangkok, Thailand

Application and Deadlines:

28 August 2026
for those requesting financial support

30 September 2026
otherwise

DIRECTORS:

I. Antoniadis, LPTHE – CNRS, Sorbonne University, France
A. Chatrabhuti, Chulalongkorn University, Thailand
P. Ittisamai, Chulalongkorn University, Thailand
C. Pongkitivanichkul, Khon Kaen University, Thailand
N. Suwonjandee, Chulalongkorn University, Thailand
S. Sypsas, Universidad del Bío-Bío, Chile
D. Wongratanaphisan, Chiang Mai University, Thailand
V. Yingcharoenrat, Chulalongkorn University, Thailand

ICTP SCIENTIFIC CONTACT:

Paolo Creminelli, (ICTP)

FURTHER INFORMATION:

E-mail: smr4203@ictp.it
Web: http://indico.ictp.it/event/11129/
Female scientists are encouraged to apply.

GRANTS:

A limited number of grants are available to support the atte
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Th

IUPAP · PMU-B · ThEP

Dark Side of The Universe

20th International Conference

14-18 December 2026

Chulalongkorn University
Bangkok, THAILAND

A series of international workshops in cosmology and astroparticle physics bringing together theorists and experimentalists



Please join us!

Chula Chulalongkorn University · ICTP

TOPICS INCLUDE:

- Dark matter: theoretical models and detection techniques
- Astrophysical signatures of DM
- Early universe and inflation
- Large scale structure and dark ages
- New ideas and observations on dark energy
- Dark energy (DE) and dark matter from string theory
- Neutrino cosmology and its interplay with DM and DE
- Gravitational waves

SCIENTIFIC COMMITTEE:

C. Boehm (Sydney U.)
C. Caprini (CERN)
M. Kunz (Geneva U.)
M. Musso (U. Salamanca)
M. Sakellariadou (King's Coll. London)
G. Servant (DESY and Hamburg U.)

ORGANIZING COMMITTEE:

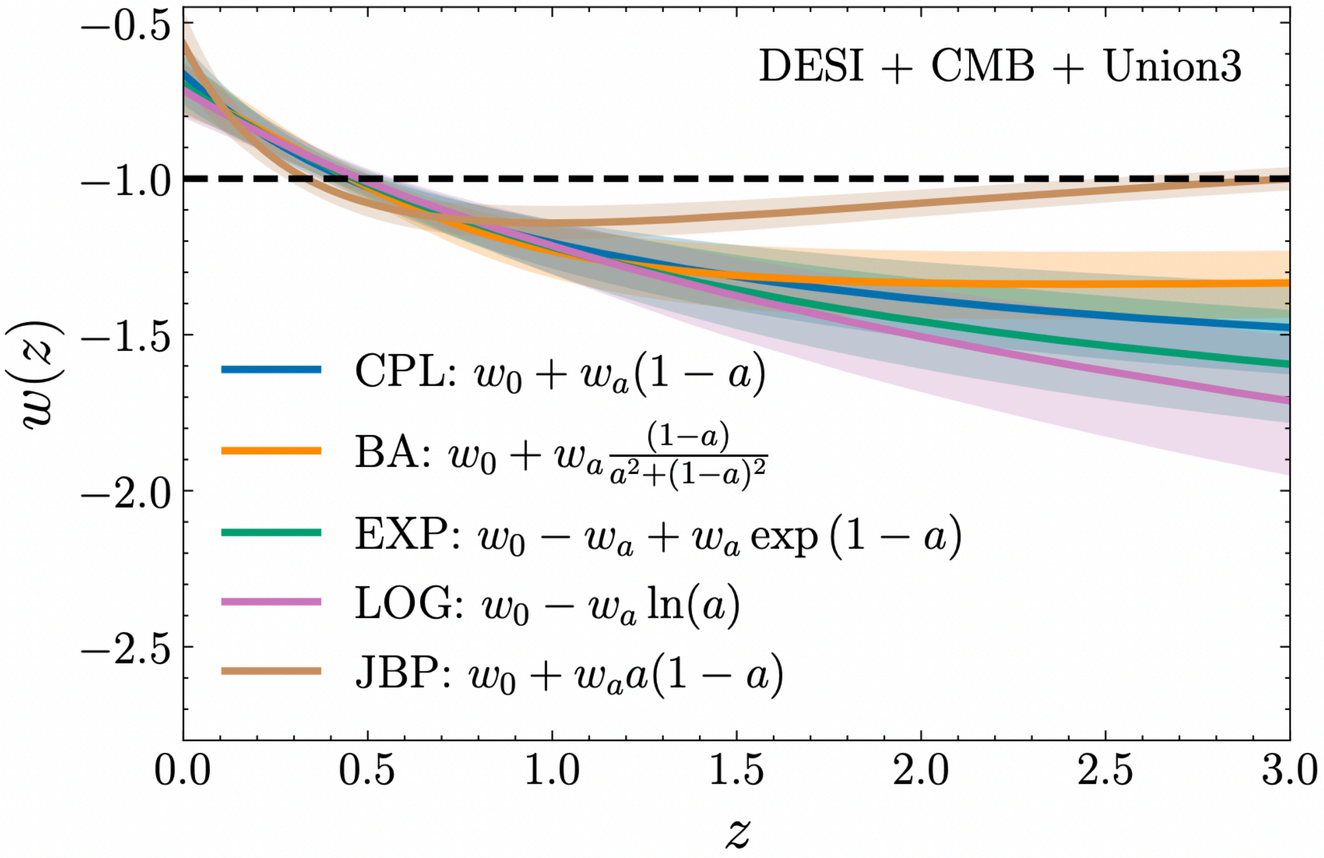
I. Antoniadis (Sorbonne U.)
A. Chatrabhuti (Chulalongkorn U.)
P. Creminelli (ICTP, Trieste)
P. Ittisamai (Chulalongkorn U.)
C. Pongkitivanichkul (Khon Kaen U.)
N. Poovuttikul (Chulalongkorn U.)
N. Srimanobhas (Chulalongkorn U.)
N. Suwanjandee (Chulalongkorn U.)
S. Sypsas (Biobio U.)
V. Yingcharoenrat (Chulalongkorn U.)

Conference webpage:
https://indico.global/e/DSU2026

Backup

Perhaps DE is dynamical…

- Other parametrizations of $w(z)$ DESI 25



There exists a regime where DE behaves as a Phantom field

$$w < -1 \Rightarrow \cancel{\text{NEC}}$$

Proposed models

- Thawing DE Caldwell & Linder 05
- Emergent DE Li & Shafieloo 19+20
- Mirage DE Linder 07

⋮

Rotating background

Mukohyama, Oshita, Takahashi, Wang, VY, in prep.

- General EFT can be applied to any background metric
- Our task is to realize the coordinate system where $\bar{\Phi}$ is spatially uniform
- Stationary and axisymmetric background:

$$\bar{g}_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \bar{g}_{tt}(r,\theta)\mathrm{d}t^2 + \bar{g}_{rr}(r,\theta)\mathrm{d}r^2 + \bar{g}_{\theta\theta}(r,\theta)\mathrm{d}\theta^2 + \bar{g}_{\phi\phi}(r,\theta)\mathrm{d}\phi^2 + 2\bar{g}_{t\phi}(r,\theta)\mathrm{d}t\mathrm{d}\phi\,.$$

- For simplicity, we consider $\bar{X} = \bar{g}^{\mu\nu}\partial_\mu\bar{\Phi}\partial_\nu\bar{\Phi} = \mathrm{const}$ and $\bar{\Phi}(\tau) \propto \tau$

$$\bar{g}^{\tau\tau} = -1 \quad \Rightarrow \quad \boxed{\bar{g}^{\mu\nu}\frac{\partial\tau}{\partial x^\mu}\frac{\partial\tau}{\partial x^\nu} = -1}$$

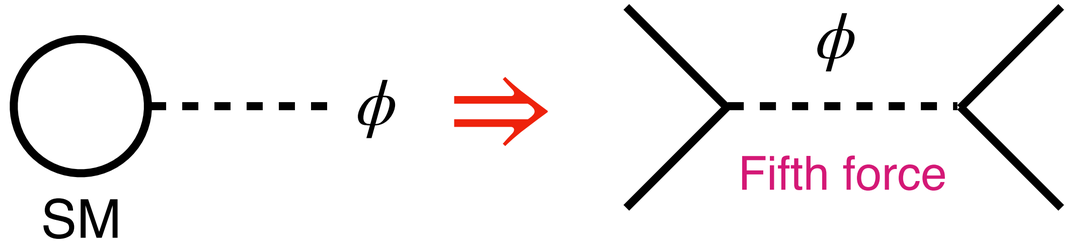
Hamilton-Jacobi eq for a timelike test particle

Solve these eqs. to obtain coordinate transformations where EFT is constructed

(The form of EFT is the same)

Couplings of DE with matter

- For our DE to contribute to the CC. problem



- Traditional scalar-tensor theories

$$S[g_{\mu\nu},\phi]=\int \mathrm{d}^4x\sqrt{-g}\left[f(\phi)R-\frac{1}{2}(\partial\phi)^2-V(\phi)\right] \quad \Leftrightarrow \quad \chi=f(\phi) \qquad f'(\phi)^2=\chi/w(\chi)$$

Non-minimally coupling btw. ϕ and $g_{\mu\nu}$

Jordan-Brans-Dicke theory

- Add matter couplings $S_{\rm J}=\int \mathrm{d}^4x\sqrt{-g}\left[f(\phi)R-\frac{1}{2}(\partial\phi)^2-V(\phi)\right]+S[\psi_{\rm m};g_{\mu\nu}]$

Minimally coupled

Jordan frame

Einstein eq. is modified

$g^{\rm E}_{\mu\nu}=f(\phi)g_{\mu\nu}$

$V(\phi)/f^2$

$$\Rightarrow \quad S_{\rm E}=\int \mathrm{d}^4x\sqrt{-g_{\rm E}}\left[R_{\rm E}-Z(\phi)g^{\mu\nu}_{\rm E}\partial_\mu\phi\partial_\nu\phi-V_{\rm E}(\phi)\right]+S\left[\psi_{\rm m};f^{-1}(\phi)g^{\rm E}_{\mu\nu}\right]$$

Einstein frame

Gravity is standard, but matter is coupled with ϕ via $\phi T_{\rm m}$

Quintessence coupling with matter

- In Einstein frame: gravity is standard $S_{\rm m}^{\rm E}\left[\psi_{\rm m}; \underbrace{f^{-1}(\phi) g_{\mu\nu}^{\rm E}}_{A(\phi)^2 g_{\mu\nu}^{\rm E}}\right]$

$$T_{\rm E}^{\mu\nu} = \frac{2}{\sqrt{-g_{\rm E}}}\frac{\delta S_{\rm m}^{\rm E}}{\delta g_{\mu\nu}^{\rm E}} \quad \Rightarrow \quad \nabla_\mu^{\rm E} T_{\rm E}{}^{\mu}{}_{\nu} = \frac{A'(\phi)}{A(\phi)} T_{\rm E} \nabla_\nu \phi \quad \text{with} \quad T_{\rm E} = g_{\rm E}^{\mu\nu} T_{\mu\nu}^{\rm E}$$

Exercise: Show that $\Box_{\rm E}\phi - V_{\rm E}'(\phi) = -\frac{A'(\phi)}{A(\phi)} T_{\rm E}$ ϕ is sourced by $T_{\rm E}$

Solution: $g_{\mu\nu}^{\rm J} = A(\phi)^2 g_{\mu\nu}^{\rm E}$ $S[g_{\mu\nu}^{\rm J}] = S_{{\rm g}\phi}[g_{\mu\nu}^{\rm J}, \phi] + S_{\rm m}[g_{\mu\nu}^{\rm J}, \psi_{\rm m}]$

$$\Rightarrow \quad \delta S_{\rm m}[g_{\mu\nu}^{\rm J}, \psi_{\rm m}] = \frac{1}{2}\int {\rm d}^4x \sqrt{-g_{\rm J}}\, T_{\rm J}^{\mu\nu} \delta g_{\mu\nu}^{\rm J}$$

$$\delta g_{\mu\nu}^{\rm J} = \delta(A^2 g_{\mu\nu}^{\rm E}) = \underline{2AA' g_{\mu\nu}^{\rm E}\delta\phi} + A^2 \delta g_{\mu\nu}^{\rm E}$$

To Einstein frame $\rightarrow$ $T_{\rm J}^{\mu\nu} = A^{-6} T_{\rm E}^{\mu\nu}$

$$\delta S_{\rm m}[A^2 g_{\mu\nu}^{\rm E}, \psi_{\rm m}] = \int {\rm d}^4x \sqrt{-g_{\rm E}}\, \underline{\frac{A'}{A}} T_{\rm E} \delta\phi$$

Matter-DE coupling

Quintessence coupling with matter

- Geodesic in Einstein frame $\mathrm{d}s^2 = -(1+2\Phi)\mathrm{d}t^2 + (1-2\Psi)\mathrm{d}\vec{x}^2$

$$\ddot{\vec{x}} = -\vec{\nabla}\Phi_{\mathrm{E}} - \beta\vec{\nabla}\phi \qquad \beta \equiv \frac{A'}{A} \qquad \text{and} \qquad \nabla^2\Phi_{\mathrm{E}} = 4\pi G M \delta^{(3)}(\vec{x}) \Rightarrow \Phi_{\mathrm{E}} = -\frac{GM}{r}$$

Fifth force

- For ϕ, in quasi-static limit $\nabla^2\phi - m_\phi^2\phi = \beta M\delta^{(3)}(\vec{x}) \Rightarrow \phi = -\frac{\beta M}{4\pi r}e^{-m_\phi r}$
- From $g_{\mu\nu} = A^2 g^{\mathrm{E}}_{\mu\nu} \Rightarrow \delta g_{\mu\nu} = 2AA'\bar{g}^{\mathrm{E}}_{\mu\nu}\phi + A^2\delta g^{\mathrm{E}}_{\mu\nu}$

$$\Phi = \Phi_{\mathrm{E}} + \beta\phi \qquad \text{and} \qquad \Psi = \Psi_{\mathrm{E}} - \beta\phi \qquad \text{with} \qquad \Phi_{\mathrm{E}} = \Psi_{\mathrm{E}}$$

$$\Phi = -\frac{GM}{r}\left(1 + \frac{\beta^2}{4\pi G}\right) \qquad \Psi = -\frac{GM}{r}\left(1 - \frac{\beta^2}{4\pi G}\right) \Rightarrow \gamma = \frac{\Psi}{\Phi} \neq 1$$

Gravitational slip

$|\gamma - 1| \lesssim 10^{-5}$

Shapiro time delay

Conformal transf. and $f(R)$

- Under $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ $\quad \sqrt{-\tilde{g}} = \Omega^4 \sqrt{-g}$ $\quad \tilde{R} = \Omega^{-2}\left[R - 6g^{\mu\nu}\dfrac{\nabla_\mu \nabla_\nu \Omega}{\Omega}\right]$

Wald's GR book, app.D

- $f(R)$ gravity $\quad S = \int \mathrm{d}^4x\sqrt{-g}\, f(R) + S_{\mathrm{m}}[g_{\mu\nu}; \psi_{\mathrm{m}}]$

Introduce the auxiliary field Φ $\Rightarrow$ $S = \int \mathrm{d}^4x\sqrt{-g}\left[f(\Phi) + \dfrac{\mathrm{d}f}{\mathrm{d}\Phi}(R - \Phi)\right] + S_{\mathrm{m}}[g_{\mu\nu}; \psi_{\mathrm{m}}]$

Φ is non-dynamical $\Rightarrow$ EoM of Φ gives $\Phi = R$ with $f'' \neq 0$

This can be recast into the form $\quad S = \int \mathrm{d}^4x\sqrt{-g}\left[F(\Phi)R - V(\Phi)\right] + S_{\mathrm{m}}[g_{\mu\nu}; \psi_{\mathrm{m}}]$

$V(\Phi) = -f(\Phi) + \Phi f'(\Phi)$

Use conformal transf. To go to Einstein frame

Example of using the constraint $c_{\rm GW}^2 = 1$

- Expand the Horndeski action on $\Phi(t)$: $S[h_{ij}] = \frac{M^2}{8}\int d^4x\, a^3(t)\left[\dot{h}_{ij}^2 - \frac{c_{\rm GW}^2}{a^2(t)}(\partial_k h_{ij})^2\right]$

$$1 - c_{\rm GW}^2 = \frac{2m_4^2}{M^2}$$

$$M^2 \equiv M_{\rm Pl}^2 + 2m_4^2$$

$$m_4^2 = -\left[2XG_{4X} + XG_{5\Phi} + (H\dot{\Phi} - \ddot{\Phi})XG_{5X}\right] + X^2F_4 - 3H\dot{\Phi}X^2F_5$$

$$X = (\partial_\mu \Phi)^2$$

- Impose $m_4^2 = 0$

$$\dot{h}_{ij}^2 - \frac{1}{a^2(t)}(\partial_k h_{ij})^2$$

$$2G_{4X} - XF_4 + G_{5\Phi} = 0$$

$$L_2 = G_2(\Phi, X) \qquad L_3 = G_3(\Phi, X)\Box\Phi \qquad \dot{h}_{ij}^2$$

$$L_4 = G_4(\Phi, X)R - 2G_{4X}(\Phi, X)[(\Box\Phi)^2 - \Phi_{\mu\nu}\Phi^{\mu\nu}] - F_4(X, \Phi)\epsilon^{\mu\nu\rho}{}_\sigma\epsilon^{\mu'\nu'\rho'\sigma}\Phi_\mu\Phi_{\mu'}\Phi_{\nu\nu'}\Phi_{\rho\rho'}$$

$$L_5 = G_5(\Phi, X)G_{\mu\nu}\Phi^{\mu\nu} + \frac{1}{3}G_{5X}(\Phi, X)[(\Box\Phi)^3 - 3(\Box\Phi)\Phi_{\mu\nu}\Phi^{\mu\nu} + 2\Phi_{\mu\nu}\Phi^{\sigma\mu}\Phi^{\nu}{}_\sigma]$$

$$- F_5(\Phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\Phi_\mu\Phi_{\mu'}\Phi_{\nu\nu'}\Phi_{\rho\rho'}\Phi_{\sigma\sigma'}$$

Creminelli and Vernizzi 17, Ezquiaga and Zumalacárregui 17, Baker et al. 17, Sakstein and Jain 17