

Schwarzschild black hole surrounded by Dehnen dark matter halo

Supakchai Ponglertsakul

Based on 2602.03349 [gr-qc]: with: David Senjaya and Thanaporn Chuensuksan

5 July 2026, AGWAM

Plan

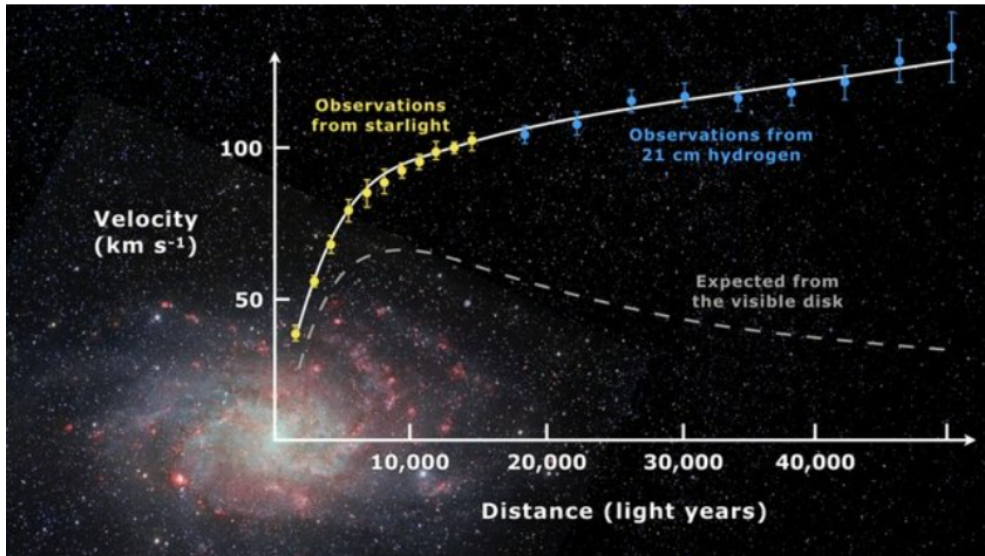
1. Motivation

2. Project

- 2.1 The solution
- 2.2 Black hole thermodynamics
- 2.3 Null geodesics
- 2.4 Eikonal QNMs

3. Conclusions

Galaxy rotation's curve



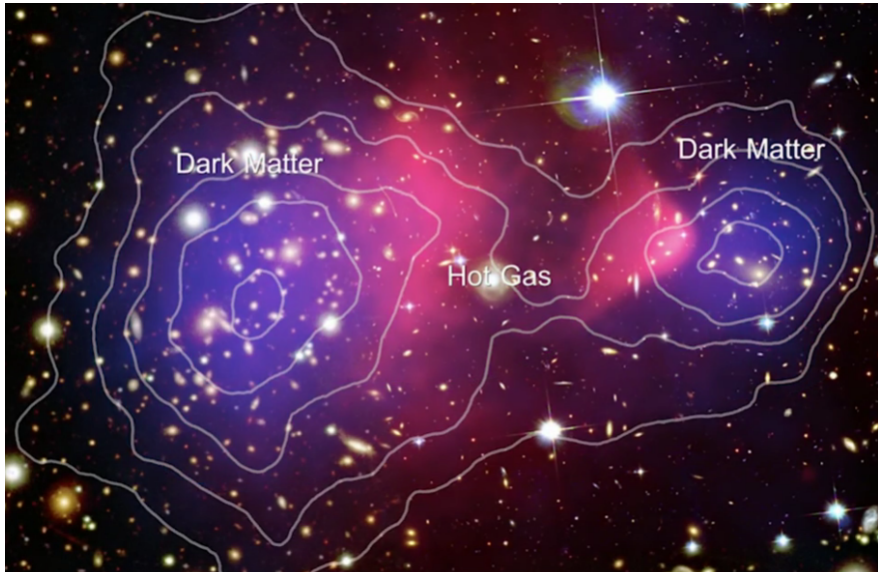
Bullet cluster



Figure: NASA's Chandra-Observatory

Bullet cluster

Cr: CAASTO



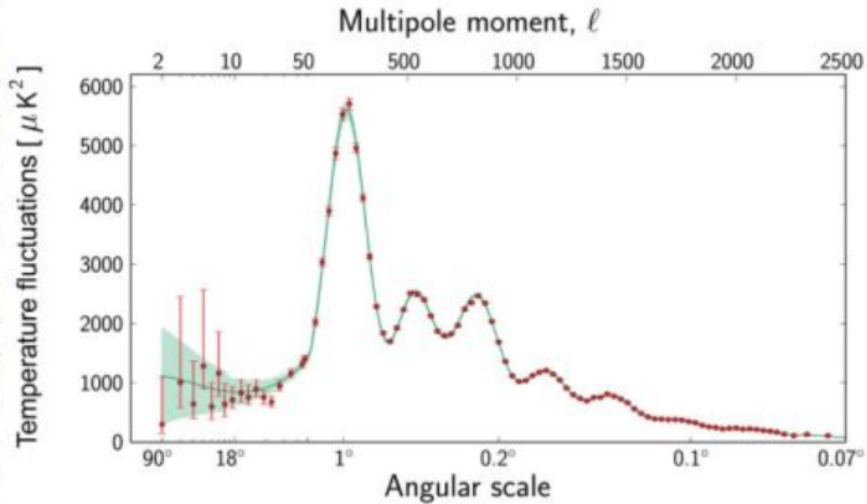


Figure: Planck

DM properties¹

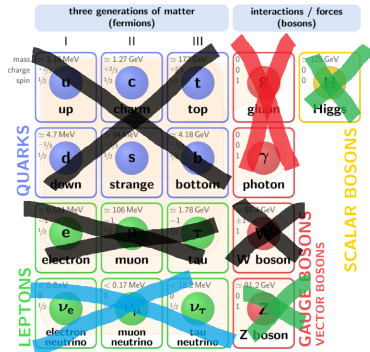
- Massive
- Dark, i.e., electrically neutral
- Weakly interact with normal matter
- Cold*, slow-moving
- Stable on cosmic scale

¹Credit: Patipan's talk @WE-Heraeus

DM properties¹

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Cr: TikZ.net



None of these matches any (known) Standard Model particles.

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Cold vs Warm DM

Cold DM

- Large-scale structure: well explained
- Missing satellite problem: predicts (more) dwarf galaxies orbiting large galaxies.
- Core-cusp problem: predicts a sharp spike at the galactic center (a cusp).

Warm DM

- Solve the core-cusp problem
- Matches large scale structure
- Slower-clumping matter predicts first stars and galaxies formed much later than in the CDM Model.

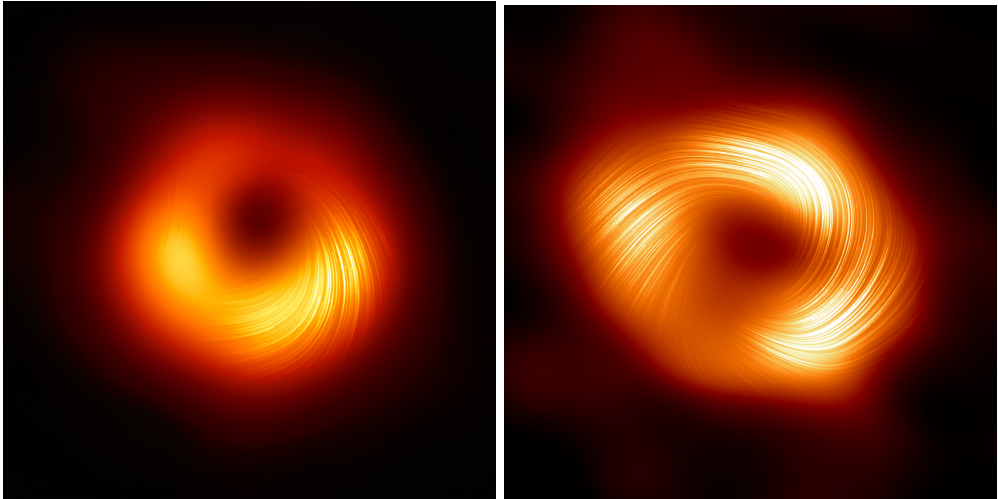


Figure: EHT collaboration

Black holes and dark matter

- They must co-exist.
- Although dark matter does not emit light, it can still be accreted by black holes.
- DM influences the growth of black holes and the evolution of galaxies.

Question

How do we model a dark matter halo?

Black holes and dark matter

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Question

How do we model a dark matter halo?

Answer

- By constructing its “density profile”, usually depends on characteristic density and some length scale.
- It tells us how dark matter is generally distributed around the galactic center.

Navarro-Frenk-White profile³

- Found by large-N bodies simulation of *cold dark matter*.

²Gondolo and Silk, Phys.Rev.Lett. 83 (1999) 1719-1722

³Astrophys.J.462:563-575,1996

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- DM annihilation rate (if exists) is extremely increased. Thus, end-product particles such as neutrinos should be observed at the galactic center.
- Mass of DM spike should alter stellar orbit around the center.

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The profiles

$$\rho(r) = \rho_0 \left(\frac{r}{r_0} \right)^{-\gamma} \left[1 + \left(\frac{r}{r_0} \right)^\alpha \right]^{(\gamma-\beta)/\alpha}.$$

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Cored

- Pseudo isothermal
- Burkert (2, 3, 1)
- Perfect sphere (2, 4, 0)
- Plummer sphere (2, 5, 0)

Cuspy

- NFW (1, 3, 1)
- Einasto
- Hernquist (1, 4, 1)
- Jaffe (1, 4, 2)
- Dehnen (1, 4, γ)

What and Why Dehnen⁴?

The profiles

$$\rho(r) = \rho_0 \left(\frac{r}{r_0} \right)^{-\gamma} \left[1 + \frac{r}{r_0} \right]^{(\gamma-4)}.$$

$$M(r) = 4\pi \int_0^r \rho(r') r'^2 dr' \propto \frac{1}{(\gamma-3)}.$$

- $\rho \sim r^{-4}, r \rightarrow \infty$ and $\rho \sim r^{-\gamma}, r \rightarrow 0$: Thus $\gamma = 0$ gives a cored profile.

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- It is flexible and can give rise to various profiles, whether cuspy or cored.
- It was originally developed to describe light distribution in galaxies i.e., stellar (mass) distribution.
- Its flexibility has proven useful when modeling cuspy or cored DM profiles.

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What and Why Dehnen?

- For dwarf galaxies, cuspy/cored DM profile is *debatable*.

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- Dwarf galaxies: Mrk 462, Henize 2-10, Leo I host SMBH, $M \sim 2 \times 10^5 M_{\odot}, 1 \times 10^6 M_{\odot}, 3.3 \times 10^6 M_{\odot}$.

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- Ultra-faint dwarf galaxies ($r \sim 98 ly, L = 10^{2-3} L_{\odot}$) $\rightarrow$ **highest** halo-mass to baryon-mass ratio, thus the most dark matter dominated system $\rightarrow$ a good laboratory to explore DM⁵

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What and Why?

- Burkert *1905.11803*
- Navarro-Frenk-White *2303.08183*
- Hernquist (1, 4, 1) *2303.17666*
- Einasto: $\rho = \rho_0 e^{-2\alpha^{-1}((r/r_0)^\alpha - 1)}$ *2307.13553*

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- Dehnen (1, 4, 0), (1, 4, 5/2), (1, 4, 1/2) *2407.0287, 2411.01145, to appear (Senjaya)*
- Jaffe (1, 4, 2) *2505.20031*
- Several static/rotating BH solutions including regular black holes are found with various DM profiles.

Our project

(1, 4, 3/2)

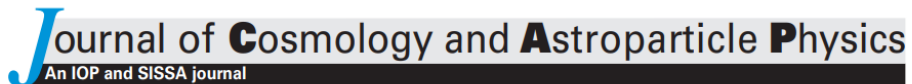
To construct an exact solution for a BH in a Dehnen-type DM halo and investigate several properties such as QNMs, thermodynamics, null geodesics, shadow and gravitational lensing under influence of (ρ_0, r_0) .

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Why $\gamma = 3/2$?

- Provides a steeper inner cusp, more centrally concentrated than the lower γ .
- Still yields a finite and physically reasonable DM halo mass ($\gamma < 3$).



Black hole space-time in dark matter halo

Zhaoyi Xu,^{a,b,c} Xian Hou,^{a,b,c} Xiaobo Gong^{a,b,c,d} and Jiancheng Wang^{a,b,c}

The solution

- A general static spherically symmetric spacetime can be written as

$$ds^2 = -[f(r) + F(r)] dt^2 + \frac{dr^2}{[g(r) + G(r)]} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad (1)$$

- When $F = 0, G = 0$, the metric above yields pure DM spacetime.
- For an isolated spacetime (no DM), we demand that $f = g = 1$ i.e., $g_{tt} = -(1 + F)$, and

$$F = G = -\frac{r_s}{r}.$$

The solution

The DM profile

$$\rho(r) = \rho_0 \left(\frac{r}{r_0} \right)^{-3/2} \left[1 + \frac{r}{r_0} \right]^{-5/2}.$$

- In spherically symmetric spacetime, the mass distribution of DM is determined by

$$M_{DM} = 4\pi \int_0^r \rho(\hat{r}) \hat{r}^2 d\hat{r} = \frac{8}{3} \pi \rho_0 r_0^3 \left(1 + \frac{r_0}{r} \right)^{-\frac{3}{2}}. \quad (2)$$

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- In pure DM spacetime i.e., $F = G = 0$, the tangential velocity of test particle moving under the influence of DM mass distribution relates to the redshift function f (Xu et.al.) as (if we impose $f = g$)

$$v_t^2(r) = \frac{M_{DM}}{r} = r \frac{d}{dr} \left(\ln \sqrt{f(r)} \right). \quad (3)$$

The solution

- Then, we can solve for $f = e^{\frac{32}{3}\pi\rho_0 r_0^2 \left(1 + \frac{r_0}{r}\right)^{-\frac{1}{2}}}$.

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(1, 4, 3/2)

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The solution

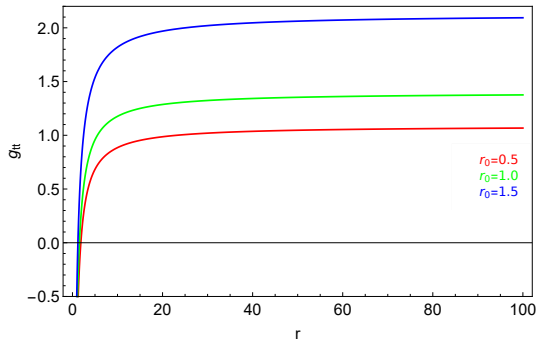
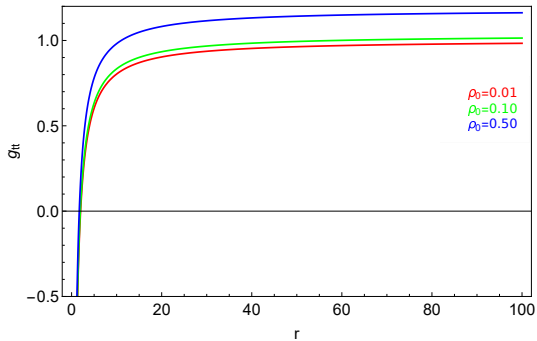
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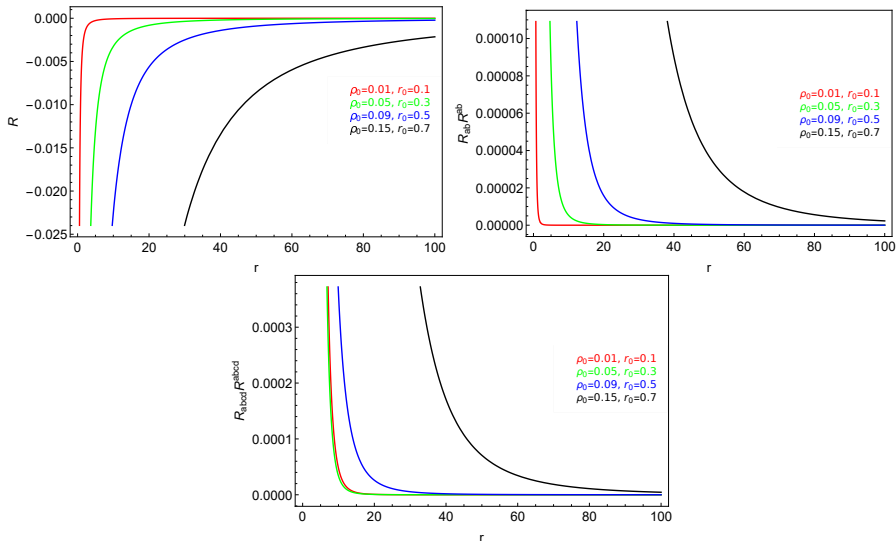
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The metric function



Curvature scalars

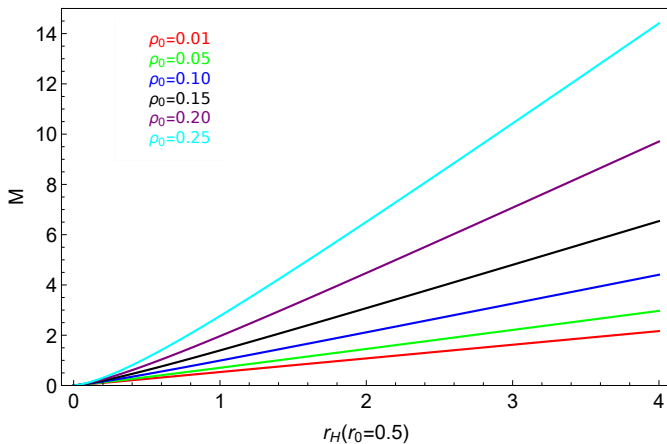


- $\{R, R_{ab}R^{ab}, K\} \rightarrow \{0, 0, \frac{12r_s^2}{r^6}\}$ (In the Schwarzschild limit)

Thermodynamics

- The total mass is

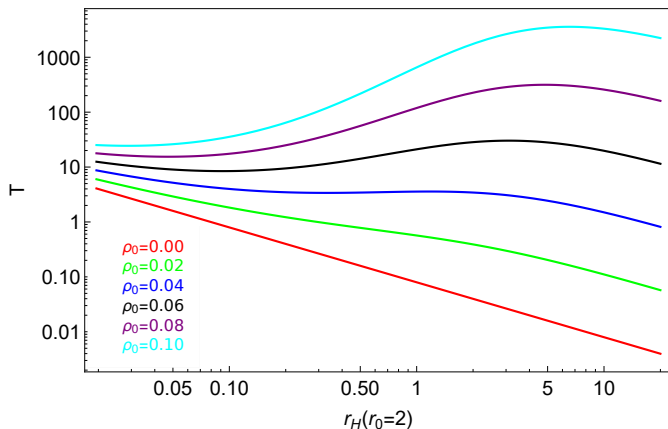
$$M = \frac{r_H}{2} e^{\frac{32}{3} \pi \rho_0 r_0^2} \left(1 + \frac{r_0}{r_H}\right)^{-1/2}. \quad (4)$$



Temperature

- The temperature of the black hole is defined by

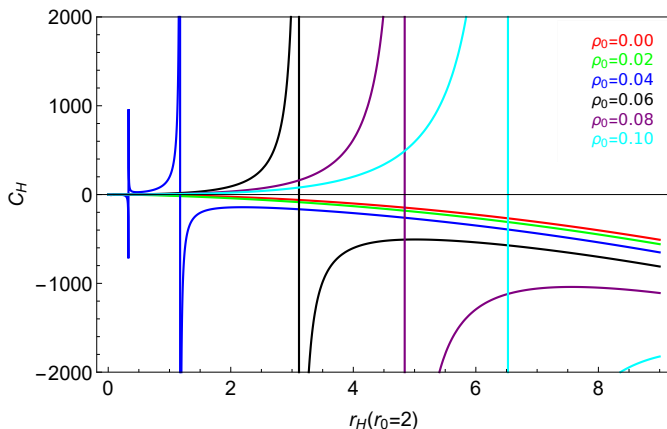
$$T = \frac{f' + F'}{4\pi} \Big|_{r=r_H} = e^{\frac{32}{3}\pi\rho_0 r_0^2 \left(1 + \frac{r_0}{r_H}\right)^{-1/2}} \left[\frac{1}{4\pi r_H} + \frac{4}{3} \frac{\rho_0 r_0^3}{\sqrt{r_H(r_0 + r_H)^3}} \right].$$



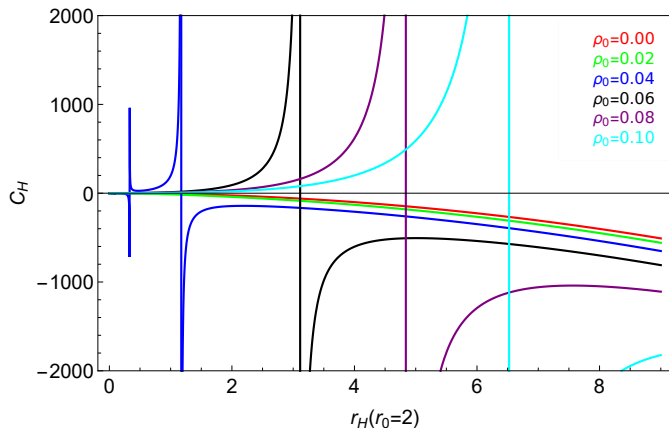
Heat capacity

- The heat capacity of the black hole is defined by

$$C_H = \frac{\partial M}{\partial T} = - \frac{6\pi r_H^2(r_0 + r_H) \left\{ 3(r_0 + r_H)^2 + 16\pi\rho_0 r_0^3 \sqrt{r_H(r_0 + r_H)} \right\}}{9(r_0 + r_H)^3 - 256\pi^2 \rho_0^2 r_0^6 r_H - 24\pi\rho_0 r_0^3 (r_0 - 2r_H) \sqrt{r_H(r_0 + r_H)}}.$$



Heat capacity

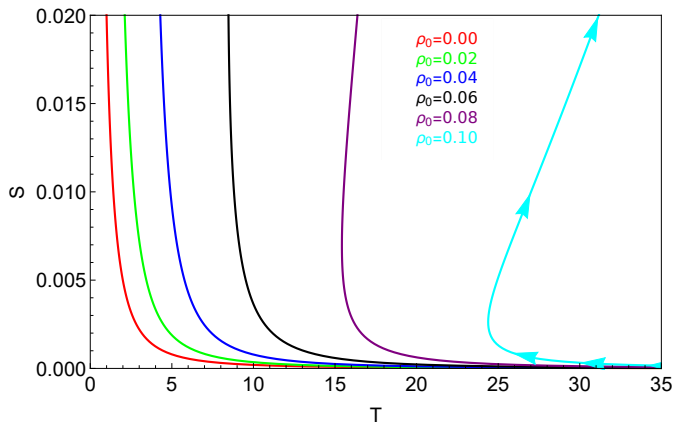


- At $\rho_0 = 0.02$, C_H is always negative.
- An intermediate sized BH is stabilized. *In (1, 4, 0) model, only small BHs are stable.*

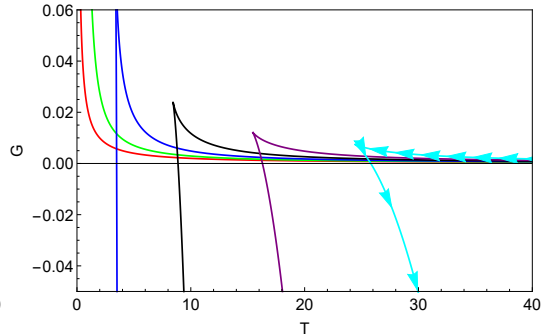
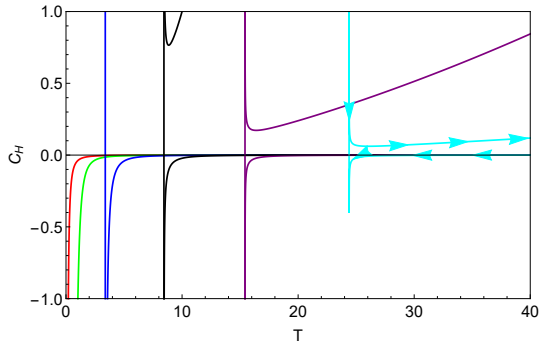
Thermodynamics

- The Gibb's free energy of the black hole is defined by

$$G = M - TS = e^{\frac{32}{3}\pi\rho_0 r_0^2 \left(1 + \frac{r_0}{r_H}\right)^{-1/2}} r_H \left[\frac{1}{4} - \frac{4}{3}\pi\rho_0 r_0^3 \sqrt{\frac{r_H}{(r_H + r_0)^3}} \right].$$



Gibbs free energy



- Inflection points signal second-order phase transition.

Null geodesics

- We consider radial null geodesics on the equatorial plane ($\theta = \pi/2$)

$$\dot{r}^2 + V_{eff} = \frac{1}{b^2},$$
$$V_{eff} = \frac{f + F}{r^2} = \frac{1}{r^2} \left[e^{\frac{32}{3}\pi\rho_0 r_0^2 \left(1 + \frac{r_0}{r}\right)^{-\frac{1}{2}}} - \frac{r_s}{r} \right].$$

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Photon radius

$$\frac{d}{dr} V_{\text{eff}}(r_{\text{ph}}) = 0,$$

$$V_{\text{eff}}(r_{\text{ph}}) = \frac{1}{b_c^2}.$$

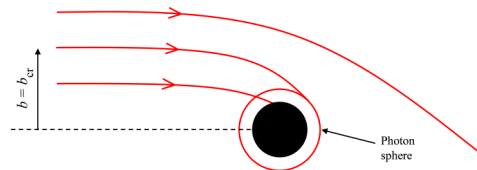


Figure: 1905.06615

Null geodesics

- The first condition yields (for small ρ_0)

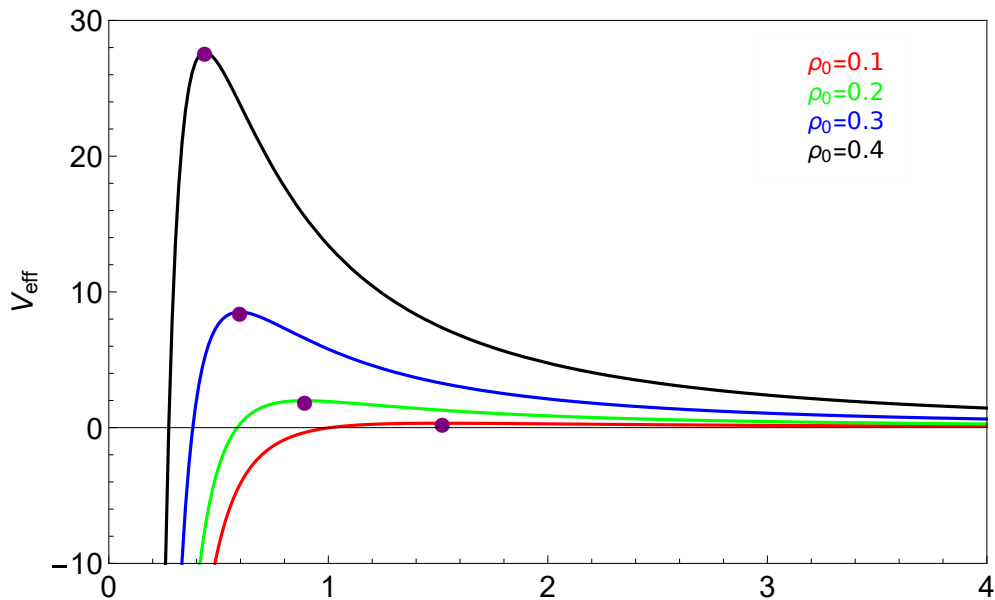
$$0 = \frac{3r_s - 2r_{ph}}{r_{ph}^4} - \frac{16\pi r_0^2}{3r_{ph}^4} (4r_{ph} + 3r_0) \left(\frac{r_{ph}}{r_{ph} + r_0} \right)^{3/2} \rho_0 + \mathcal{O}(\rho_0)^2.$$

- The second condition gives

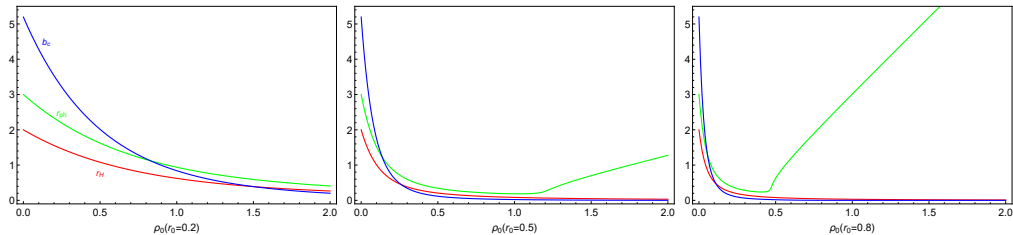
$$b_c = \pm \frac{r_{ph}^{3/2}}{\sqrt{r_{ph} e^{32\pi r_0^2 \rho_0 \left(\frac{r_{ph}}{r_{ph} + r_0} \right)^{1/2}} - r_s}}.$$

- At $\rho_0 = 0$, $r_{ph} = \frac{3}{2}r_s$ and $b_c = 3\sqrt{3}$ agree with the Schwarzschild case.

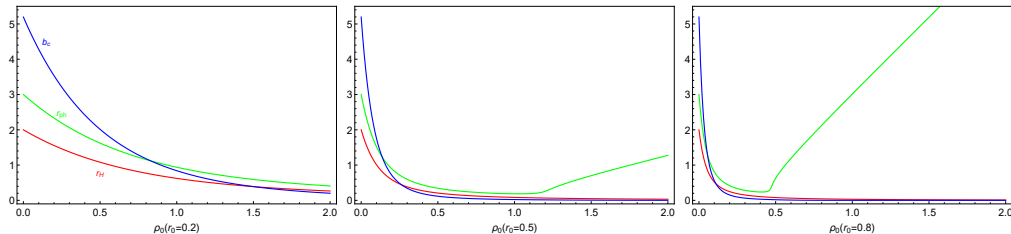
Null geodesics



Null geodesics



Null geodesics



- For small $\rho_0 r_0^3$, photon maintains its circular orbit closer to the black hole.
- At sufficiently large $\rho_0 r_0^3$, we can show that $r_{ph} \sim 8\pi \rho_0 r_0^3$.

- As observed by a distant observer, the black hole's shadow radius is given by⁶

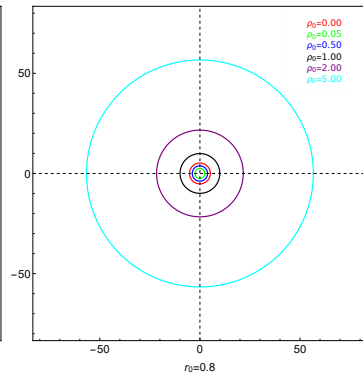
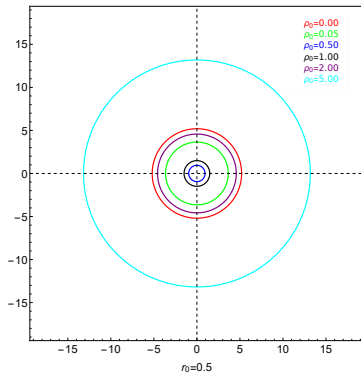
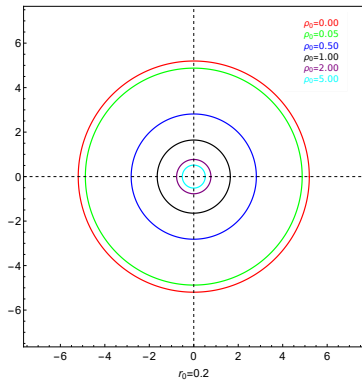
$$R_s = b_c \sqrt{g_{rr}(r \rightarrow \infty)} \approx \sqrt{\frac{r_{ph}^2}{g_{rr}(r \rightarrow \infty)}} e^{\frac{16}{3} \pi \rho_0 r_0^2}.$$

- For small $\rho_0 r_0^3$, it is

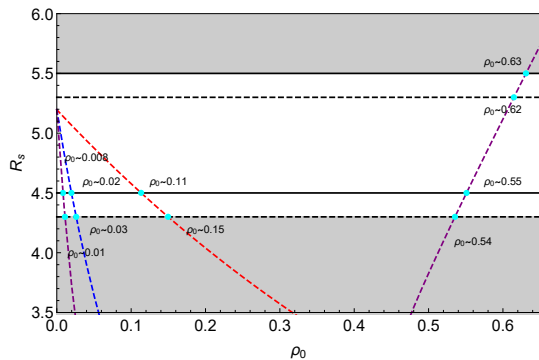
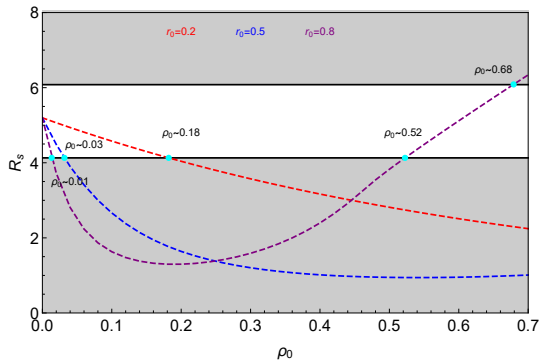
$$R_s \approx \frac{\sqrt{3}}{2} \left[3r_s + 16\pi\rho_0 r_0^3 \left(1 - 2\frac{r_s}{r_0} \right) \right].$$

⁶Pantig Phys. Dark Univ. 24, 2024, 101550

Shadow radius



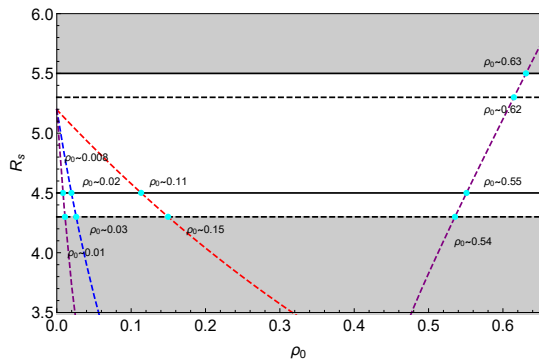
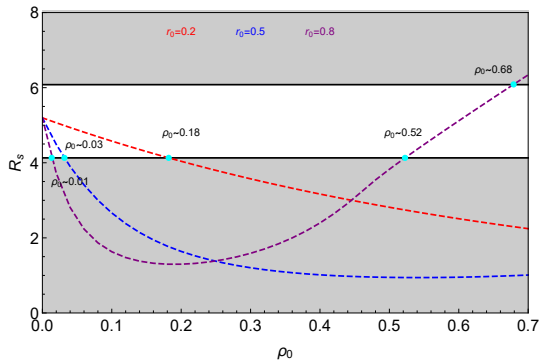
Constrains from M87 and Sgr A*



- Left: M87 from the EHT
- Right: Sgr A* from the Keck observatory (solid lines) and the VLTI (dashed lines).

⁷Vagnozzi et.al. CQG 40 (2023)

Constraints from M87 and Sgr A*



- Left: M87 from the EHT
- Right: Sgr A* from the Keck observatory (solid lines) and the VLT (dashed lines).
- Shadow radius is mildly sensitive to spin.⁷

⁷Vagnozzi et.al. CQG 40 (2023)

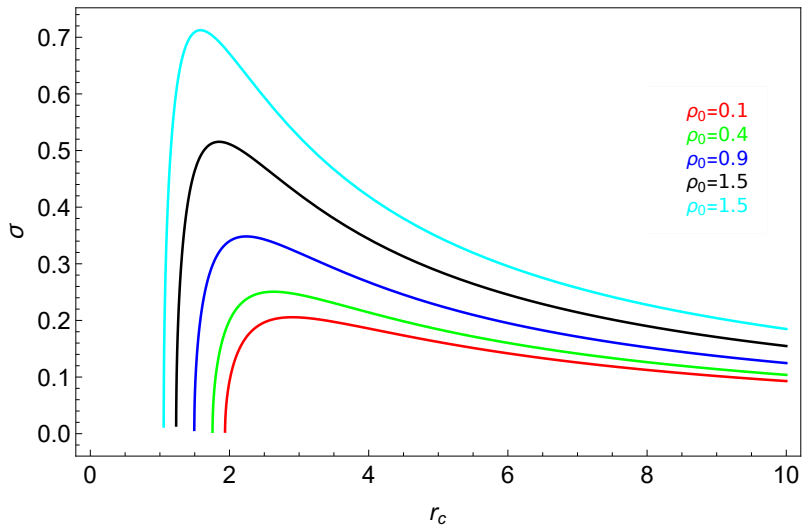
Lyapunov exponent

- It measures how fast two neighboring photon trajectories converge or diverge over time.

$$\sigma = \sqrt{\frac{(e^{\Delta} r_c - r_s) [(2r_c^2 - 4M_{DM}^2)(r_0 + r_c) + M_{DM} r_c (r_0 + 4r_c)]}{2e^{-\Delta} (r_0 + r_c) r_c^5}}. \quad (5)$$

- $\Delta \equiv \frac{4M_{DM}(r_0+r_c)}{r_0 r_c}$ and $V(r_c) = V'(r_c) = 0$.
- When $\Delta = 0$, $\sigma = \frac{1}{3\sqrt{3}}$ agrees with the Schwarzschild case.

Lyapunov exponent



Scalar perturbations and the Eikonal approximation

- The radial wave equation is,

$$\frac{d^2 R}{dr_*^2} + \left[\omega^2 - \frac{h}{r^2} \{ \ell(\ell + 1) + rh' \} \right] R = 0. \quad (6)$$

- $h \equiv f + F$. For $\ell \rightarrow \infty$, the quasinormal normal frequencies are obtained

$$\omega_{QNM} = \ell \sqrt{\frac{h_c}{r_c^2}} - i \left(n + \frac{1}{2} \right) \sigma. \quad (7)$$

- **Real part:** angular velocity of unstable circular orbit and shadow radius
- **Imaginary part:** Lyapunov exponent, since $\sigma > 0$ then $\text{Im}(\omega) < 0$.

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- Real part:** angular velocity of unstable circular orbit and shadow radius
- Imaginary part:** Lyapunov exponent, since $\sigma > 0$ then $\text{Im}(\omega) < 0$.
- For small $\rho_0 r_0^3$, it agrees with the Schwarzschild limit

$$\omega_{QNM} = \frac{\ell}{3\sqrt{3}M} - \frac{i \left(n + \frac{1}{2} \right)}{3\sqrt{3}M}. \quad (8)$$

Conclusions

- We **construct** spherically symmetric black hole immersed in Dehnen DM profile $(1, 4, 3/2)$.
- The solution possesses regular event horizon and essential singularity.
- Thermodynamically stable configurations **are possible** (provided the BH is neither too small nor too large).
- The shadow radius initially *decreases* with ρ_0 then *increases* linearly with ρ_0 .
- Instability of null circular orbit *increases* with ρ_0 .

What's left to be done?

- Gravitational perturbation
- Optical appearance with luminous accreting matter
- Rotating solution

The End