

Supermassive Black Holes Formations via Bondi Accretion of Self-Interacting Dark Matter

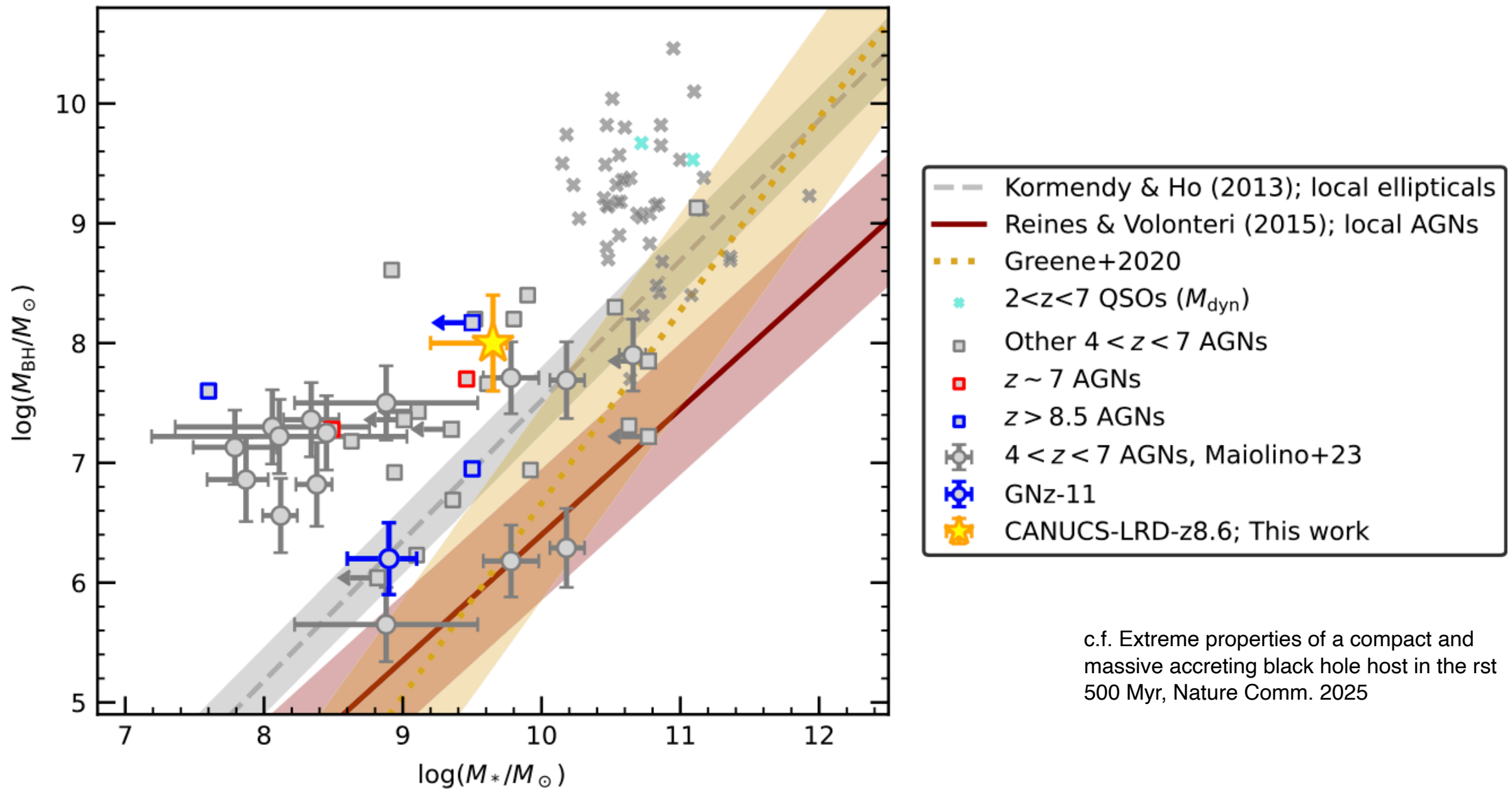
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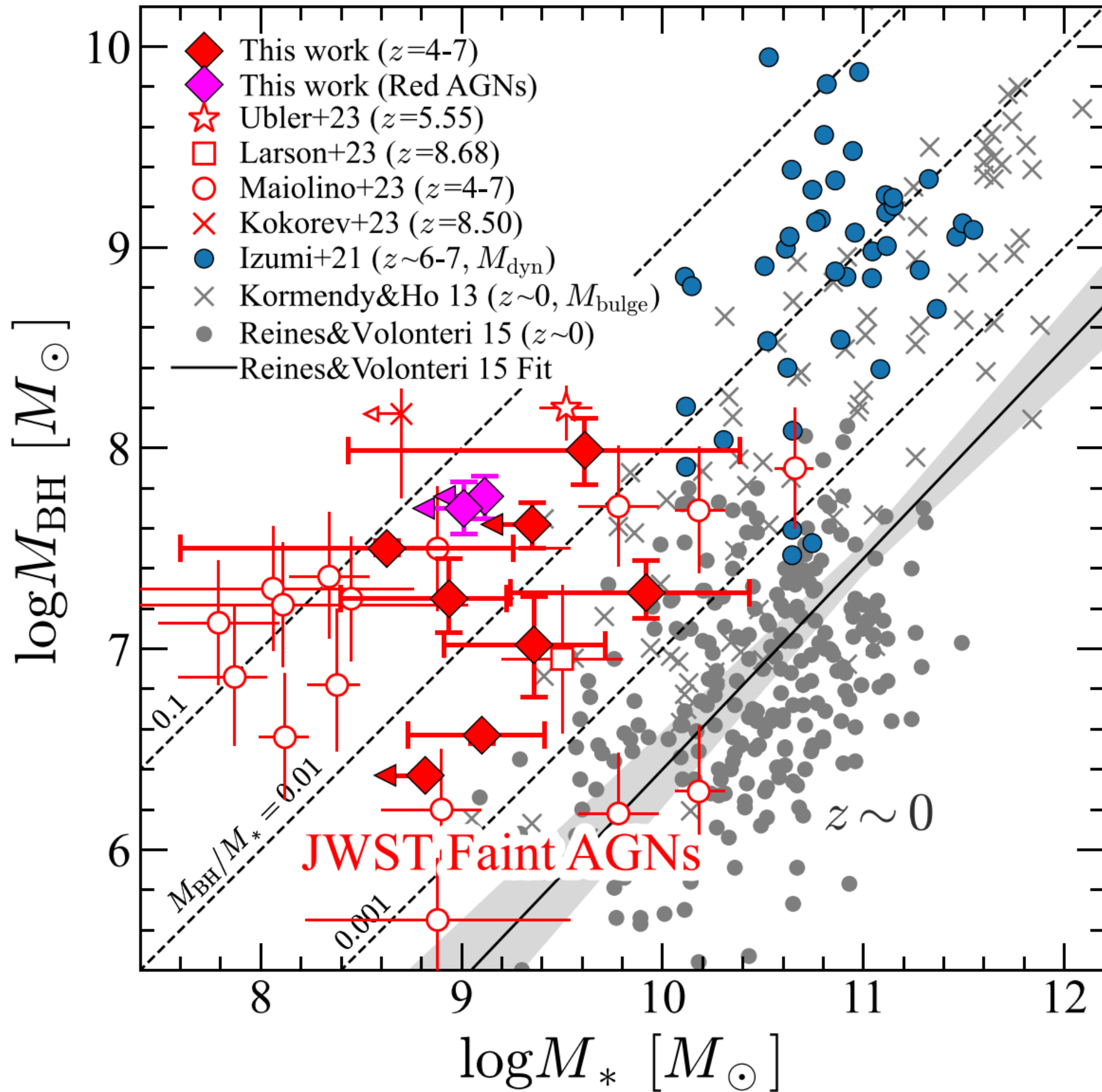
Based on a work with Chian-Shu Chen (Tamkang U) on ApJL (2026/04)

JWST's Early AGN Census

- AGN = an active galactic nucleus whose luminosity is dominated by **accretion onto a compact central object (BH)**:
 1. Low luminosity $\rightarrow$ IMBHs ($10^3 - 10^5 M_{\odot}$)
 2. Moderate luminosity $\rightarrow 10^5 - 10^7 M_{\odot}$
 3. Quasars $\rightarrow$ SMBH ($10^8 - 10^{10} M_{\odot}$)
- A set of coordinated programs—CEERS, JADES, PRIMER, and related ERS/GO programs aim to identify AGNs at $z \geq 5$ using JWST's unprecedented sensitivity in the near-IR and mid-IR.
- Observed results imply:
 - AGN density is underestimated by early surveys.
 - $M_{\text{BH}}/M_{\text{stellar}}$ at high $z \geq 6$ is significantly higher (10-100 times) than the local relation $\Rightarrow$ Early SMBH growth is more common than expected.

Outpaced Growth of BH over host galaxy





c.f. A JWST/NIRSpec First Census of Broad-line AGNs at $z=4-7$: Detection of 10 Faint AGNs with $M_{\text{BH}} \sim 10^6 - 10^8 M_{\odot}$ and Their Host Galaxy Properties, ApJ 2023.

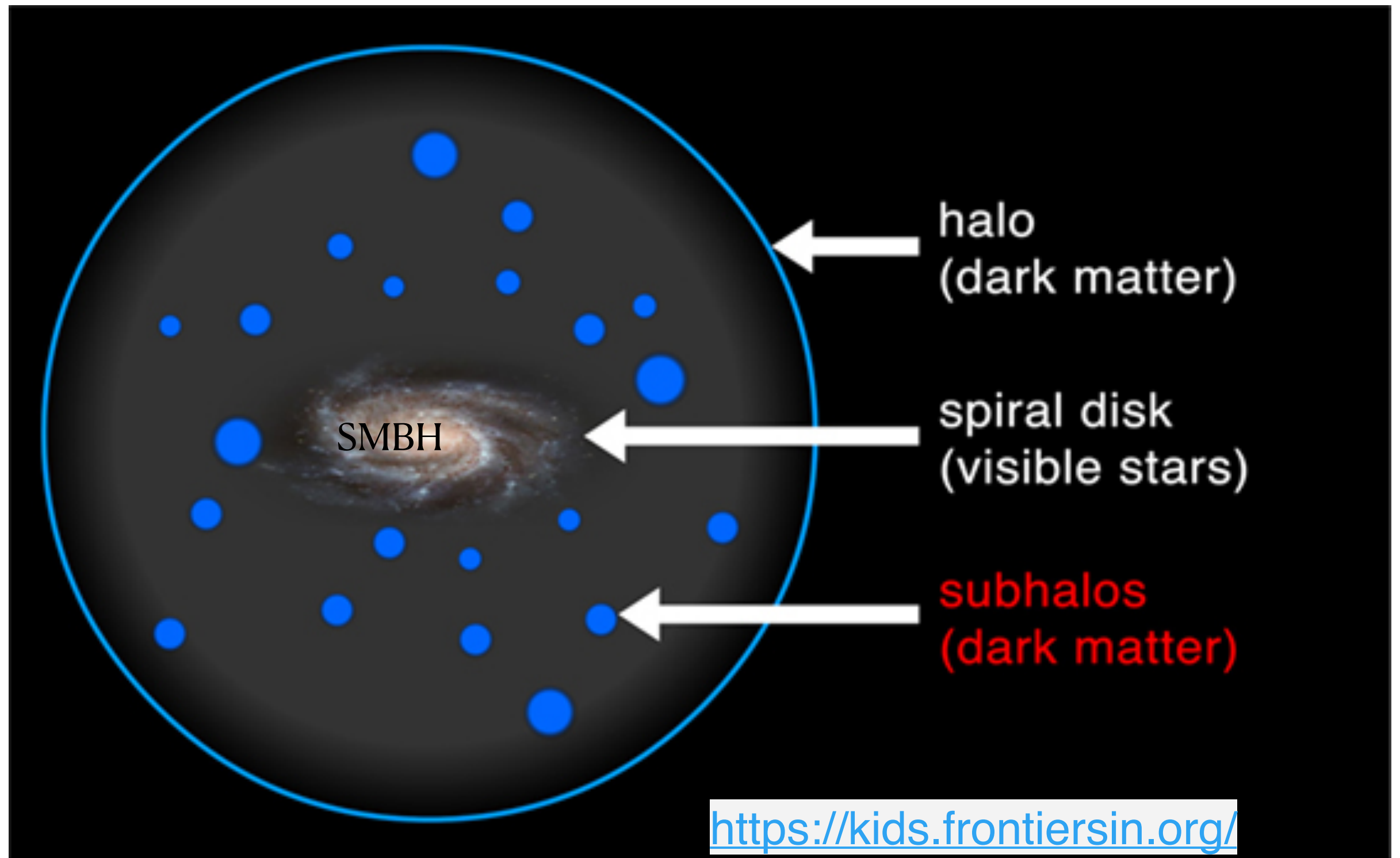
Eddington vs Super-Eddington

- The most common growth of a central BH is by Eddington accretion: the outward radiation pressure balances the inward pull of gravity.
 1. Eddington limit $\dot{M} = \Gamma_{\text{Edd}} M$, with $\Gamma_{\text{Edd}} = 4\pi G_N m_p / \eta c \sigma_T \approx 4.5 \times 10^{-8} \text{yr}^{-1}$.
 2. The Eddington limit is the maximal Eddington accretion rate causing exponential growth of the central mass, i.e., about 45 Myr for an e-fold growth.
 3. To grow from a stellar seed ($10 - 100 M_\odot$) to $10^9 M_\odot$ requires about 16-18 e-folds, or 0.7-0.8 Gyr.
- However, we see SMBH by $z > 7$, or 0.5-0.7 Gyr. Or, by Eddington accretion, a stellar mass BH can only grow to $10^5 - 10^6 M_\odot$.
- Thus, to explain JWST's results, we need to consider some super-Eddington mechanism or with heavier seeds.

Bondi accretion

- Eddington accretion is the dominant accretion mechanism, but not the only one.
- Another common spherical accretion mechanism is Bondi accretion, driven by the gravitational pull of the surrounding hydrodynamic medium within the Bondi radius $r_B = \frac{2M}{a_\infty^2}$, where the infall speed equals the sound speed.
- (Non-relativistic) Bondi accretion rate $\dot{M}_{\text{Bondi}} = 16\pi\gamma(\text{EoS})\rho \frac{(GM)^2}{a^3} := KM^2$ with EoS = equation of state for the surrounding medium.
- Note that $\dot{M}_{\text{Bondi}}/\dot{M}_{\text{Edd}} \sim \mathcal{O}(M)$. Thus, Bondi accretion will start to dominate over Eddington's if $M \geq M_\star := \Gamma_{\text{Edd}}/K$. However, in practice,
 - A. For the intergalactic medium ($n \sim 10^{-6} \text{ cm}^{-3}$, $a \sim 12 \text{ km/s}$), $\dot{M}_{\text{Bondi}} \sim 10^{-12} \dot{M}_{\text{Edd}}$
 - B. For the molecular cloud ($n \sim 100 \text{ cm}^{-3}$, $a \sim 0.3 \text{ km/s}$), $\dot{M}_{\text{Bondi}} \sim 10^{-3} \dot{M}_{\text{Edd}}$
 - C. For cold dark matter (CDM), $\dot{M}_{\text{Bondi}} \ll \dot{M}_{\text{Edd}}$ as $\rho_{\text{CDM}} \ll \rho_{\text{baryon}}$ in the halo core.
 - D. For relativistic dark matter of sound speed $\sim \mathcal{O}(c)$, $\dot{M}_{\text{Bondi}} \sim 1/c^3 \ll \dot{M}_{\text{Edd}}$.

Galactic Structures with DM & SMBH



Cold Dark Matter

- The most popular model is the CDM (slow-moving, non-relativistic, non-(EM-)interacting, or simply “uncharged dust”).
- It matches CMB anisotropies and BAO in the Λ CDM model, predicts large-scale structure (galaxy clusters, cosmic web), and reproduces hierarchical structure formation (small halos $\rightarrow$ galaxies $\rightarrow$ clusters).
- It, however, faces tension on galactic and sub-galactic scales — small-scale issues:
 1. It predicts cuspy core profiles ($\sim 1/r$) in dwarf galaxies, not the observationally flat core.
 2. It predicts too many dwarf satellites around galaxy-sized halos as compared with observations.
 3. It predicts subhalos that are too dense to host the observed faint dwarfs.
 4. It lacks model diversity to yield widely varying rotation curves for a given halo mass.

Self-Interacting Dark Matter (SIDM)

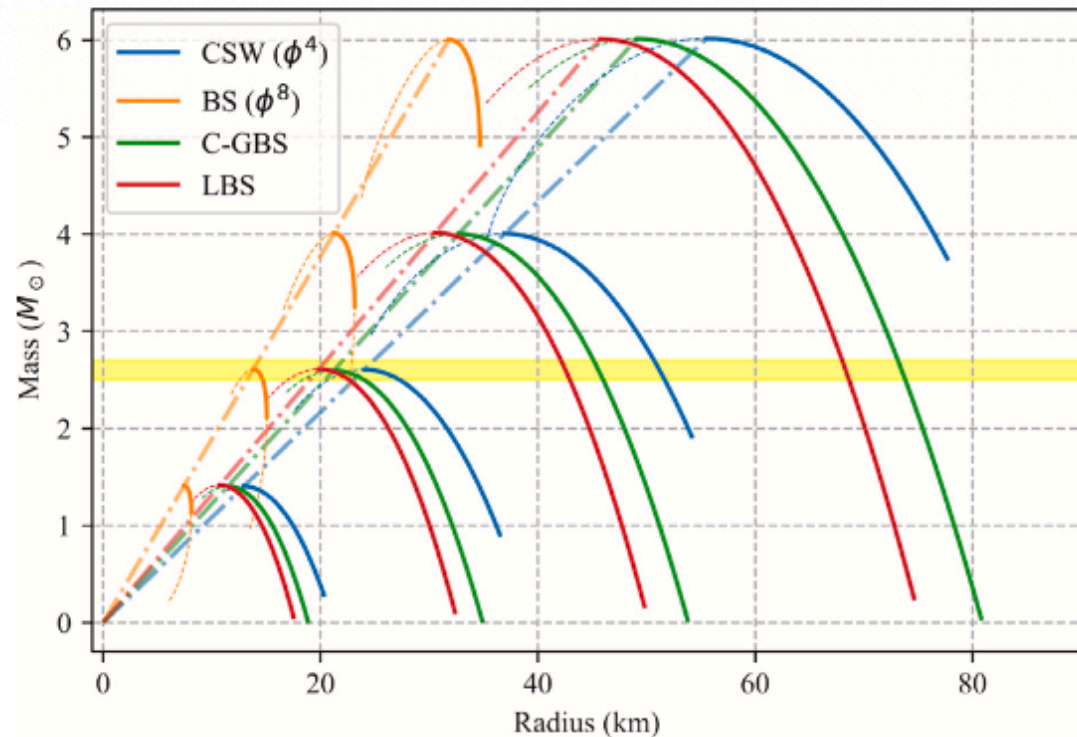
- Resolving the above issues requires two scales (cosmic and galactic) in DM model.
- The simplest way is to introduce the self-interaction characterized by σ/m .
- Resolution of the small-scale issues requires $\sigma/m \sim 0.1 - 1 \text{ cm}^2 \text{ g}^{-1}$ (cf. Spergel & Steinhardt 2000).
- This constraint still allows for a wide variety of SIDM models.
- The simplest SIDM is the ϕ^4 model: $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$ with $\sigma_{\phi^4} = \frac{\lambda^2}{64\pi m^2}$.
- It was first proposed by Colpi et al '86 to obtain bosonic stars as a TOV configuration with mass comparable with $M_{\text{Chandrasekhar}} := M_{\text{pl}}^3/m^2$, i.e., $M_{\text{TOV}} \sim \lambda^{1/2} M_{\text{Chandrasekhar}}$, due to self-repulsive force, instead of fermionic degeneracy pressure.
- Most of the small-scale issues should be considered in the hydrodynamic regime, for which the SIDM should be characterized by its equation of state (EoS) $p(\rho)$ in the self-gravitating phase.

EoS of Self-Gravitating SIDM

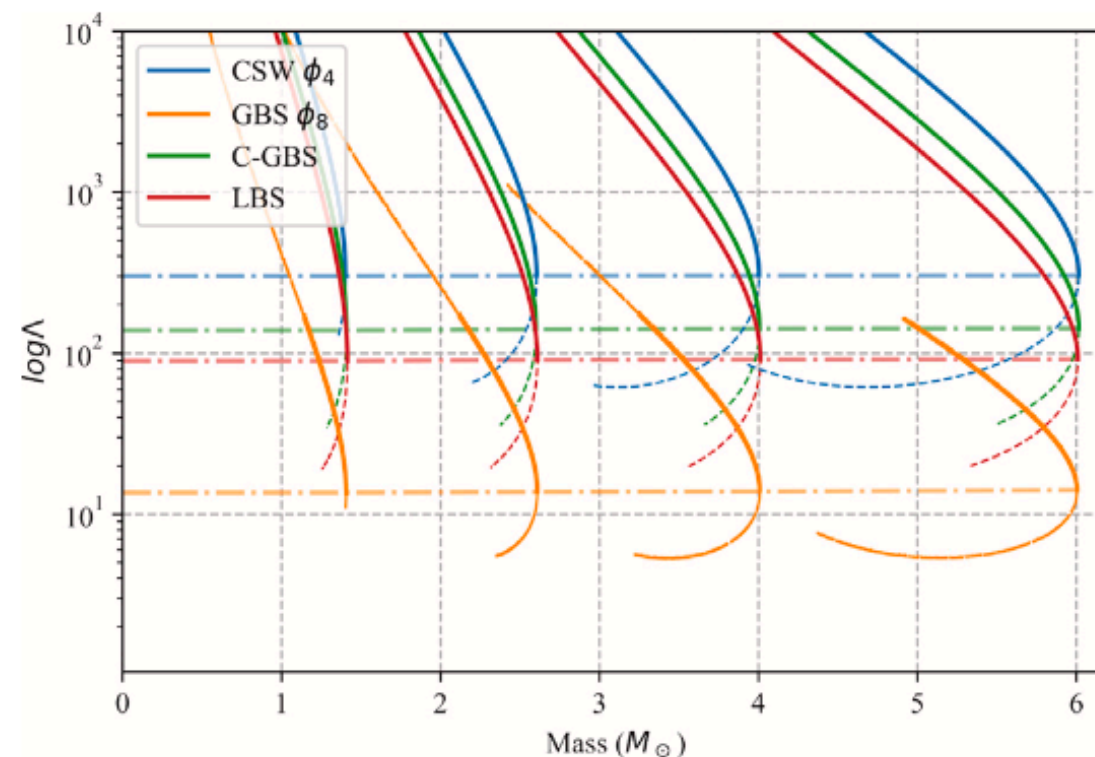
- EoS depends on the phase of matter, e.g., H₂O has different EoS for ice, water, and steam phases.
- For astrophysical or cosmological considerations, we consider the EOS of SIDM with $\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi - \frac{1}{2}m^2|\phi|^2 - U(|\phi|)$ in the self-gravity phase.
- Hold the SIDM fluid in a gravitational background $ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2d\Omega$. (Cf. Zhang et al, 2023)
 1. Set $\phi(t, r) = e^{-i\omega t}\Phi(r)$ to derive $T^\mu_\nu = \text{diag}(-\rho, p, p_\perp, p_\perp)$ with $x = mr$, $\Omega = \omega/m$, $\sigma = \Phi/M_{\text{pl}}$. The results are $\rho, p = m^2M_{\text{pl}}^2\left[\frac{1}{2}\left(\frac{\Omega^2}{B} + 1\right)\sigma^2 + \frac{|\partial_x\sigma|^2}{2A} \pm \frac{U(M_{\text{pl}}\sigma)}{m^2M_{\text{pl}}^2}\right]$, $p_\perp = p - \frac{m^2M_{\text{pl}}^2|\partial_x\sigma|^2}{A}$.
 2. Further constraint by scalar field equation: $\partial_x\left(x^2\sqrt{\frac{B}{A}}\partial_x\sigma\right) + x^2\sqrt{AB}\left[\left(\frac{\Omega^2}{B} - 1\right)\sigma - \frac{U'(M_{\text{pl}}\sigma)}{m^2M_{\text{pl}}}\right] = 0$.
 3. Take the hydrodynamical limit $\Lambda := \frac{\sigma_*^2}{\sigma^2} = \frac{x^2}{x_*^2} \gg 1$, so that $\frac{U(M_{\text{pl}}\sigma)}{m^2M_{\text{pl}}^2} \rightarrow \frac{U_*(\sigma_*)}{\Lambda}$, and $\left(\frac{\Omega^2}{B} - 1\right)\sigma_* = U'_*(\sigma_*)$, $\rho = \rho_0\left[\frac{1}{2}\sigma_*U'_*(\sigma_*) + U_*(\sigma_*) + \sigma_*^2\right]$, $p = p_\perp = \rho_0\left[\frac{1}{2}\sigma_*U'_*(\sigma_*) - U_*(\sigma_*)\right]$.
- For ϕ^4 -SIDM, $\frac{\rho}{\rho_B} = \frac{3p}{\rho_B} + 4\sqrt{\frac{p}{\rho_B}}$ with $\rho_B := \frac{3m^4}{2\lambda} = \frac{3.48}{\lambda}\left(\frac{m}{\text{GeV}}\right)^4 \times 10^{20} \text{ kg m}^{-3}$.

Dark Stars as BH Mimickers Cf. Zhang et al, 2023

- The Dark Stars of SIDM can be compact enough to mimic black holes and help to resolve the mass gap of black holes: absence of astrophysical-origin BHs in the mass range of $2.5 - 5M_{\odot}$ and $50 - 130M_{\odot}$.
- TOV equations $\frac{dp}{dr} = -(\rho + p)\frac{M + 4\pi r^3\rho}{r(r - 2M)}$, $\frac{dM}{dr} = 4\pi r^2\rho$ are invariant under the scalings:
 $\rho \rightarrow k_s^{-2}\rho$, $p \rightarrow k_s^{-2}p$, $M \rightarrow k_s M$, $r \rightarrow k_s r$.



Mass-Radius



TLN-Mass

ϕ^4 -SIDM Self-Gravitating Hydrodynamics

- ϕ^4 -SIDM is a relativistic fluid in the self-gravitating regime with EoS $\frac{\rho}{\rho_B} = \frac{3p}{\rho_B} + 4\sqrt{\frac{p}{\rho_B}}$.
 1. Adiabatic sound speed square $a^2 := \left(\frac{\partial p}{\partial \rho}\right)_{\text{adiabatic}} = \frac{1}{3} \left(1 - \frac{1}{\sqrt{1 + 3\rho/4\rho_B}}\right) \leq 1/3$.
 2. Energy conservation relates mass density ρ_0 and energy density ρ by $\left(\frac{\partial \rho}{\partial \rho_0}\right)_{\text{adiabatic}} = \frac{p + \rho}{\rho_0} := h$, i.e., enthalpy equation.
 3. The above twos yield $\frac{\rho_0}{\rho_{0,\infty}} = \frac{a^2}{a_\infty^2} \sqrt{\frac{(1 - a^2)(1 - 3a_\infty^2)^3}{(1 - a_\infty^2)(1 - 3a^2)^3}}$, and Bondi radius $r_B := \frac{2M}{a_\infty^2}$.
- To be free of the small-scale issue requires $0.1 \text{ cm}^2 \text{ g}^{-1} \lesssim \frac{\sigma_{\phi^4}}{m} = \frac{\lambda^2}{64\pi m^3} \lesssim \mathcal{O}(1) \text{ cm}^2 \text{ g}^{-1} \implies 30 \left(\frac{m}{\text{GeV}}\right)^{3/2} \lesssim \lambda \lesssim 90 \left(\frac{m}{\text{GeV}}\right)^{3/2}$
- For simplicity, setting $\lambda \simeq 50(m/\text{GeV})^{3/2}$, it then gives $\rho_B := \frac{3m^4}{2\lambda} = 2.2 \times 10^{-4} \left(\frac{m}{\text{eV}}\right)^{5/2} \text{ kg m}^{-3}$.

Bondi Accretion of ϕ^4 -SIDM

- Due to the relativistic nature of matter, Bondi accretion should also be relativistic and governed by the Euler and continuity equations, i.e., $\nabla_\nu T^{\mu\nu} = 0$ and $\nabla_\mu(\rho_0 u^\mu) = 0$.
- From relativistic Bernoulli equation $\left(\frac{p+\rho}{\rho_0}\right)^2 \left(1 + u^2 - \frac{2M}{r}\right) = \left(\frac{p_\infty + \rho_\infty}{\rho_{0,\infty}}\right)^2$, we can derive the flow equation of poloidal velocity: $\frac{du_{\text{pol}}}{d\lambda} = \frac{N(r; u_{\text{pol}}, h)}{D(r; u_{\text{pol}}, h)}$ with $u_{\text{pol}}^2 := g_{rr}(u^r)^2$.
- Sonic horizon defined by $D(r_s) = 0$, along with its smoothness by $N(r_s) = 0$ give $u_s^2 = \frac{a_s^2}{1 + 3a_s^2} = \frac{M}{2r_s}$.
- Combining the above gives $a_\infty^2 = \frac{a_s^2(9a_s^2 - 1)}{2 - 3a_s^2 + 9a_s^4}$, implying $1/9 \leq a_s^2 \leq 1/3$, $0 \leq a_\infty^2 \leq 1/3$.
- Moreover, the Bondi accretion rate turns to $\dot{M}_{\text{Bondi}} = 4\pi r_s^2 \rho_{0,s} u_s = 2\pi \frac{(1 + 3a_s^2)^3}{a_s^3 (9a_s^2 - 1)} M^2 \rho_{0,\infty}$.
 - The Bondi accretion rate seems divergent when $a_s \rightarrow 1/3$.
 - This motivates our work on the formation of SMBHs via Bondi-dominated accretion of SIDM.

Universal Bondi Accretion

Consider the Bondi accretion of ϕ^4 -SIDM in the critical regime with $\epsilon := a_s^2 - \frac{1}{9} \ll 1$.

1. It gives $r_s \simeq 24M$ and $\dot{M}_{\text{Bondi}} = \frac{128\pi}{9} \frac{1}{\epsilon} M^2 \rho_{0,\infty}$ (seemingly divergent!)
2. However, using the former relations among $\rho_0, \rho_{0,\infty}, \rho_B, a_s$ and ϵ , we obtain the following relations:

$$\epsilon \simeq \frac{2}{9} \frac{\rho_\infty}{\rho_B} \simeq \frac{16}{27\sqrt{3}} \frac{\rho_{0,\infty}}{\rho_{0,s}} \simeq \frac{16}{9} a_\infty^2.$$
3. These give $r_B \simeq \frac{9M}{8} \epsilon^{-1}$, which are large enough to cover the whole halo, lift the supply issue.
4. Moreover, $\rho_\infty \simeq \rho_{0,\infty} := \rho_{\text{DM}}$, $\because p_\infty \sim \mathcal{O}(\epsilon^2)$. It then gives $\epsilon \simeq \frac{2}{9} \frac{\rho_{\text{DM}}}{\rho_B}$ and $r_B \simeq \frac{81}{16} \frac{\rho_B}{\rho_{\text{DM}}} M$.
5. Finally, it gives a universal Bondi accretion rate $\dot{M}_{\text{Bondi}} = 64\pi\rho_B \frac{G^2 M^2}{c^3} := KM^2$, which is independent of $\rho_{0,\infty}$.

○ As choosing $\lambda \simeq 50(m/\text{GeV})^{3/2}$, then $\dot{M}_{\text{Bondi}} = 1.45 \times 10^{13} \left(\frac{M}{M_\odot} \right)^2 \left(\frac{m}{\text{GeV}} \right)^{5/2} \frac{M_\odot}{\text{yr}}$.

○ This relates the astrophysical Bondi rate to the microscopic mass of a scalar particle.

Constraint by Dark Matter Halos

- Typical ambient DM density around the halos: $\rho_{\text{DM}} = 5 \times 10^{-22} (1+z)^3 \Delta_{\text{vir}} \text{ kg m}^{-3}$.
 - The typical virial overdensity $\Delta_{\text{vir}} \simeq 300$ from spherical-collapse simulation.
 - Then, the Bondi radius $r_B(z) = \frac{81}{16} \frac{\rho_B}{\rho_{\text{DM}}} M = 3.55 \times 10^2 (1+z)^{-3} \left(\frac{m}{\text{eV}} \right)^{5/2} \left(\frac{M(z)}{M_\odot} \right) \text{ pc}$, and
 - $\epsilon(z, m) = 1.52 \times 10^{-16} \frac{(1+z)^3}{(m/\text{eV})^{5/2}}$.
- Ensuring a viable super-Eddington mechanism requires
 $r_B(z=7) \geq \text{core radius} \simeq 6 \text{ kpc} \implies m \geq 9.44 \times 10^{-3} \text{ eV} \simeq 10^{-2} \text{ eV}.$

Transition Mass $M_\star = \Gamma/K$ at Which Relativistic SIDM Bondi Inflow
Overtakes Baryonic Eddington Growth

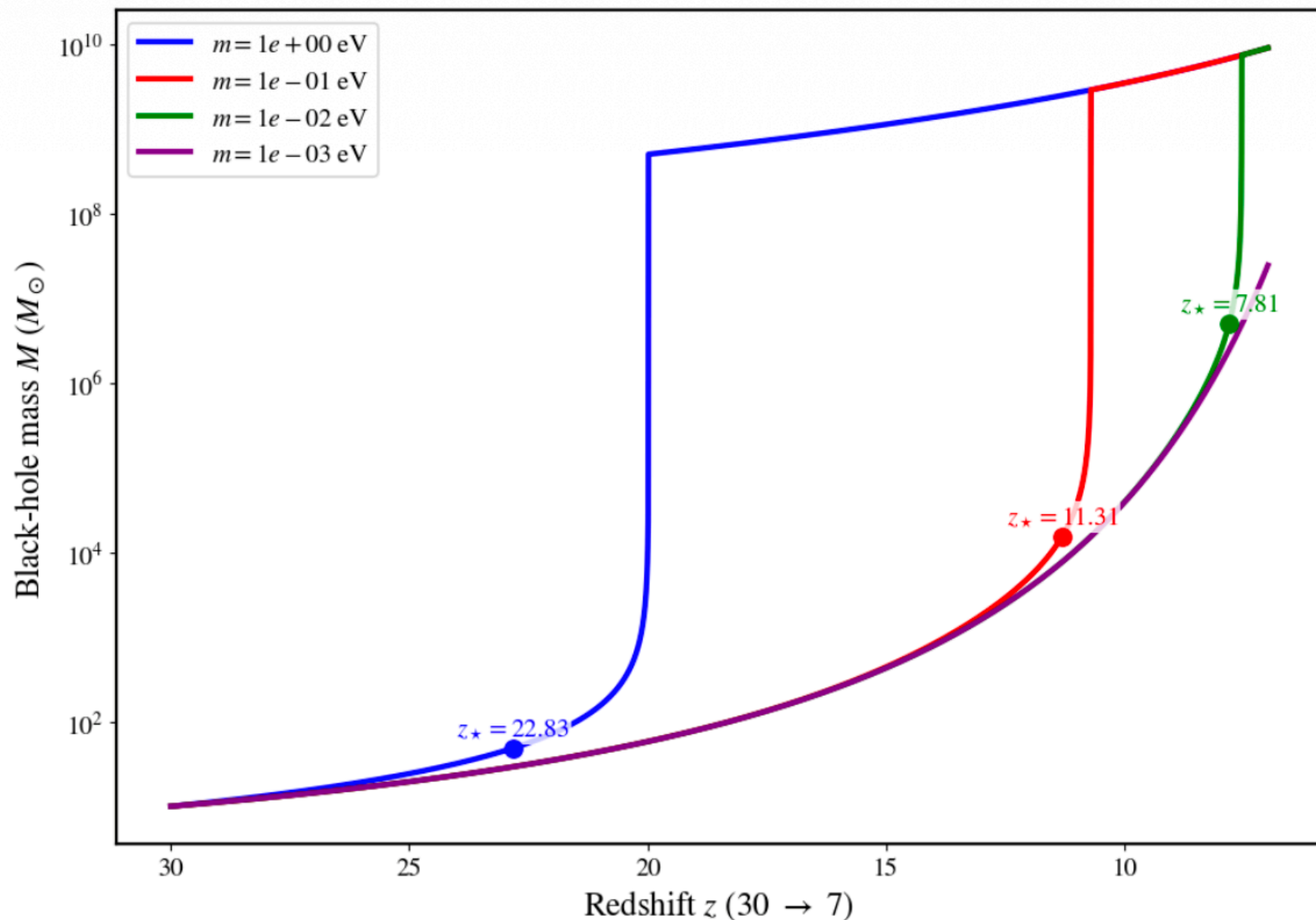
$m \text{ (eV)}$	$M_\star (M_\odot)$	$z_\star$	Growth Regime
10^{-3}	1.5307×10^9	none	Bondi never dominates
10^{-2}	4.8403×10^6	7.8	Bondi dominates around $z \sim 11$
10^{-1}	1.5307×10^4	11.31	Early Bondi onset
1	4.8403×10^1	22.83	Bondi dominates almost immediately

$z_\star$ = onset of Bondi-dominated phase

Bondi-aided Eddington Accretion

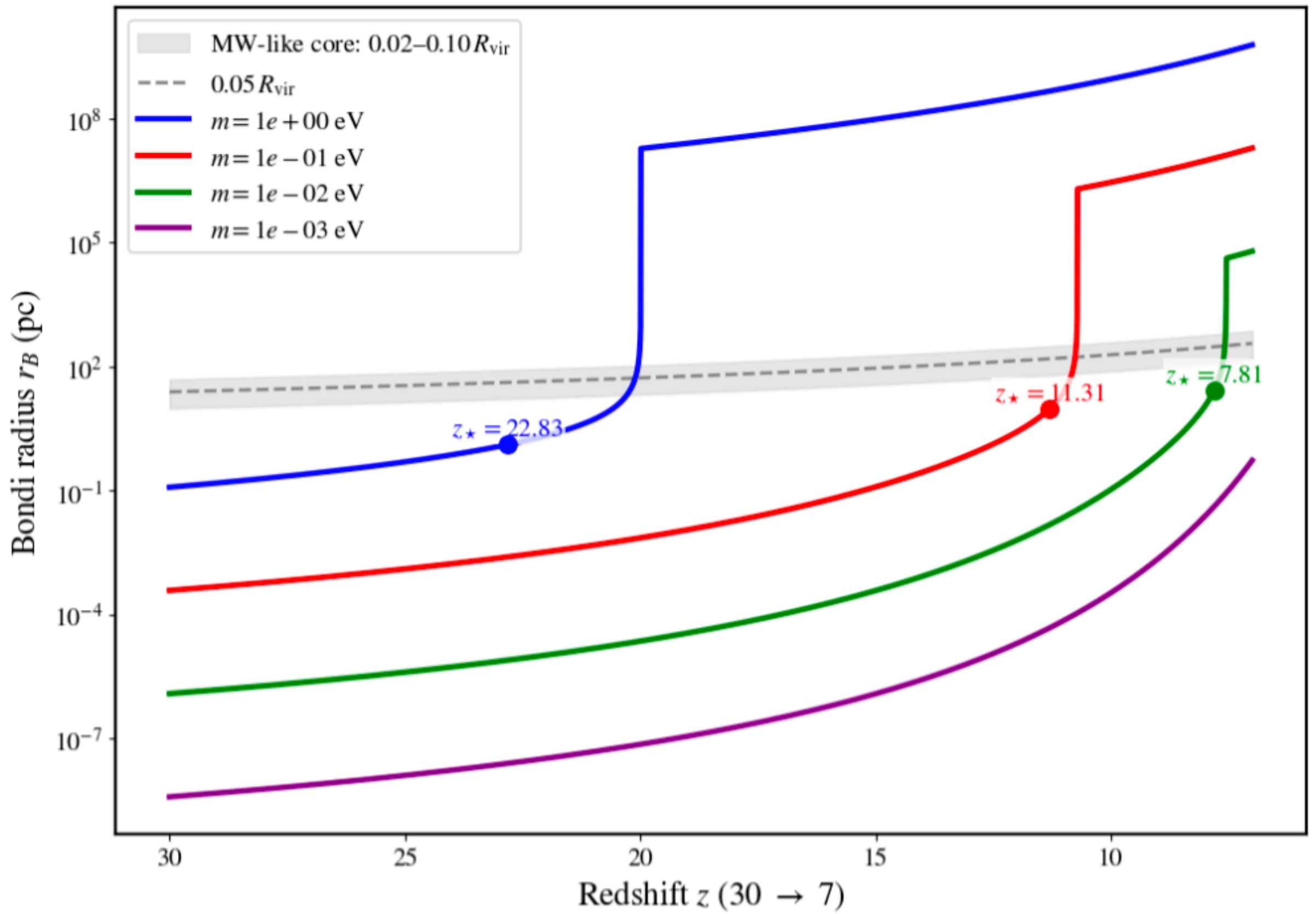
- Solution of $\dot{M} = \dot{M}_{\text{Edd}} + \dot{M}_{\text{Bondi}} = \Gamma M + KM^2$ takes a simple form

○ $M(z) = \frac{M_0 e^{\Gamma \tau(z)}}{1 - \frac{K}{\Gamma} M_0 (e^{\Gamma \tau(z)} - 1)}$ with $\int_z^{z_i} \frac{dz'}{(1+z')H(z')}$ and $H(z) = H_0 \sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_N}$.



Sharply exit when the inner core is exhausted!

$$H_0 = 67.4, \Omega_m = 0.315, \Omega_\Lambda = 0.685, \text{ and } \Omega_\gamma = 9.2 \times 10^{-5}$$



Short Summary

- Heavier SIDM gives a sharper growth and a lower $M_\star$, reaching heavier SMBHs.
- We have assumed a limited supply of inner-core SIDM for Bondi accretion.
- We see that Bondi accretion, once it dominates over the Eddington one, quickly engulfs most of the inner-core SIDM, then abruptly exits to re-enter the universal Eddington-dominated phase.
 - In the numerical plots, we further restrict the final BH mass to be about 90 % of the host halo's total mass, which is derived from the Press-Schechter formalism to be $M_{\text{ref}}|_{z=10} \simeq 5 \times 10^9 M_\odot$.
 - Our results indicate that SMBH formation can end around $z = z_B \simeq 25 - 10$, earlier than the observed time around $z \simeq 7$ by JWST.
 - The subsequent cosmic evolution of halo mass assembly through hierarchical mergers will scale the halo mass by $(1 + z)^{-\alpha}$, but not increase the mass of the central SMBH. This will reduce the mass ratio $M_{\text{BH}}(z_B)/M_{\text{halo}}(z)$ by $\left(\frac{1+z}{1+z_B}\right)^\alpha \simeq 0.3 - 0.03$ for $\alpha = 3, z = 7, z_B = 25 - 10$.
- Our environment-universal super-Eddington channel is distinct from other environmental enhancement proposals, such as catastrophic gas inflow in metal-poor halos or deep central potentials of fuzzy-DM solitons.

PBH Seeds to Mass Function at $z=7$

- Our mass-growth formula is universal and can be applied to any black hole at any time.
- We consider three PBH profiles of stellar masses:

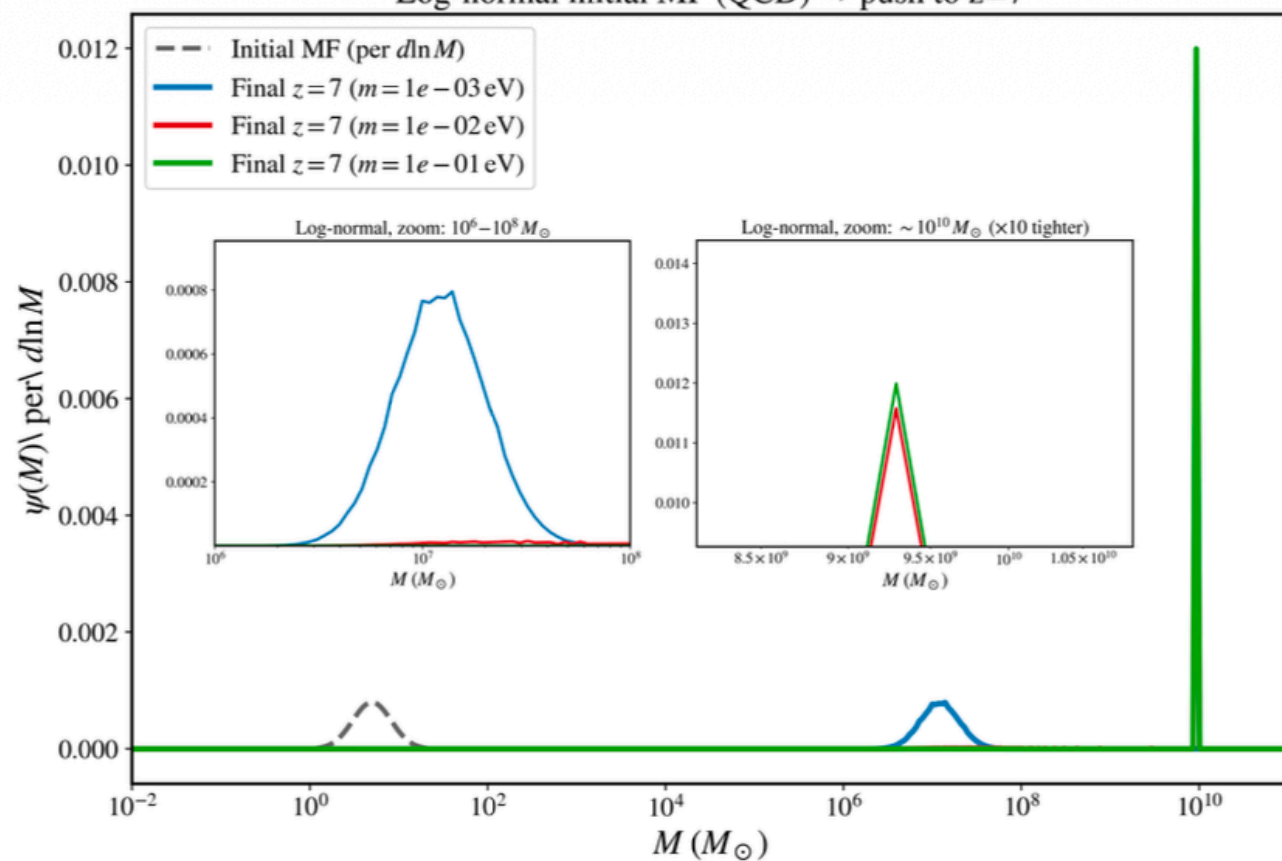
A. Lognormal: $\psi_0^{\text{LN}}(M_0) = \frac{f_{\text{PBH}}}{\sqrt{2\pi}\sigma M_0} \exp\left[-\frac{\left(\ln(M_0/M_c)\right)^2}{2\sigma^2}\right]$ with $M_c = 5M_\odot$, $\sigma = 0.5$.

B. Power law: $\psi_0^{\text{PL}}(M_0) = \frac{(1-\alpha)f_{\text{PBH}}}{M_{\text{max}}^{1-\alpha} - M_{\text{min}}^{1-\alpha}} M_0^{-\alpha}$ with $\alpha = 2$, $M_{\text{min}} = 0.1M_\odot$, $M_{\text{max}} = 100M_\odot$.

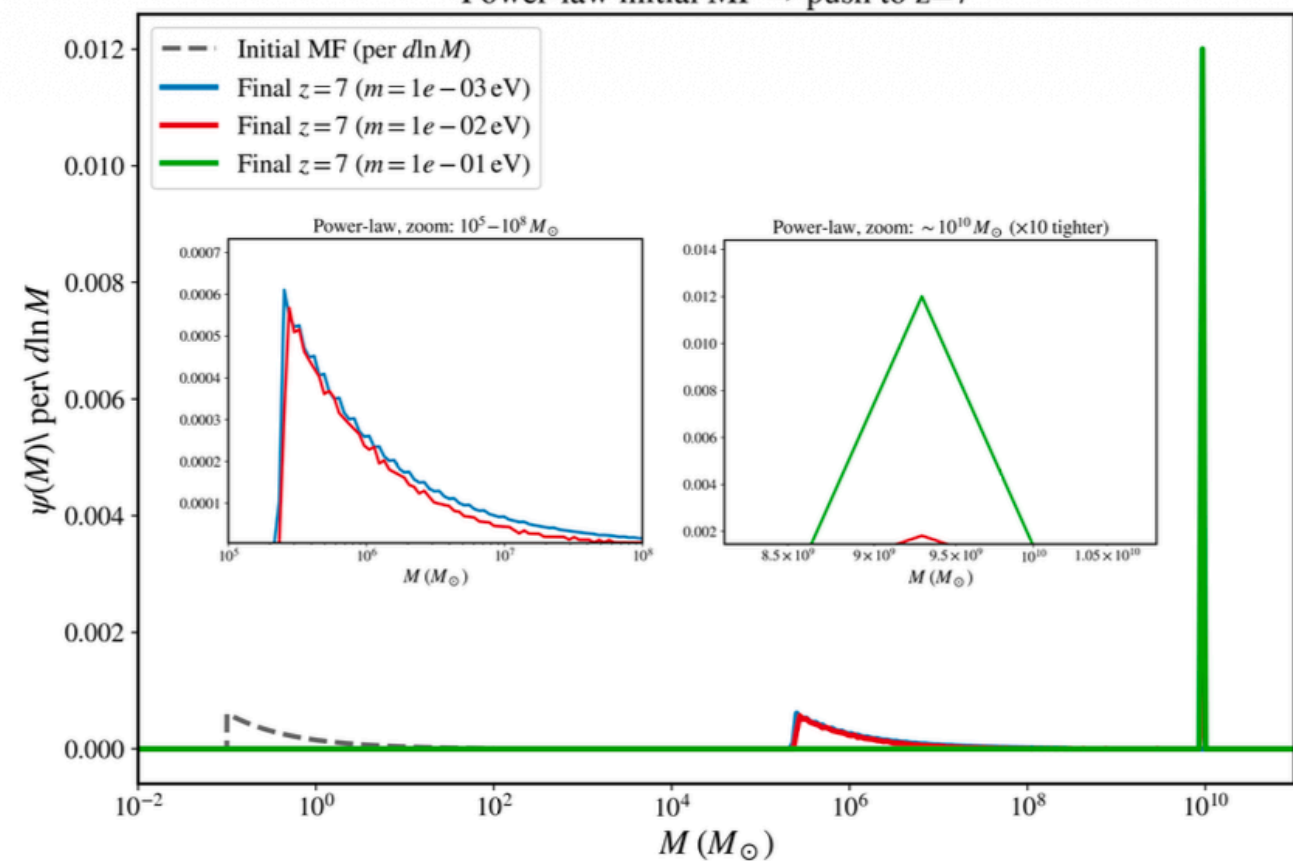
C. Critical collapse: $\psi_0^{\text{CC}}(M_0) = \tilde{A} M_0^{\frac{1}{\gamma}-1} \exp\left[-\beta(M_0)/M_c\right]^{1/\gamma}$ with $\beta = 1$, $\gamma = 0.36$, $\tilde{A}$ fixed by normalization.

- Choosing $f_{\text{PBH}} = 10^{-3}$ for subdominant PBH abundance, consistent with the limits from CMB anisotropies and spectral-distortion constraints.

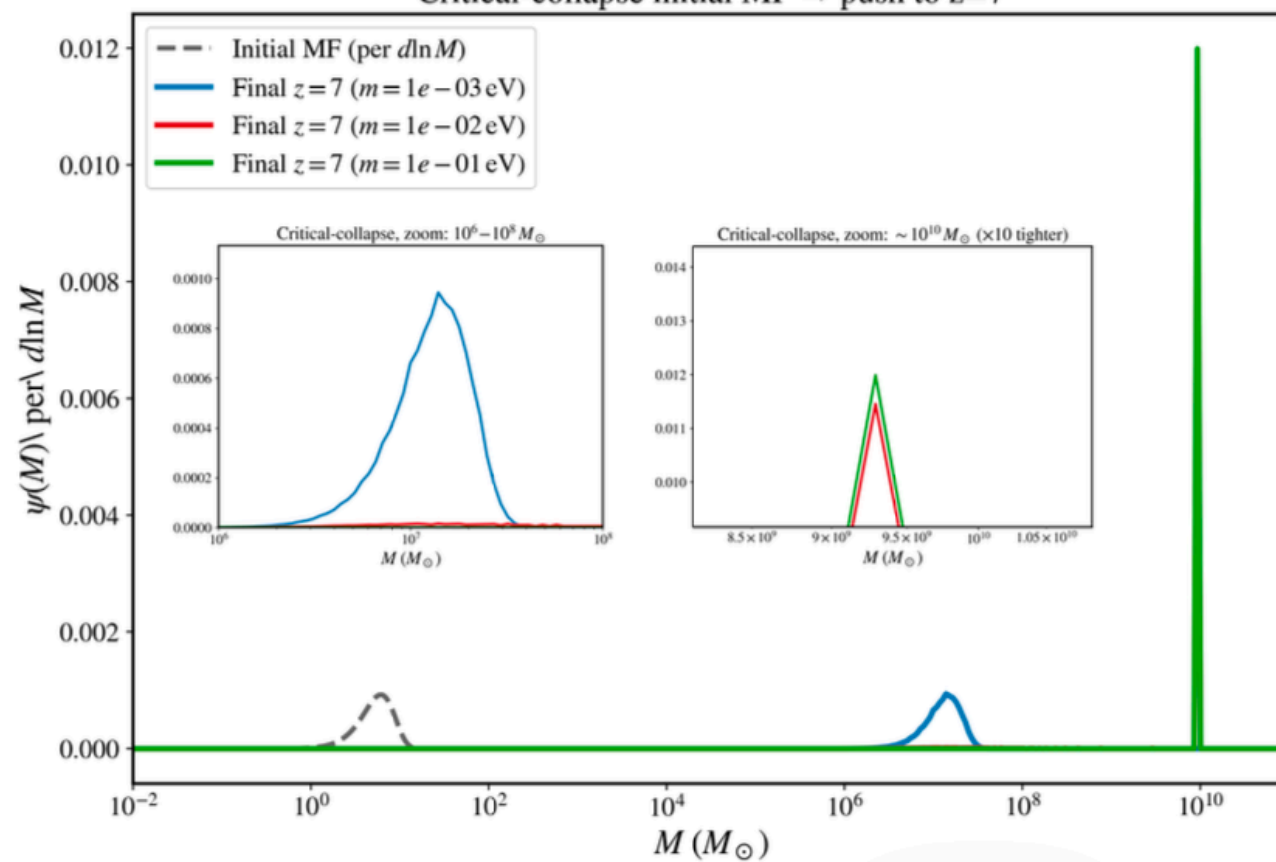
Log-normal initial MF (QCD) $\rightarrow$ push to $z=7$



Power-law initial MF $\rightarrow$ push to $z=7$



Critical-collapse initial MF $\rightarrow$ push to $z=7$



Goldilocks Problem & Its Resolution

- ϕ^4 -SIDM gives a velocity-independent cross-section $\sigma_{\phi^4} = \frac{\lambda^2}{64\pi m^2}$, and cannot explain the multiscale tension, known as the Goldilocks problem of DM halos:
 1. Cluster-scale observations require lower values of σ/m ,
 2. Galaxy-scale observations require higher values of σ/m .
- 2. Two possible new ingredients to ϕ^4 -SIDM to reconcile the Goldilocks problem and formation of SMBHs:
 - A. Introduce another SIDM with the required velocity cross-section to resolve the Goldilocks problem but not contribute significantly to Bondi accretion.
 - B. Introduce a heavy mediator field φ coupled to the ϕ^4 -SIDM: $\mathcal{L}' = \frac{1}{2}(\partial\varphi)^2 - m_\varphi^2\varphi^2 - \lambda_3\varphi\phi^2$ (cf. Tsai et al 2022).
 1. By integrating out φ , we obtain an effective ϕ^4 coupling $\lambda'(p) = \lambda + \mathcal{O}(1)\frac{\lambda_3^2}{p^2 + m_\varphi^2}$.
 2. It can solve the Goldilocks problem, i.e., larger/smaller λ' (cross-section) for smaller/larger p^2 (velocity).
 3. It will not change the EoS of ϕ^4 -SIDM, as it is obtained in the homogeneous limit, but replacing λ with $\lambda'(p = 0)$.

Conclusion

- In this talk, we introduce a new application of dark matter to the formation of supermassive black holes (SMBHs).
- A key ingredient is adopting the hydrodynamic point of view for dark matter in a strong gravity regime, which yields a nontrivial equation of state (EoS). This contrasts with the conventional treatment of dark matter in particle physics.
- The novel EoS of self-interacting dark matter (SIDM) results in an environment-universal Bondi accretion rate, which can lead to the formation of SMBHs in a very early universe around $z = 25 - 10$.
- Our environment-universal super-Eddington mechanism predicts a unique mass function for black holes across all times, based on an initial mass function of primordial black holes.
- This helps explain the recent discoveries of SMBHs by the James Webb Space Telescope.
- Most importantly, the novel Bondi accretion rate is tightly tied to the mass of the SIDM. It implies that the microscopic properties of dark matter can be inferred from astrophysical observations of SMBHs.