



# Spinning Extreme-Mass-Ratio-Inspirals into an Astrophysical Black Hole: Onset of Chaos at Astrophysically Realistic Spin

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Surajit Das, [USTC](#), Hefei, China

Collaborators:

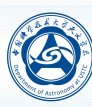
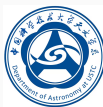
Tatsuya Daniel, [McGill University](#), Montreal, Canada

Tucker Manton, [HIAS-UCAS](#), Hangzhou, China

Yi-Fu Cai, [USTC](#), Hefei, China

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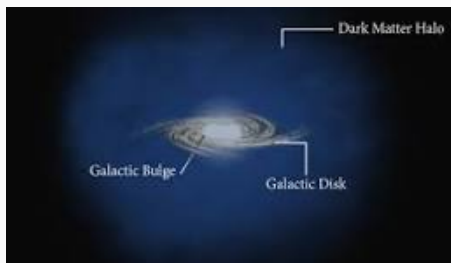
**中国科学技术大学粒子宇宙学国际中心**

International Initiative of Particle Cosmology at University of Science and Technology of China

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- ☉ **Binary system:** Supermassive black hole ( $M_{\text{BH}}$ ) as primary, orbited by a stellar-mass compact object ( $\mu$ ) as secondary.
- ☉ **Formation environment:** Dense nuclear star clusters surrounding galactic centers.
- ☉ **Mass hierarchy:** Extreme mass ratio spanning  $\mu/M_{\text{BH}} \sim 10^{-7} - 10^{-4}$ .
- ☉ **Gravitational wave band:** Emit in the  $10^{-4} - 10^{-1}$  Hz frequency range.
- ☉ **Orbital evolution:** The smaller body completes  $\sim 10^5$  revolutions in the strong-field region.
- ☉ **Waveform diversity:** Exhibit rich orbital dynamics and complex gravitational wave signatures.
- ☉ **Detections:** EMRIs are prime targets for LISA, TianQin, & Taiji.

# In Reality, Nothing is Vacuum



## The Einstein Cluster Configuration

- ☉ Anisotropic fluid: possible circular geodesics up to scale  $a_0$  with mass  $M_H$  and vanishing radial pressure
- ☉  $T^\mu_\nu = \text{diag}(-\rho(r), 0, P_t, P_t)$
- ☉ Spherically symmetric metric
$$ds^2 = -f(r)dt^2 + \frac{dr^2}{1-2m(r)/r} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$
- ☉ Scaled-density profile to mimic observations:
$$\rho(r) \rightarrow \Theta(r - \nu M_{\text{BH}}) \rho(r), \quad \nu \geq 2$$

## Hernquist-Inspired Closed-Form Analytic Solution

- ⊙  $m(r) = M_{\text{BH}} + \frac{M_{\text{H}} r^2}{(a_0 + r)^2} \left(1 - \frac{2M_{\text{BH}}}{r}\right)^2$
- ⊙  $f(r) = \left(1 - \frac{2M_{\text{BH}}}{r}\right) e^{\Gamma}$
- ⊙  $\Gamma(r, a_0, M_{\text{H}}, M_{\text{BH}}) = \sqrt{\frac{M_{\text{H}}}{(2a - M_{\text{H}} + 4M_{\text{BH}})}} \left[ -\pi + 2 \tan^{-1} \left( \frac{r + a - M_{\text{H}}}{\sqrt{M_{\text{H}}(2a - M_{\text{H}} + 4M_{\text{BH}})}} \right) \right]$

[Cardoso *et al.*, PRD Letters (2022), PRL(2022)]

## Physical Properties

- ⊙ Horizon at  $r = 2M_{\text{BH}}$
- ⊙ Asymptotically flat spacetime
- ⊙ Compactness parameter:  
 $M_{\text{H}}/a_0 \lesssim 10^{-4}$  to mimic galaxies
- ⊙ Extension to numerical solutions with various matter profiles to accretion disks, galactic matter, etc.

## Key Insight

The framework provides exact fully-relativistic solutions for black holes embedded in galactic-scale matter distributions, with a closed-form analytic expression.

# Spinning Secondary at the Pole-Dipole Level

## ⊙ MPD Equations:

$$\frac{Dp^\alpha}{D\tau} = -\frac{1}{2}R^\alpha{}_{\beta\gamma\delta}v^\beta S^{\gamma\delta},$$
$$\frac{DS^{\alpha\beta}}{D\tau} = p^\alpha v^\beta - p^\beta v^\alpha$$

## ⊙ TD SSC: $p_\alpha S^{\alpha\beta} = 0$

## ⊙ Integrals of Motions: $p_\alpha p^\alpha = -\mu^2$ , $S^2 = \frac{1}{2}S_{\alpha\beta}S^{\alpha\beta}$

## ⊙ Astrophysically Relevant Spin: Spin term in the MPD equations scales as $S/(\mu M_{\text{BH}})$ , must be bounded by $\mu^2 \implies \frac{S}{\mu M_{\text{BH}}} \leq \frac{\mu^2}{\mu M_{\text{BH}}} = \frac{\mu}{M_{\text{BH}}} \approx$ EMRIs limit

## ⊙ Conserved Quantities in Spinning Dynamics:

$$C_\xi = \xi^\alpha p_\alpha - \frac{1}{2}\nabla_\beta \xi_\alpha S^{\alpha\beta}$$

↓

### Time Translational Symmetry

Spin-orbital energy  $E$

### Rotational Symmetry

Total spin-orbital angular momentum  $J_z$

# Our Numerical Setup with Vector Formulation

- ⊙ **Numerical errors:** In the limit  $S \rightarrow 0$ ,  $p^\alpha \rightarrow v^\alpha$ , but in actual computations it suffers from significant numerical round-off errors
- ⊙ **Spin vector 1-form:**  $S_\alpha = -\frac{1}{2}\eta_{\alpha\beta\gamma\delta}p^\beta S^{\gamma\delta}$
- ⊙ **Velocity 4-vector  $\parallel$  Momentum 4-vector:**  
 $v^\alpha = N(p^\alpha + w^\alpha)$  ,  $w^\alpha = -{}^*R^{\alpha\beta\gamma\delta} s_\beta p_\gamma s_\delta$  ;  $N$  is not constant rather dynamical and fixed from  $v_\alpha v^\alpha = -\mu^2$
- ⊙ **1-form MPD equations:**

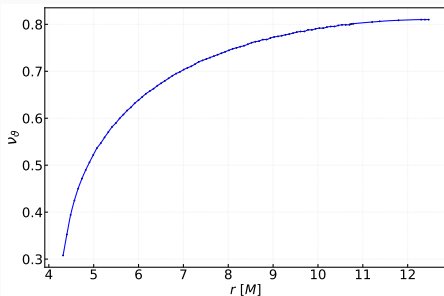
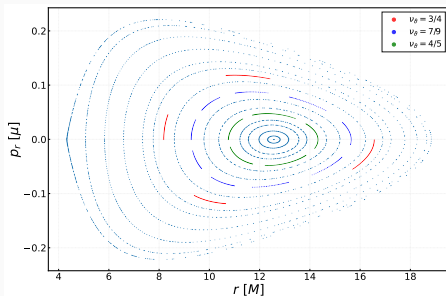
$$\frac{Dp_\alpha}{D\tau} = -R_{\alpha\beta}^*{}^{\sigma\delta} v^\beta p_\sigma S_\delta,$$
$$\frac{DS_\alpha}{D\tau} = -p_\alpha \left( R^{\nu\gamma\delta}{}_\beta S_\nu v^\beta p_\gamma S_\delta \right)$$

- ⊙ **5 Constraints in Hand:**  $E$  conservation,  $J_z$  conservation, Hamiltonian conservation, Spin conservation, and TD SSC conservation, during orbital evolution
- ⊙ **Full Set of DVs:**  
 $(t(\tau), \phi(\tau), r(\tau), \theta(\tau), p_t(\tau), p_\phi(\tau), p_r(\tau), p_\theta(\tau), s_t(\tau), s_\phi(\tau), s_r(\tau), s_\theta(\tau))$   
[Hartl, PRD+ (2003)]

## Appropriate Spinning Trajectories: Pseudo-Effective Potential

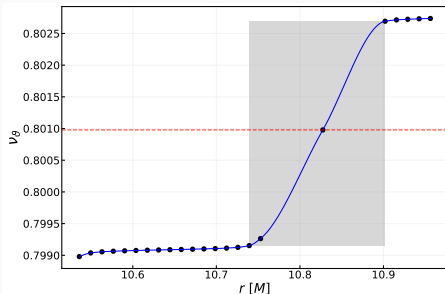
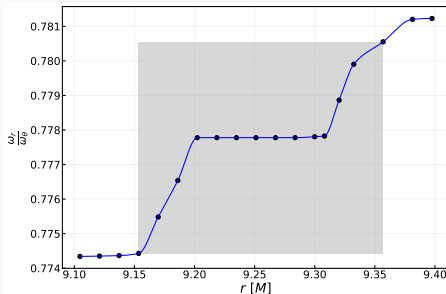
- ② No true effective potential exists for a spinning particle due to the presence of spin-curvature coupling
- ② To get a bounded motion in  $(r - \theta)$  plane, we should set  $p_r = 0 = p_\theta$ , by defining  $p_t = -\sqrt{\text{Redshift}(r)} \cosh X(r, \theta)$ ,  $p_\phi = r \sin \theta \sinh X(r, \theta)$  [Suzuki *et al.*, PRD (1996)]
- ②  $X(r, \theta)$  will be solved from the spin constraint  $S_{\alpha\beta} S^{\alpha\beta} = 2S^2$
- ② The solution of  $X(r, \theta)$  defines the possible bounded region of the non-plunging spinning particle
- ② Asking for a possible “(Pseudo-) effective potential” and the straightforward answer:  $\mathcal{V}_{\text{eff}}^2(r, \theta, S, J_z, M_{\text{BH}}, M_{\text{H}}, a)^{(\pm)} \leq E^2$
- ②  $\mathcal{V}_{\text{eff}}^{(\pm)} \implies$  Maximum radial location & unstable maxima location  $\implies$  possible space locations for a non-plunging bounded spinning non-geodesics
- ② For Astrophysically Relevant  $S = 5 \times 10^{-6} \mu M_{\text{BH}}$ , below  $M_{\text{H}}/a \simeq 10^{-3}$  the dynamics remains unchanged

# Diagnosis of Non-Integrability: Poincaré Surface of Section



- ☉ Identified at-least 3 resonances as  $3/4, 7/9, 4/5$  for **Astrophysically Relevant**  $S = 5 \times 10^{-6} \mu M_{\text{BH}}$
- ☉ Away from resonances, the non-resonant **KAM** curves surround a central invariant point
- ☉ Resonant points are surrounded by **Birkoff chains**, correspond to unstable periodic points
- ☉ Spinning resonant orbits (“periodic orbits”) share same **rational ratio**  
 $\nu_{\vartheta} \equiv \omega_r / \omega_{\vartheta}$

# Observational Imprints of Spin-Environment Induced Non-Integrability



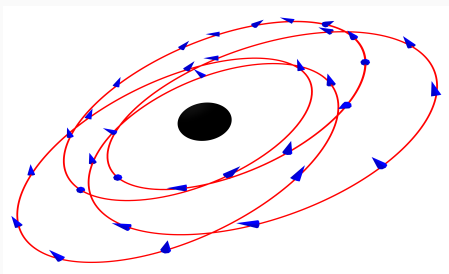
- ⊙ Nonintegrable systems  $\implies$  discontinuities in the monotonicity of a rotation curve through the formation of **non-analytic plateaus** with nonzero widths and area
- ⊙ **Sudden jumps** in the monotonicity of a rotation curve appear as trajectory pass through unstable periodic points [Deich et al., PRD (2022)]
- ⊙ The area of the plateau  $\implies$  the area of the resonant  $\implies$  general measure of the **timescales involved in resonances** of nonintegrable EMRIs.

## Conclusions & Future Perspective

- ② Spin-environment induced perturbations generate resonances in phase space as Birkoff chains/thin chaotic layers.
- ② A pronounced chaotic sea is also found close to the unstable separatrix of the pseudo-effective potential in the strong-field regime [Zelenka *et al.*, PRD (2020)]
- ② Thousands of EMRIs will be observable during space-based detectors' mission lifetime [Gair *et al.*, J. Phys. (2017)]
- ② Integrable deviations due to nonlinear spin-orbit coupling from astrophysical spacetime with matter halos produce smooth, predictable modifications to EMRI waveforms [Apostolatos *et al.*, PRL (2009), GLG *et al.*, PRD (2010)]
- ② The presence of chaos leaves a distinctive imprint on the GWs signal—observable as sudden frequency shifts [Destounis *et al.*, PRL (2021), PRD (2021), PRD (2023)].
- ② The characteristic frequency glitches may serve as unambiguous indicators of chaotic dynamics in EMRIs
- ② Precise waveform templates are essential for extracting EMRI parameters and testing fundamental physics.

## Acknowledgments

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**khàawp-khun khráp!**  
**Thanks a lot for your attention!**

- © For comments, suggestions and discussions in our work, please feel free to contact me at: [surajitdas@mail.ustc.edu.cn](mailto:surajitdas@mail.ustc.edu.cn)