

Dark Energy & Gravitational Waves

Vicharit Yingcharoenrat

Chulalongkorn University

School@WEHS-Narit: October 17th, 2025

Outline

- Basic cosmology and dark energy
- Simplest dark energy model
- Introduction to screening mechanism
- Dark energy models as EFT

Friedmann's Universe

- **Physics** — General Relativity

Friedmann's Universe

- **Physics** — General Relativity $M_{\text{Pl}}^2 = 1/(8\pi G)$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R + L_m \right] \Rightarrow R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{T_{\mu\nu}}{M_{\text{Pl}}^2}$$

Friedmann's Universe

- **Physics** — General Relativity $M_{\text{Pl}}^2 = 1/(8\pi G)$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R + L_m \right] \Rightarrow R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{T_{\mu\nu}}{M_{\text{Pl}}^2}$$

- **Observations** — Isotropy & Homogeneity in space — **Cosmological Principles**
on super large scales $\sim 200 - 300 \text{ Mpc}$ (1 pc $\sim 10^{13} \text{ km}$)

Friedmann's Universe

- **Physics** — General Relativity $M_{\text{Pl}}^2 = 1/(8\pi G)$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R + L_m \right] \Rightarrow R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{T_{\mu\nu}}{M_{\text{Pl}}^2}$$

- **Observations** — Isotropy & Homogeneity in space — **Cosmological Principles**
on super large scales $\sim 200 - 300 \text{ Mpc}$ ($1 \text{ pc} \sim 10^{13} \text{ km}$)

Friedmann-Robertson-Walker (FRW) metric:

$$ds^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$$

Translational symmetry: everywhere in space looks the same — $T_4: x^i \rightarrow x^i + c^i$

Rotational symmetry: every direction looks the same — $\text{SO}(3): x^i \rightarrow O^i_j x^j$

Friedmann's Universe

- Matter Lagrangian — Perfect Fluid: $T^\mu_\nu = \text{diag}(-\rho(t), p(t), p(t), p(t))$

Friedmann's Universe

- **Matter Lagrangian** — Perfect Fluid: $T^\mu_\nu = \text{diag}(-\rho(t), p(t), p(t), p(t))$
- Continuity eqn. $\nabla_\mu T^\mu_\nu = 0$:

Friedmann's Universe

- **Matter Lagrangian** — Perfect Fluid: $T^\mu_\nu = \text{diag}(-\rho(t), p(t), p(t), p(t))$

- Continuity eqn. $\nabla_\mu T^\mu_\nu = 0$:

$$p = w\rho$$

$$\dot{\rho} + 3H(1 + w)\rho = 0 \quad H = \frac{\dot{a}(t)}{a(t)}$$

Friedmann's Universe

- **Matter Lagrangian** — Perfect Fluid: $T^\mu_\nu = \text{diag}(-\rho(t), p(t), p(t), p(t))$

- Continuity eqn. $\nabla_\mu T^\mu_\nu = 0$:

$$p = w\rho$$

$$\dot{\rho} + 3H(1 + w)\rho = 0 \quad H = \frac{\dot{a}(t)}{a(t)}$$


$$\rho(t) \propto a(t)^{-3(1+w)}$$

Friedmann's Universe

- **Matter Lagrangian** — Perfect Fluid: $T^\mu_\nu = \text{diag}(-\rho(t), p(t), p(t), p(t))$

- Continuity eqn. $\nabla_\mu T^\mu_\nu = 0$:

$$p = w\rho$$

$$\dot{\rho} + 3H(1 + w)\rho = 0 \quad H = \frac{\dot{a}(t)}{a(t)}$$


$$\rho(t) \propto a(t)^{-3(1+w)}$$

Rad ($w = 1/3$): $\rho_R \propto a^{-4}$

Dust ($w = 0$): $\rho_M \propto a^{-3}$

Cosmological Constant ($w = -1$): $\rho_\Lambda = \text{const.}$

Friedmann's Universe

- **Matter Lagrangian** — Perfect Fluid: $T^\mu_\nu = \text{diag}(-\rho(t), p(t), p(t), p(t))$

- Continuity eqn. $\nabla_\mu T^\mu_\nu = 0$:

$$p = w\rho$$

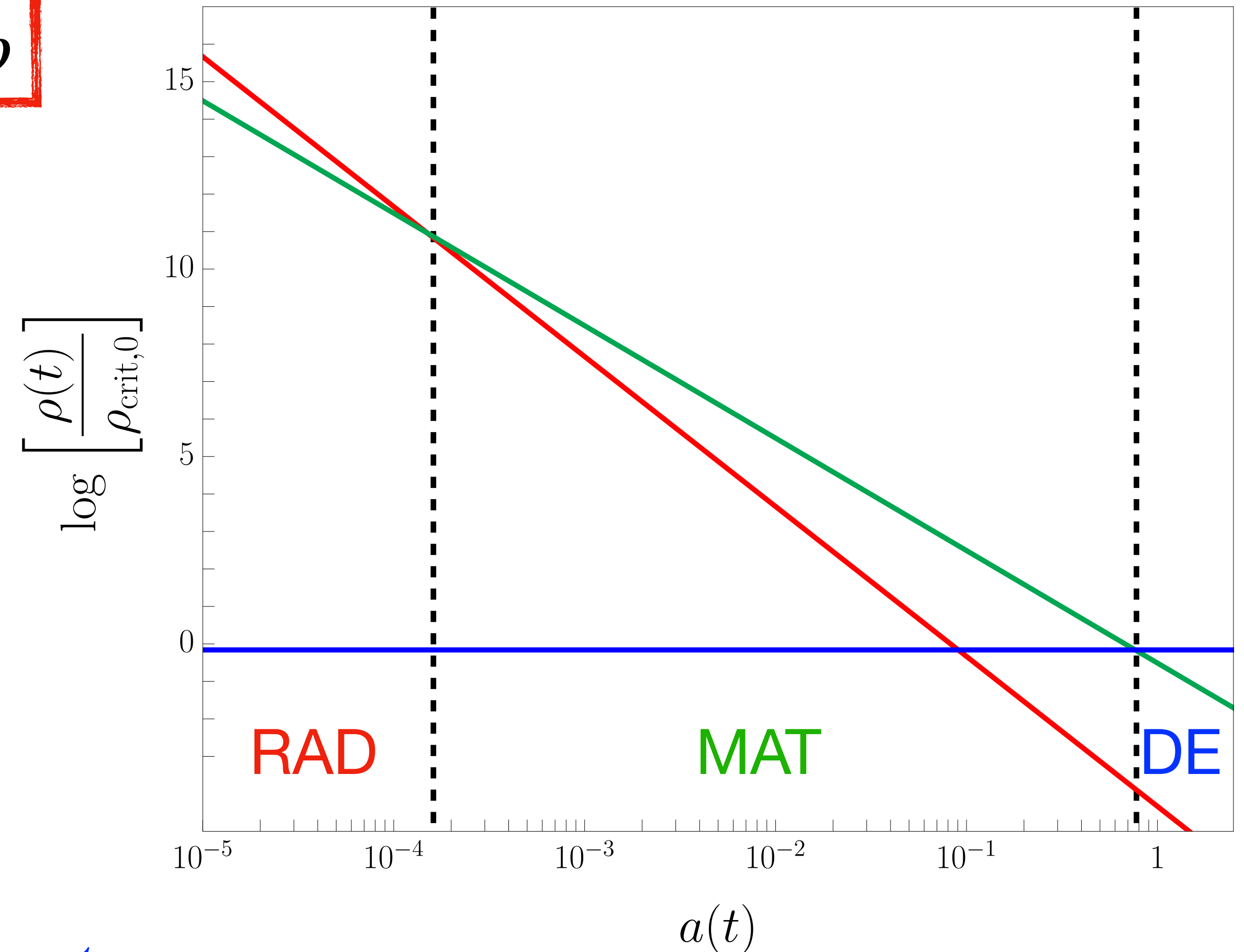
$$\dot{\rho} + 3H(1 + w)\rho = 0 \quad H = \frac{\dot{a}(t)}{a(t)}$$

$$\rho(t) \propto a(t)^{-3(1+w)}$$

Rad ($w = 1/3$): $\rho_R \propto a^{-4}$

Dust ($w = 0$): $\rho_M \propto a^{-3}$

Cosmological Constant ($w = -1$): $\rho_\Lambda = \text{const.}$



Friedmann's Universe

- Friedmann equations

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

Friedmann's Universe

- Friedmann equations

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) \quad \ddot{a} > 0 \quad \Rightarrow \quad P < -\frac{1}{3}\rho \quad \text{or} \quad \boxed{w < -\frac{1}{3}}$$

Friedmann's Universe

- Friedmann equations

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) \quad \ddot{a} > 0 \quad \Rightarrow \quad P < -\frac{1}{3}\rho \quad \text{or} \quad w < -\frac{1}{3}$$

- Energy conditions: positivity conditions on energy

Friedmann's Universe

- **Friedmann** equations

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) \quad \ddot{a} > 0 \quad \Rightarrow \quad P < -\frac{1}{3}\rho \quad \text{or} \quad \boxed{w < -\frac{1}{3}}$$

- **Energy** conditions: positivity conditions on energy n^μ : timelike ℓ^μ : null

Strong energy condition (**SEC**) $\left(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}\right)n^\mu n^\nu \geq 0$

Friedmann's Universe

- **Friedmann** equations

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) \quad \ddot{a} > 0 \quad \Rightarrow \quad P < -\frac{1}{3}\rho \quad \text{or} \quad \boxed{w < -\frac{1}{3}}$$

- **Energy** conditions: positivity conditions on energy n^μ : timelike ℓ^μ : null

$$\text{Strong energy condition (SEC)} \quad \left(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}\right)n^\mu n^\nu \geq 0 \quad \Rightarrow \quad \rho + 3P \geq 0 \quad \text{and} \quad \rho + P \geq 0$$

Friedmann's Universe

- **Friedmann** equations

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) \quad \ddot{a} > 0 \quad \Rightarrow \quad P < -\frac{1}{3}\rho \quad \text{or} \quad \boxed{w < -\frac{1}{3}}$$

- **Energy** conditions: positivity conditions on energy n^μ : timelike ℓ^μ : null

$$\text{Strong energy condition (SEC)} \quad \left(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}\right)n^\mu n^\nu \geq 0 \quad \Rightarrow \quad \rho + 3P \geq 0 \quad \text{and} \quad \rho + P \geq 0$$

$$\text{Weak energy condition (WEC)} \quad T_{\mu\nu}n^\mu n^\nu \geq 0$$

Friedmann's Universe

- **Friedmann** equations

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) \quad \ddot{a} > 0 \quad \Rightarrow \quad P < -\frac{1}{3}\rho \quad \text{or} \quad \boxed{w < -\frac{1}{3}}$$

- **Energy** conditions: positivity conditions on energy n^μ : timelike ℓ^μ : null

Strong energy condition (**SEC**) $\left(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}\right)n^\mu n^\nu \geq 0 \quad \Rightarrow \quad \rho + 3P \geq 0 \quad \text{and} \quad \rho + P \geq 0$

Weak energy condition (**WEC**) $T_{\mu\nu}n^\mu n^\nu \geq 0 \quad \Rightarrow \quad \rho + P \geq 0 \quad \text{and} \quad \rho \geq 0$

Friedmann's Universe

- Friedmann equations

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \quad \ddot{a} > 0 \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) \Rightarrow P < -\frac{1}{3}\rho \quad \text{or} \quad w < -\frac{1}{3}$$

- Energy conditions: positivity conditions on energy n^μ : timelike ℓ^μ : null

$$\text{Strong energy condition (SEC)} \quad \left(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}\right)n^\mu n^\nu \geq 0 \Rightarrow \rho + 3P \geq 0 \quad \text{and} \quad \rho + P \geq 0$$

$$\text{Weak energy condition (WEC)} \quad T_{\mu\nu}n^\mu n^\nu \geq 0 \Rightarrow \rho + P \geq 0 \quad \text{and} \quad \rho \geq 0$$

$$\text{Null energy condition (NEC)} \quad T_{\mu\nu}\ell^\mu \ell^\nu \geq 0 \Rightarrow \rho + P \geq 0$$

Friedmann's Universe

- All forms of known matters (even Λ) satisfy **NEC & WEC** $\rho + P \geq 0$ and $\rho \geq 0$

Friedmann's Universe

- All forms of known matters (even Λ) satisfy **NEC & WEC** $\rho + P \geq 0$ and $\rho \geq 0$

$$\dot{H} = -4\pi G(\rho + P)$$

$$(k = 0)$$

Friedmann's Universe

- All forms of known matters (even Λ) satisfy **NEC & WEC** $\rho + P \geq 0$ and $\rho \geq 0$

$$\begin{array}{ccccc} \dot{H} = -4\pi G(\rho + P) & \Rightarrow & \dot{H} < 0 & \text{Hubble always decreasing} \\ (k = 0) & \text{NEC} & & \end{array}$$

Friedmann's Universe

- All forms of known matters (even Λ) satisfy **NEC & WEC** $\rho + P \geq 0$ and $\rho \geq 0$

$$\dot{H} = -4\pi G(\rho + P) \quad \Rightarrow \quad \dot{H} < 0 \quad \text{Hubble always decreasing}$$

$(k = 0)$ **NEC**

~~NEC~~ $\Rightarrow$ Instabilities of a system, e.g. phantom field

Friedmann's Universe

- All forms of known matters (even Λ) satisfy **NEC & WEC** $\rho + P \geq 0$ and $\rho \geq 0$

$$\dot{H} = -4\pi G(\rho + P) \quad \Rightarrow \quad \dot{H} < 0 \quad \text{Hubble always decreasing}$$

$(k = 0)$ **NEC**

~~NEC~~ $\Rightarrow$ Instabilities of a system, e.g. phantom field

Negative Λ (AdS) $\Rightarrow$ ~~WEC~~

Friedmann's Universe

- All forms of known matters (even Λ) satisfy **NEC & WEC** $\rho + P \geq 0$ and $\rho \geq 0$

$$\dot{H} = -4\pi G(\rho + P) \quad \Rightarrow \quad \dot{H} < 0 \quad \text{Hubble always decreasing}$$

$(k = 0)$ **NEC**

~~NEC~~ $\Rightarrow$ Instabilities of a system, e.g. phantom field

Negative Λ (AdS) $\Rightarrow$ ~~WEC~~

- **SEC** easily violated $\rho + 3P \geq 0$ and $\rho + P \geq 0$

$\Rightarrow$ Both Inflation and DE with $\ddot{a} > 0$ typically violate SEC

Friedmann's Universe

- All forms of known matters (even Λ) satisfy **NEC & WEC** $\rho + P \geq 0$ and $\rho \geq 0$

$$\dot{H} = -4\pi G(\rho + P) \quad \Rightarrow \quad \dot{H} < 0 \quad \text{Hubble always decreasing}$$

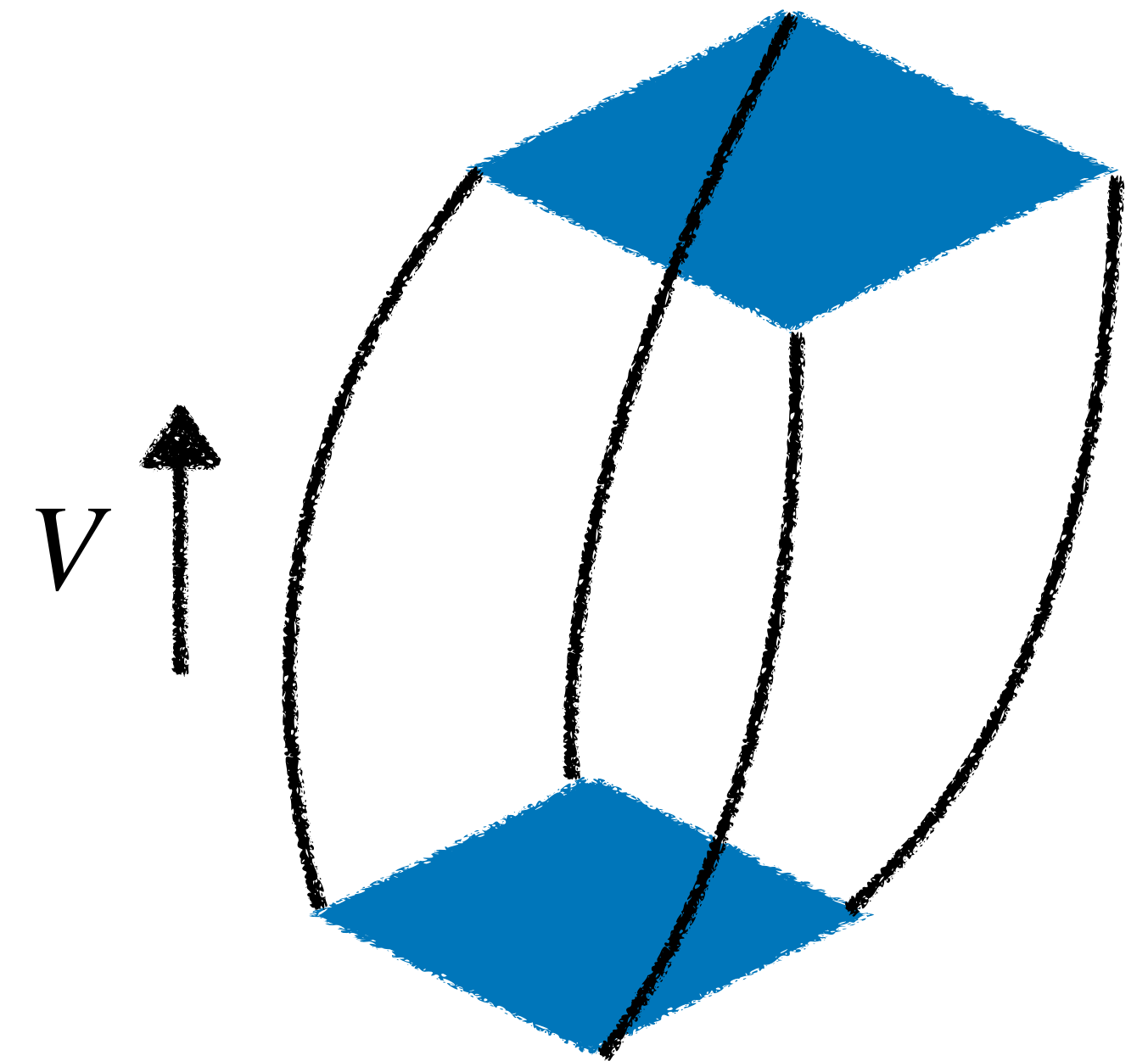
$(k = 0)$ **NEC**

~~NEC~~ $\Rightarrow$ Instabilities of a system, e.g. phantom field

Negative Λ (AdS) $\Rightarrow$ ~~WEC~~

- SEC** easily violated $\rho + 3P \geq 0$ and $\rho + P \geq 0$

$\Rightarrow$ Both Inflation and DE with $\ddot{a} > 0$ typically violate SEC



Energy cons. important role
in the Raychaudhuri eq.

Dark Energy as Λ

Λ (> 0): Cosmological constant = Dark Energy

$$H^2 = \frac{\rho_\Lambda}{3M_{\text{Pl}}^2} \quad \Rightarrow \quad a(t) \propto e^{Ht} \quad w = -1$$

Dark Energy as Λ

$\Lambda (> 0)$: Cosmological constant = Dark Energy

$$H^2 = \frac{\rho_\Lambda}{3M_{\text{Pl}}^2} \quad \Rightarrow \quad a(t) \propto e^{Ht} \quad w = -1$$

$$\rho_{\text{vac}} \sim 10^{-12} \text{ eV}^4 \quad \Rightarrow \quad \frac{M_{\text{Pl}}^4}{\rho_{\text{vac}}} \sim 10^{120}$$

Cosmological Constant
Problem

Dark Energy as Λ

$\Lambda (> 0)$: Cosmological constant = Dark Energy

$$H^2 = \frac{\rho_\Lambda}{3M_{\text{Pl}}^2} \quad \Rightarrow \quad a(t) \propto e^{Ht} \quad w = -1$$

$$\rho_{\text{vac}} \sim 10^{-12} \text{ eV}^4 \quad \Rightarrow \quad \frac{M_{\text{Pl}}^4}{\rho_{\text{vac}}} \sim 10^{120}$$

Cosmological Constant Problem

CDM: Cold Dark Matter — Non-relativistic

Formation of large scale structure

WIMPs (Weakly Interacting Massive Particles)

PBHs (Primordial Black Holes)

Axions ...

Dark Energy as Λ

$\Lambda (> 0)$: Cosmological constant = Dark Energy

$$H^2 = \frac{\rho_\Lambda}{3M_{\text{Pl}}^2} \quad \Rightarrow \quad a(t) \propto e^{Ht} \quad w = -1$$

$$\rho_{\text{vac}} \sim 10^{-12} \text{ eV}^4 \quad \Rightarrow \quad \frac{M_{\text{Pl}}^4}{\rho_{\text{vac}}} \sim 10^{120}$$

Cosmological Constant Problem

CDM: Cold Dark Matter — Non-relativistic

Formation of large scale structure

WIMPs (Weakly Interacting Massive Particles)

PBHs (Primordial Black Holes)

Axions ...



Long-lived and
No interaction with light

Dark Energy as Λ

$\Lambda (> 0)$: Cosmological constant = Dark Energy

$$H^2 = \frac{\rho_\Lambda}{3M_{\text{Pl}}^2} \Rightarrow a(t) \propto e^{Ht} \quad w = -1$$

$$\rho_{\text{vac}} \sim 10^{-12} \text{ eV}^4 \Rightarrow \frac{M_{\text{Pl}}^4}{\rho_{\text{vac}}} \sim 10^{120}$$

Cosmological Constant Problem

CDM: Cold Dark Matter — Non-relativistic

Formation of large scale structure

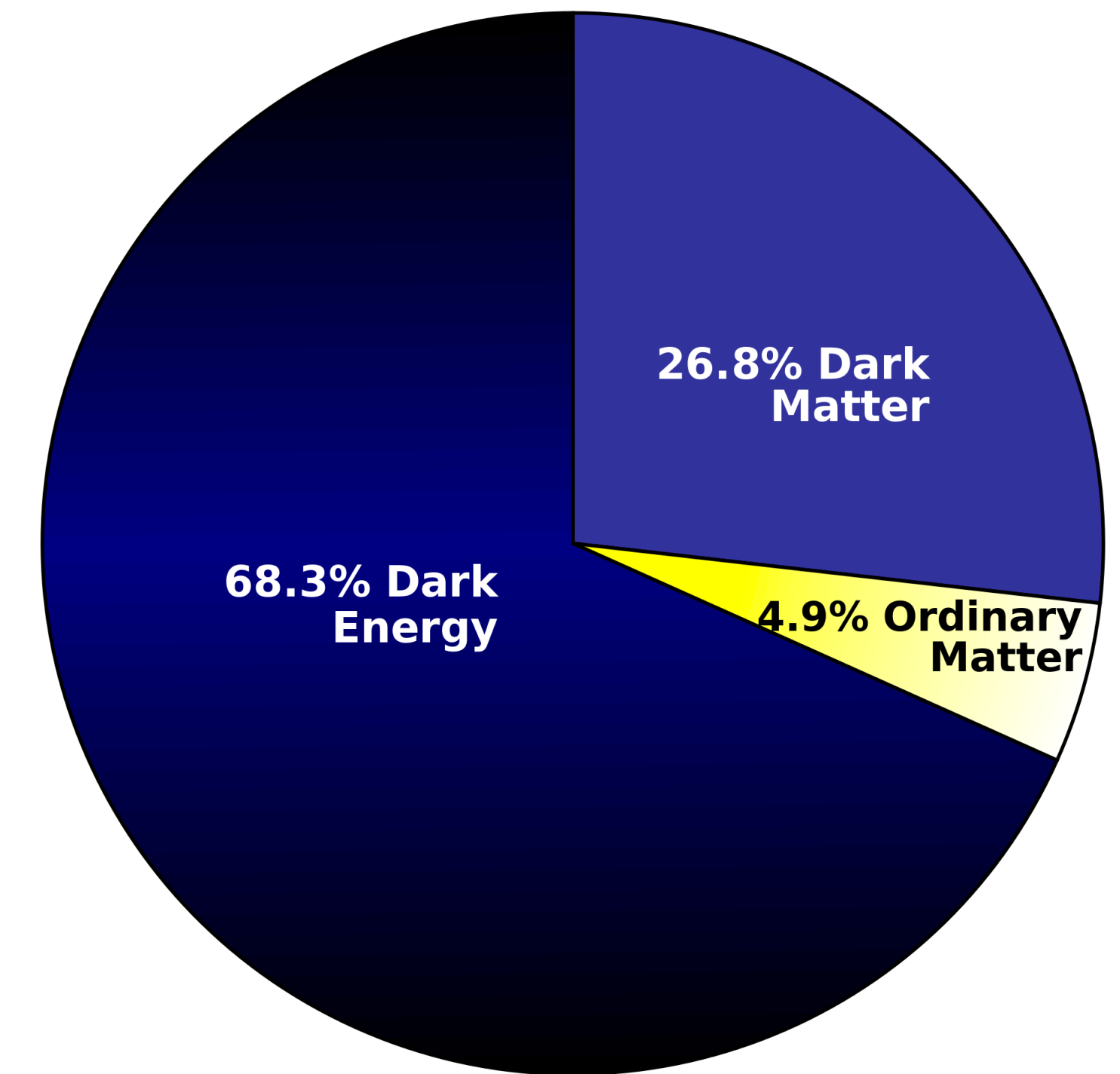
WIMPs (Weakly Interacting Massive Particles)

PBHs (Primordial Black Holes)

Axions ...

Long-lived and
No interaction with light

Present Energy Budget



Measuring w of Dark Energy

- Friedmann's eq. $\Omega_i \equiv \rho_i/\rho_c$ $w = w(z)$

Measuring w of Dark Energy

- Friedmann's eq. $\Omega_i \equiv \rho_i/\rho_c$ $w = w(z)$

$$\frac{H^2}{H_0^2} = \Omega_M(1+z)^3 + \Omega_R(1+z)^4 + \Omega_k(1+z)^2 + \Omega_{DE} \frac{\rho_{DE}(z)}{\rho_{DE,0}}$$

Measuring w of Dark Energy

- Friedmann's eq. $\Omega_i \equiv \rho_i/\rho_c$ $w = w(z)$

$$\frac{H^2}{H_0^2} = \Omega_M(1+z)^3 + \Omega_R(1+z)^4 + \underbrace{\Omega_k(1+z)^2}_{\text{Almost zero}} + \Omega_{DE} \frac{\rho_{DE}(z)}{\rho_{DE,0}}$$

Almost zero

Measuring w of Dark Energy

- Friedmann's eq. $\Omega_i \equiv \rho_i/\rho_c$ $w = w(z)$

$$\frac{H^2}{H_0^2} = \Omega_M(1+z)^3 + \Omega_R(1+z)^4 + \underbrace{\Omega_k(1+z)^2}_{\text{Almost zero}} + \Omega_{DE} \frac{\rho_{DE}(z)}{\rho_{DE,0}}$$

Caution about neutrinos!!

Almost zero

Measuring w of Dark Energy

- Friedmann's eq. $\Omega_i \equiv \rho_i/\rho_c$ $w = w(z)$

$$\frac{H^2}{H_0^2} = \Omega_M(1+z)^3 + \Omega_R(1+z)^4 + \underbrace{\Omega_k(1+z)^2}_{\text{Almost zero}} + \Omega_{DE} \frac{\rho_{DE}(z)}{\rho_{DE,0}} \quad \frac{\rho_{DE}(z)}{\rho_{DE,0}} = \exp \left[3 \int_0^z [1 + w(z')] \frac{dz'}{1+z} \right]$$

Caution about neutrinos!!

Almost zero



Measuring w of Dark Energy

- Friedmann's eq. $\Omega_i \equiv \rho_i/\rho_c$ $w = w(z)$

$$\frac{H^2}{H_0^2} = \Omega_M(1+z)^3 + \Omega_R(1+z)^4 + \underbrace{\Omega_k(1+z)^2}_{\text{Almost zero}} + \Omega_{DE} \frac{\rho_{DE}(z)}{\rho_{DE,0}} \quad \frac{\rho_{DE}(z)}{\rho_{DE,0}} = \exp \left[3 \int_0^z [1 + w(z')] \frac{dz'}{1+z} \right]$$

Caution about neutrinos!!

- $w = -1$ Λ CDM model

Measuring w of Dark Energy

- Friedmann's eq. $\Omega_i \equiv \rho_i / \rho_c$ $w = w(z)$

$$\frac{H^2}{H_0^2} = \Omega_M(1+z)^3 + \Omega_R(1+z)^4 + \underbrace{\Omega_k(1+z)^2}_{\text{Almost zero}} + \Omega_{DE} \frac{\rho_{DE}(z)}{\rho_{DE,0}} \quad \frac{\rho_{DE}(z)}{\rho_{DE,0}} = \exp \left[3 \int_0^z [1 + w(z')] \frac{dz'}{1+z} \right]$$

Caution about neutrinos!!

- $w = -1$ Λ CDM model
- Linear parametrization $w(a) = w_0 + w_a(1-a)$

$$\frac{\rho_{DE}(z)}{\rho_{DE,0}} = a^{-3(1+w_0+w_a)} e^{-3w_a(1-a)} \quad a = (1+z)^{-1}$$

$w_0 w_a$ CDM model

Measuring w of Dark Energy

- Friedmann's eq. $\Omega_i \equiv \rho_i / \rho_c$ $w = w(z)$

$$\frac{H^2}{H_0^2} = \Omega_M(1+z)^3 + \Omega_R(1+z)^4 + \underbrace{\Omega_k(1+z)^2}_{\text{Almost zero}} + \Omega_{DE} \frac{\rho_{DE}(z)}{\rho_{DE,0}} \quad \frac{\rho_{DE}(z)}{\rho_{DE,0}} = \exp \left[3 \int_0^z [1 + w(z')] \frac{dz'}{1+z} \right]$$

Caution about neutrinos!!

Almost zero

- $w = -1$ Λ CDM model

BAO + CMB + SNe data

- Linear parametrization $w(a) = w_0 + w_a(1 - a)$

$$H(z) = \left[\frac{d}{dz} \left(\frac{d_L(z)}{1+z} \right) \right]^{-1}$$

$$\frac{\rho_{DE}(z)}{\rho_{DE,0}} = a^{-3(1+w_0+w_a)} e^{-3w_a(1-a)} \quad a = (1+z)^{-1}$$

d_L = Luminosity distance

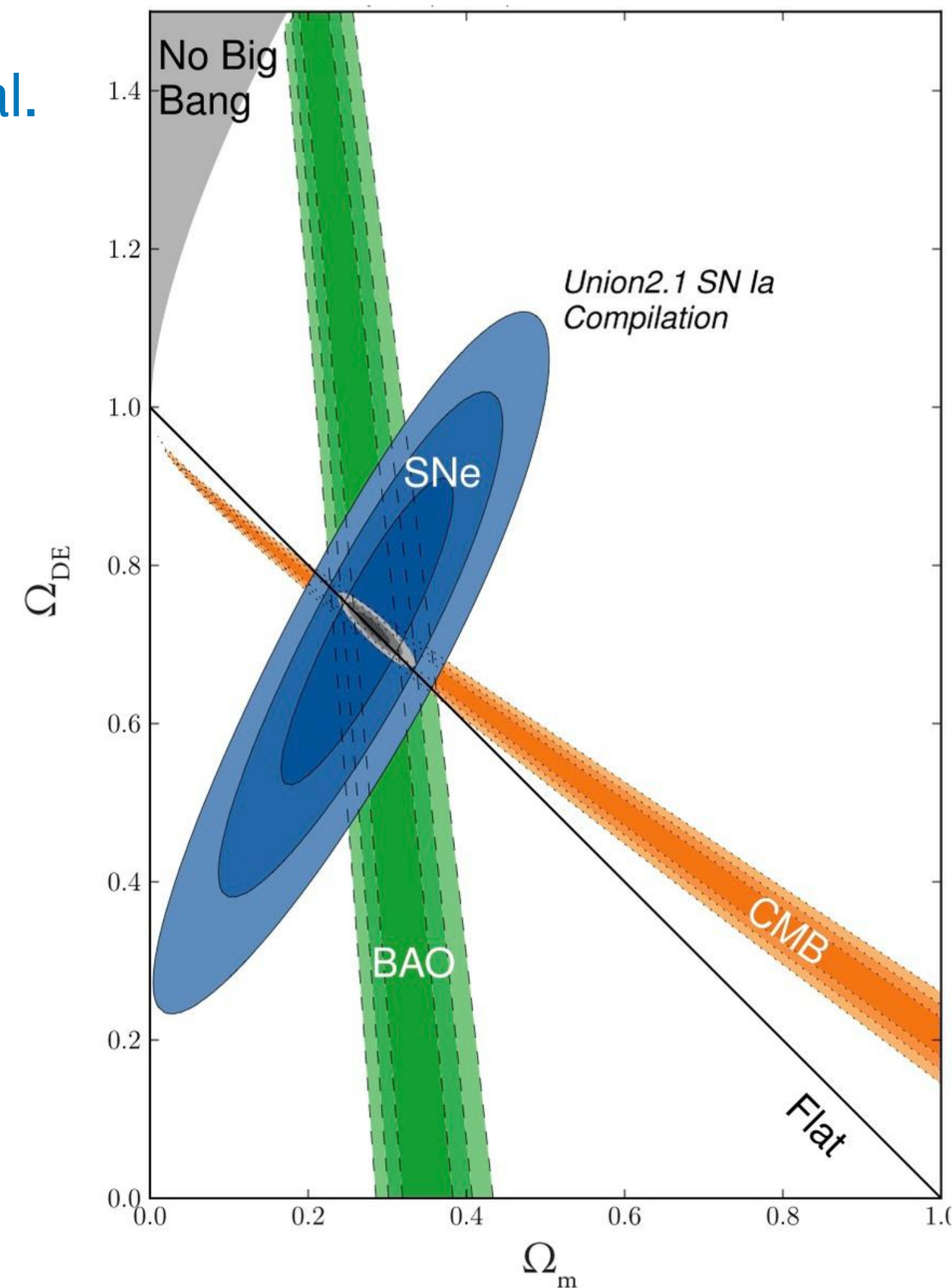
$w_0 w_a$ CDM model

Tight constraints on Ω_{DE} and $w(z)$

Dark Energy Evidences

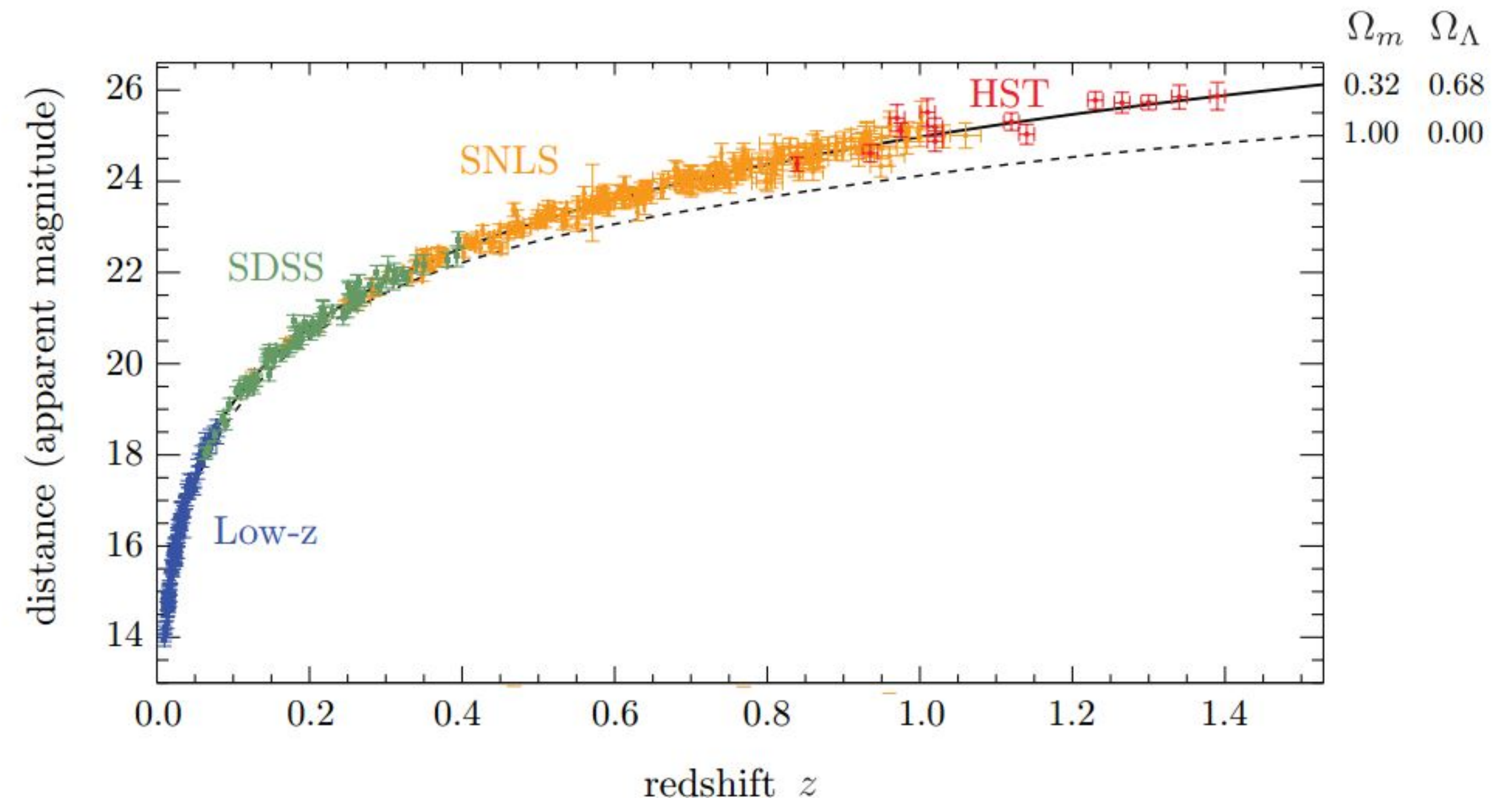
- Responsible for **accelerating** expansion of the Universe today

Suzuki et al.
2012



2011 **Nobel** Prize — Perlmutter, Schmidt and Riess

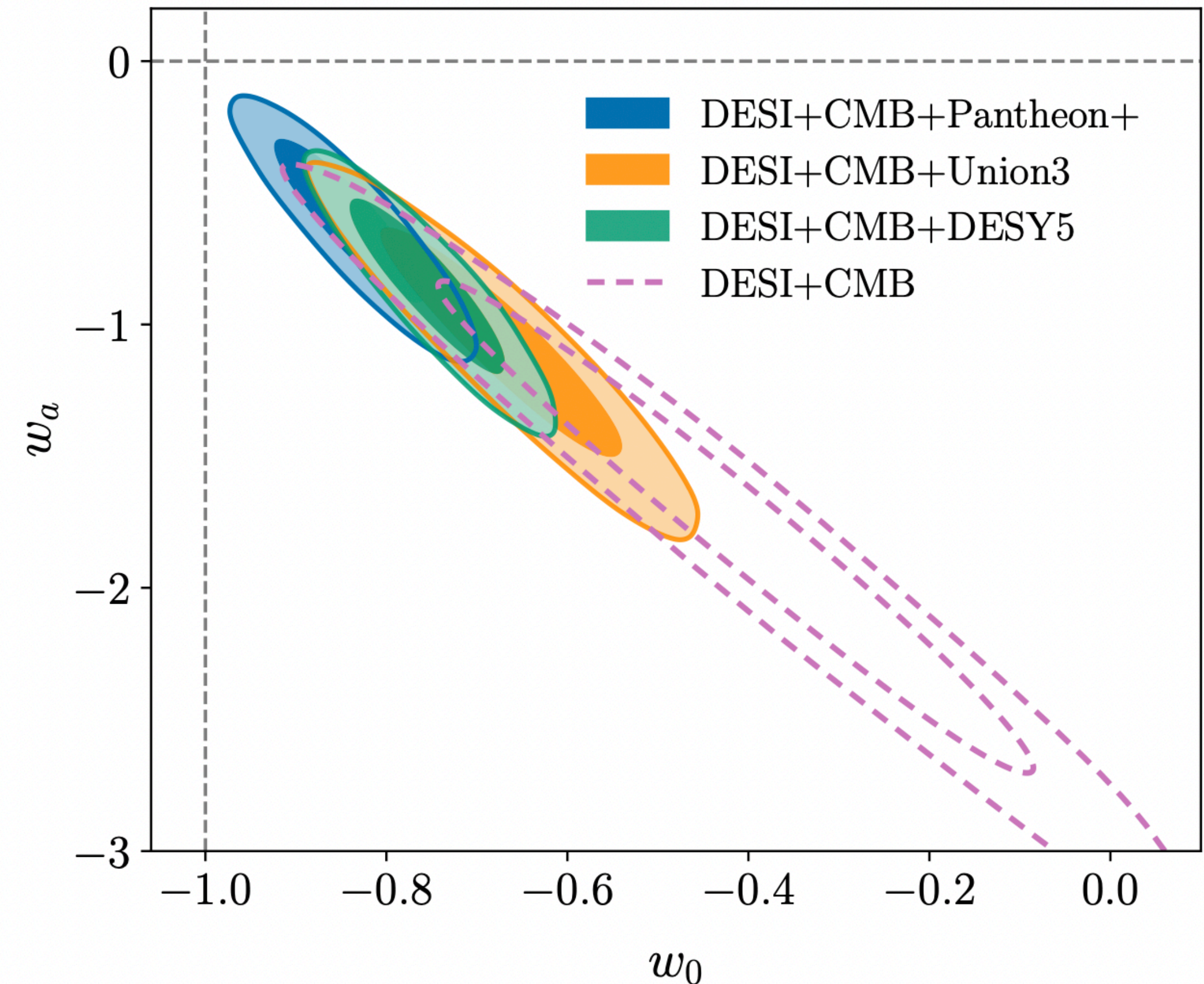
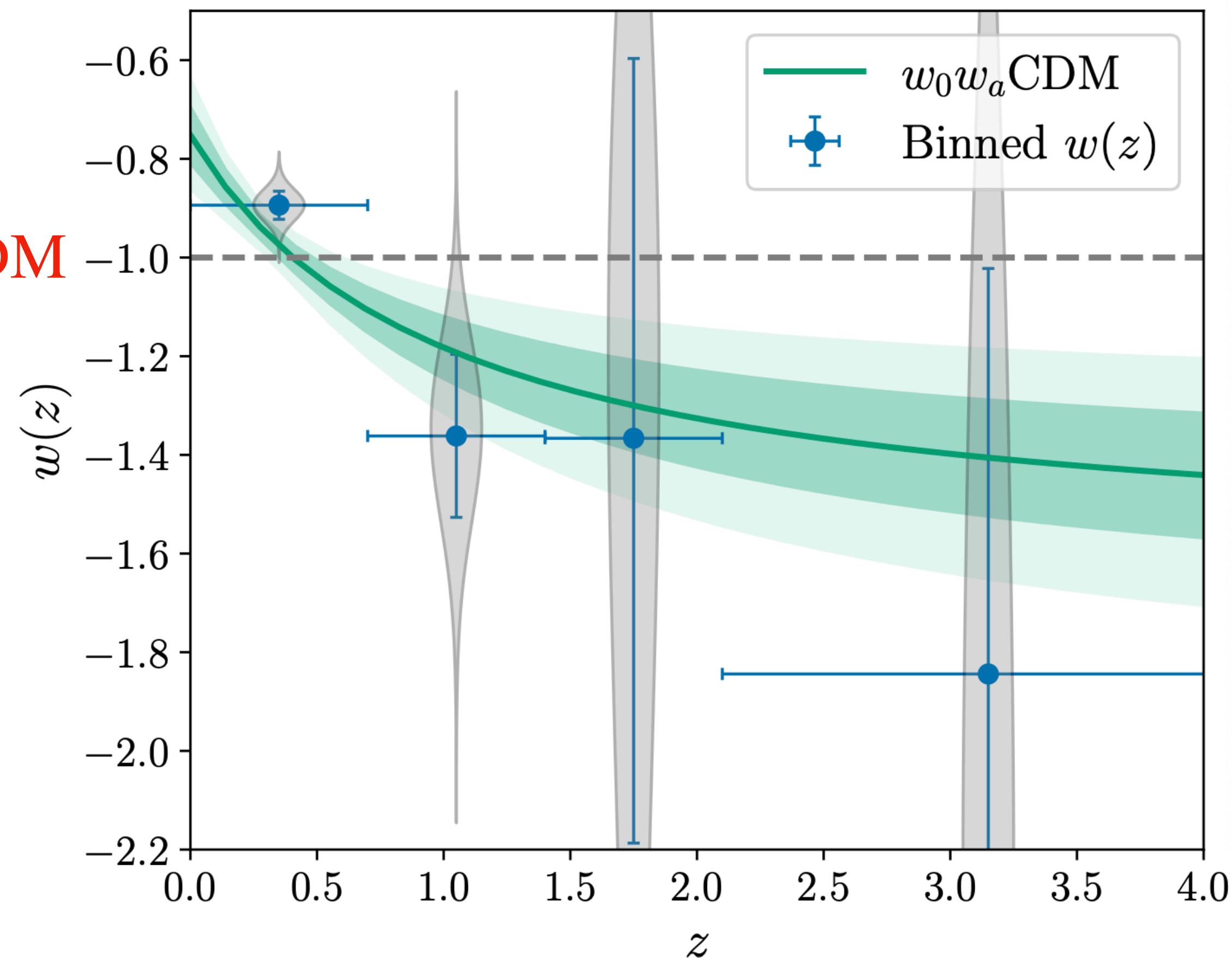
$$\Omega_{DE} \sim 0.68 \Rightarrow \rho_{DE} \sim 7 \cdot 10^{-30} \text{ g/cm}^3 \simeq 10^{-12} \text{ eV}^4$$



Perhaps DE is dynamical...

See Nandan Roy's Lecture

- DESI DR2 Results (DESI 25)



Blue: no assuming a functional form of $w(z)$

$$w_0 = -0.752 \pm 0.057$$

$$w_a = -0.86^{+0.23}_{-0.20}$$

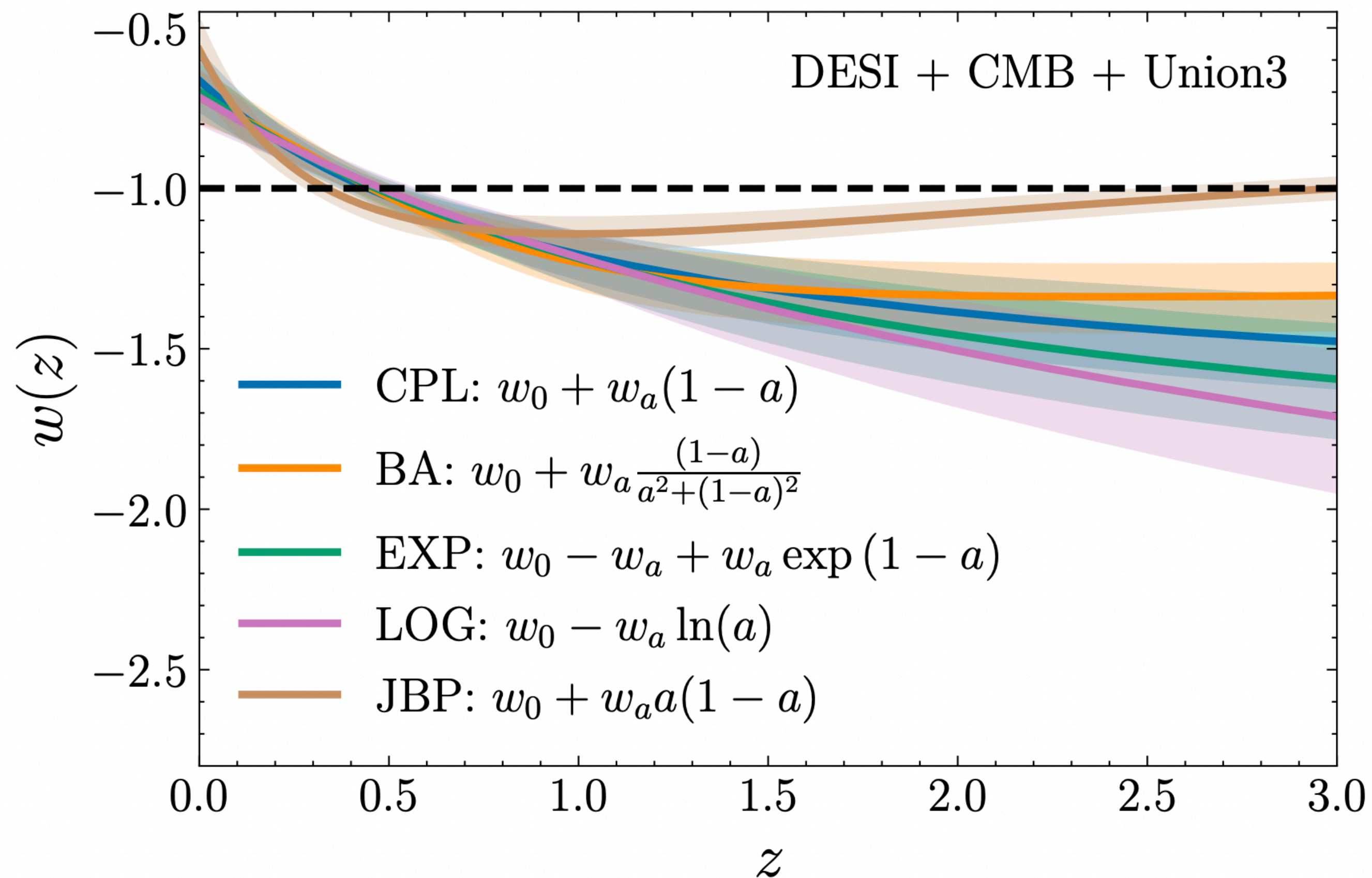
DESI + CMB
+ DESY5

Perhaps DE is dynamical...

See Nandan Roy's Lecture

- Other parametrizations of $w(z)$ DESI 25

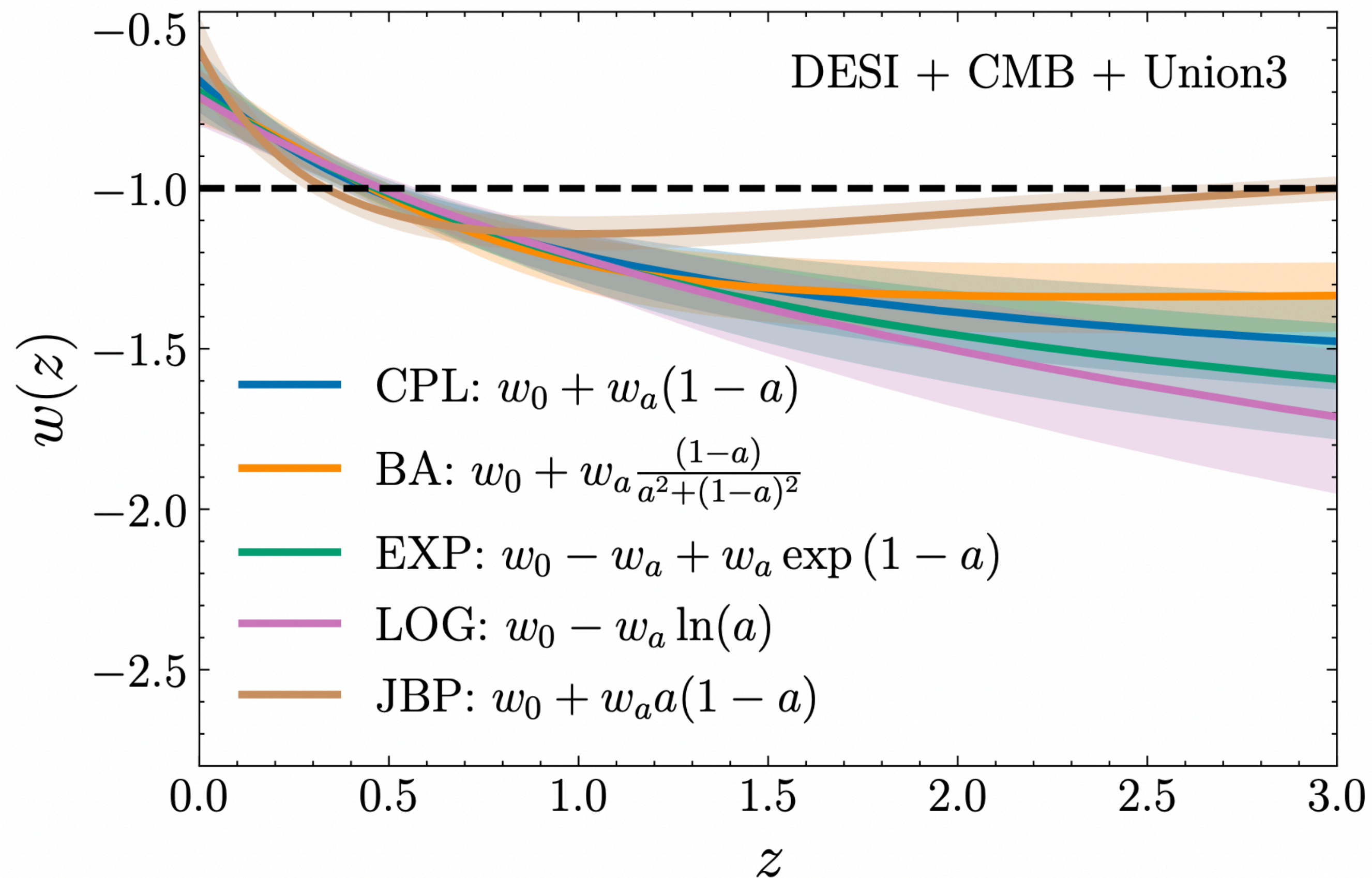
There exists a regime where DE behaves as a **Phantom** field



Perhaps DE is dynamical...

See Nandan Roy's Lecture

- Other parametrizations of $w(z)$ DESI 25



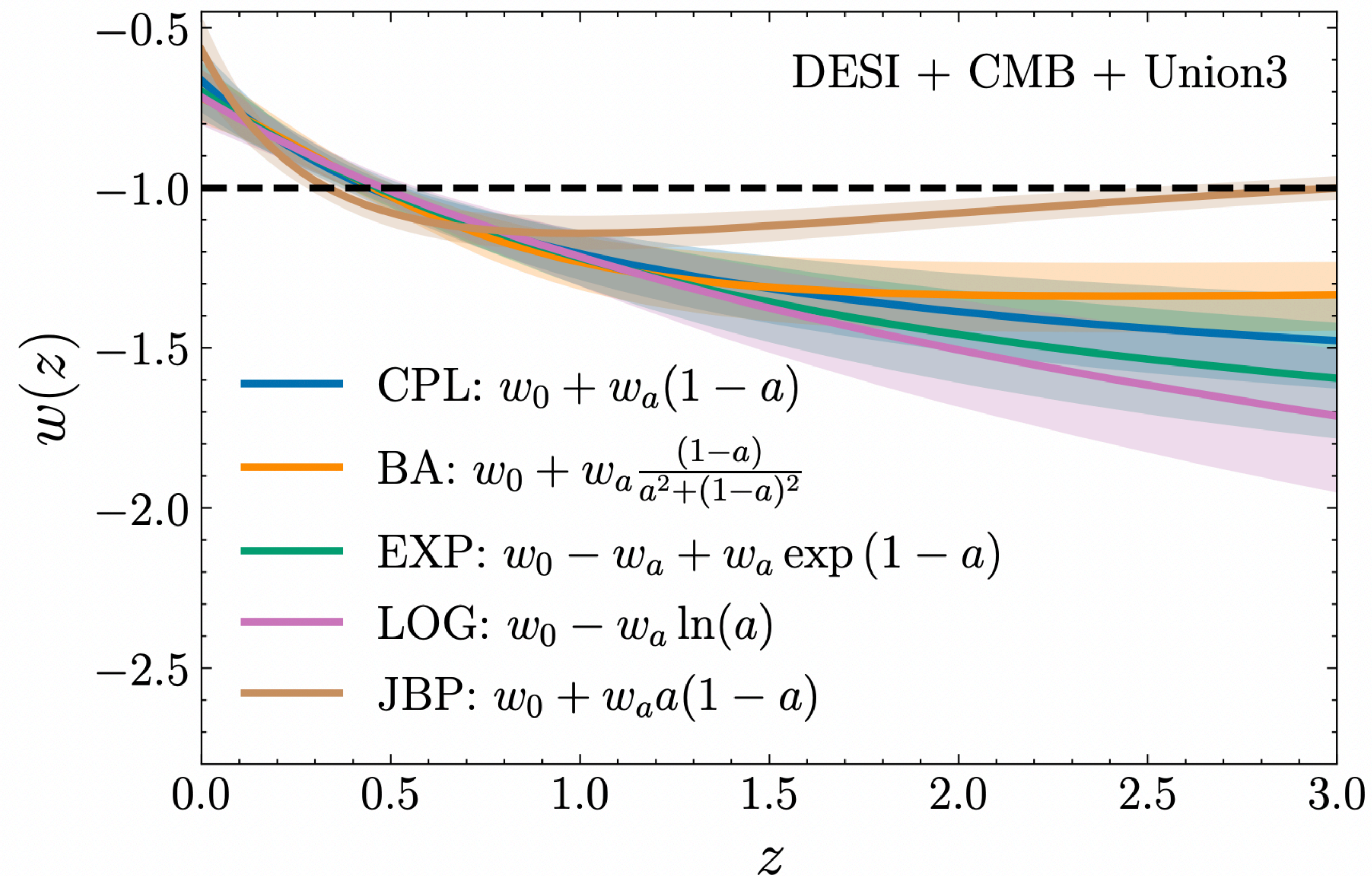
There exists a regime where DE behaves as a **Phantom** field

$$w < -1 \Rightarrow \text{NEC}$$

Perhaps DE is dynamical...

See Nandan Roy's Lecture

- Other parametrizations of $w(z)$ DESI 25



There exists a regime where DE behaves as a **Phantom** field

$$w < -1 \Rightarrow \text{NEC}$$

Proposed **models**

- Thawing DE Caldwell & Linder 05
- Emergent DE Li & Shafieloo 19+20
- Mirage DE Linder 07

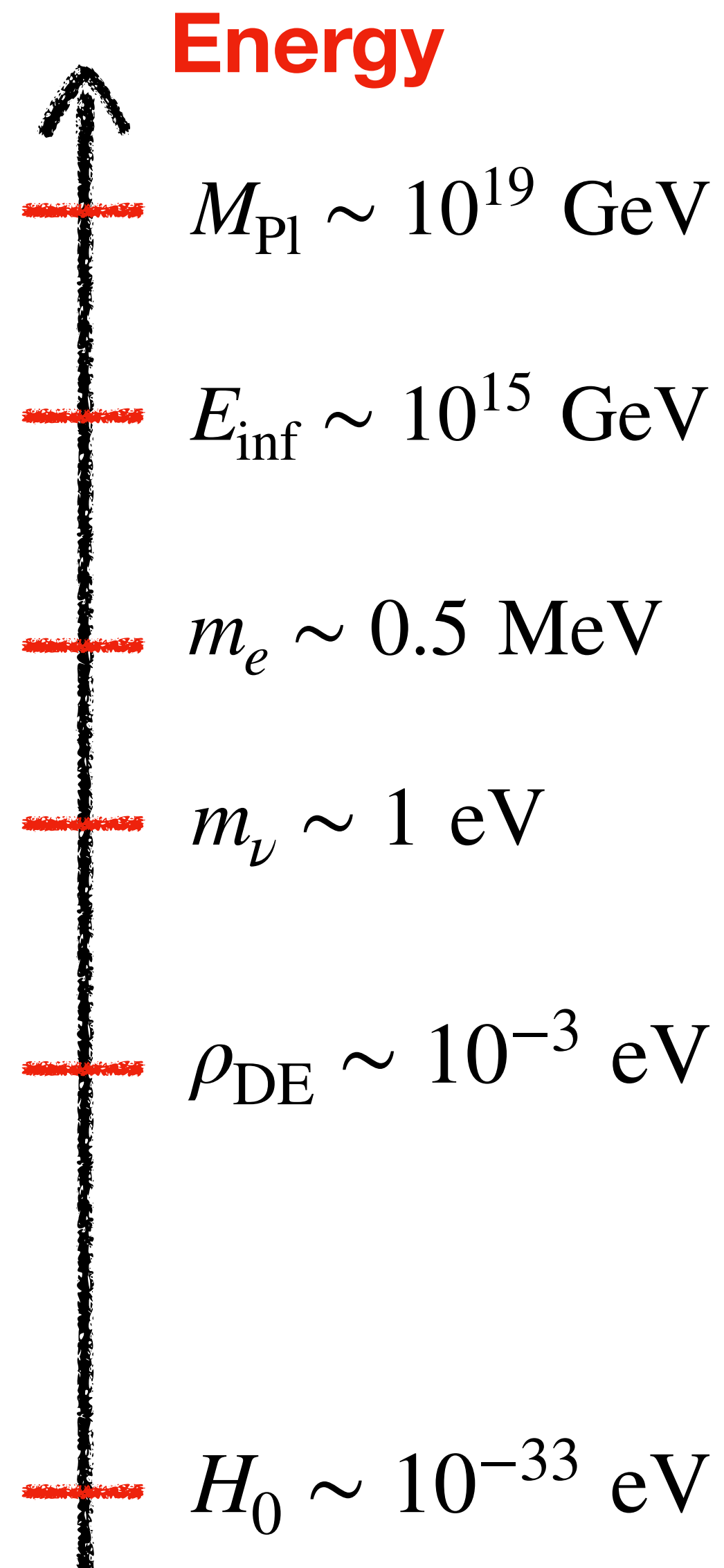


Dark Energy...

$$H_0 = (67 - 74) \frac{\text{km}}{\text{s} \cdot \text{Mpc}}$$

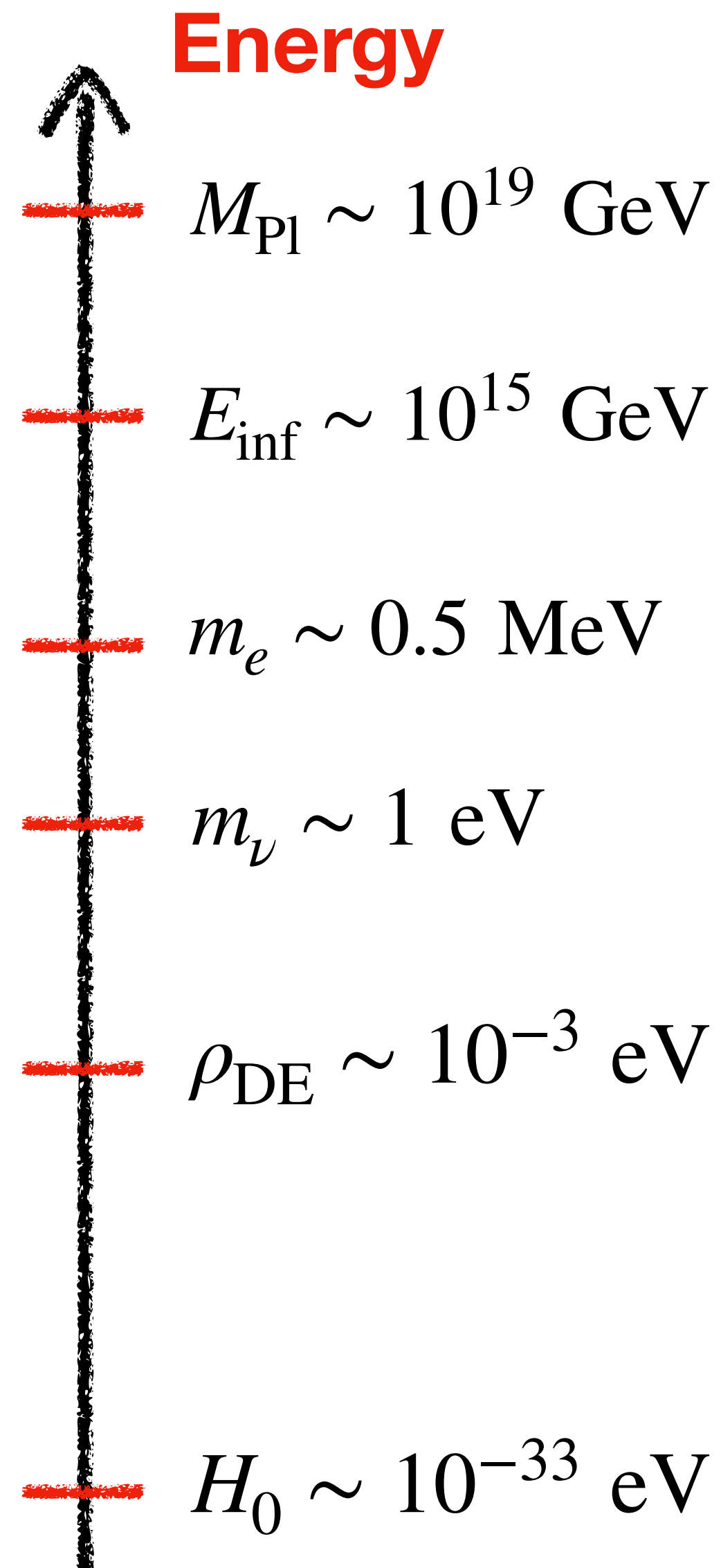
Dark Energy...

$$H_0 = (67 - 74) \frac{\text{km}}{\text{s} \cdot \text{Mpc}}$$



Dark Energy...

$$H_0 = (67 - 74) \frac{\text{km}}{\text{s} \cdot \text{Mpc}}$$



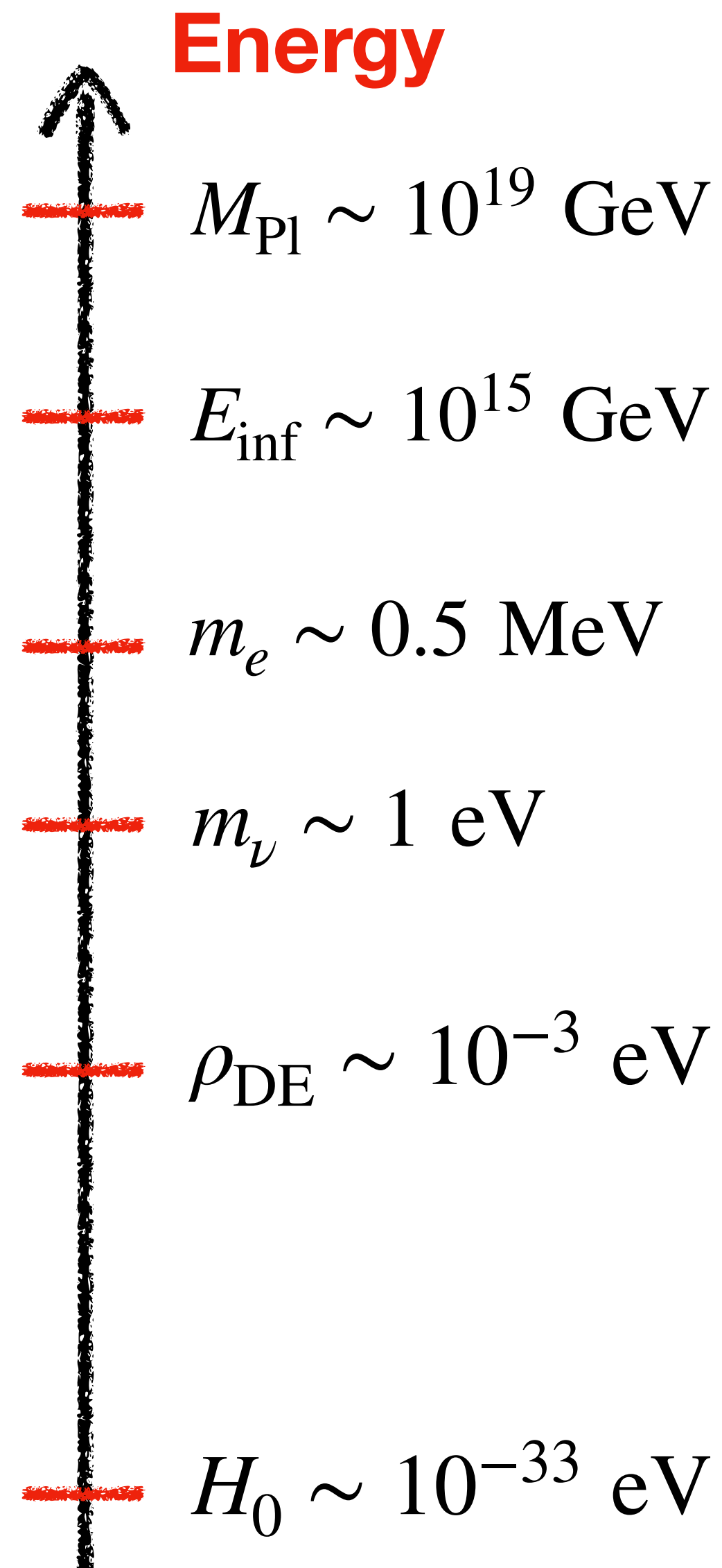
Physics of **Dark Energy** = **Low-Energy** Physics

On large scales, gravity (+ possible **fifth force**) matters

$$\mathcal{L} = \frac{M_{\text{Pl}}^2}{2} R + \mathcal{L}_\phi(\phi, \partial\phi, \partial\partial\phi) + \dots$$

Dark Energy...

$$H_0 = (67 - 74) \frac{\text{km}}{\text{s} \cdot \text{Mpc}}$$



Physics of **Dark Energy** = **Low-Energy** Physics

On large scales, gravity (+ possible **fifth force**) matters

$$\mathcal{L} = \frac{M_{\text{Pl}}^2}{2} R + \mathcal{L}_\phi(\phi, \partial\phi, \partial\partial\phi) + \dots$$

If **Dark Energy** = Cosmological Constant Λ

$$\frac{\rho_\Lambda}{M_{\text{Pl}}^4} \sim 10^{-120}$$

Quintessence Dark Energy

- Additional **scalar field**: pheno. extension of Λ CDM

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]$$

Quintessence Dark Energy

- Additional **scalar field**: pheno. extension of Λ CDM

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]$$

- **EoM** of $\phi(t)$ $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$

$$\bar{\rho}_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad \text{and} \quad \bar{P}_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

Quintessence Dark Energy

- Additional **scalar field**: pheno. extension of Λ CDM

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]$$

- **EoM** of $\phi(t)$ $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$

$$\bar{\rho}_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad \text{and} \quad \bar{P}_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \quad \Rightarrow \quad -1 \leq w_\phi \equiv \frac{\bar{P}_\phi}{\bar{\rho}_\phi} \leq 1$$

Quintessence Dark Energy

- Additional **scalar field**: pheno. extension of Λ CDM

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]$$

- **EoM** of $\phi(t)$ $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$

$$\bar{\rho}_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad \text{and} \quad \bar{P}_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \quad \Rightarrow \quad -1 \leq w_\phi \equiv \frac{\bar{P}_\phi}{\bar{\rho}_\phi} \leq 1$$

- Slow roll $\dot{\phi}^2/2 \ll V$

Quintessence Dark Energy

- Additional **scalar field**: pheno. extension of Λ CDM

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]$$

- **EoM** of $\phi(t)$ $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$

$$\bar{\rho}_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad \text{and} \quad \bar{P}_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \quad \Rightarrow \quad -1 \leq w_\phi \equiv \frac{\bar{P}_\phi}{\bar{\rho}_\phi} \leq 1$$

- Slow roll $\dot{\phi}^2/2 \ll V$ $\Rightarrow$ $H^2 \sim \frac{V}{M_{\text{Pl}}^2}$ and $H\dot{\phi} \sim V'$

Quintessence Dark Energy

- Additional **scalar field**: pheno. extension of Λ CDM

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]$$

- **EoM** of $\phi(t)$ $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$

$$\bar{\rho}_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad \text{and} \quad \bar{P}_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \quad \Rightarrow \quad -1 \leq w_\phi \equiv \frac{\bar{P}_\phi}{\bar{\rho}_\phi} \leq 1$$

- Slow roll $\dot{\phi}^2/2 \ll V \quad \Rightarrow \quad H^2 \sim \frac{V}{M_{\text{Pl}}^2} \quad \text{and} \quad H\dot{\phi} \sim V'$

$$V \sim m_\phi^2 \phi^2 \quad \Rightarrow \quad m_\phi \ll H \sim 10^{-33} \text{ eV} \quad \text{Very light field}$$

Quintessence Dark Energy

- Additional **scalar field**: pheno. extension of Λ CDM

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]$$

- EoM** of $\phi(t)$ $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$

$$\bar{\rho}_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad \text{and} \quad \bar{p}_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \quad \Rightarrow \quad -1 \leq w_\phi \equiv \frac{\bar{p}_\phi}{\bar{\rho}_\phi} \leq 1$$

- Slow roll $\dot{\phi}^2/2 \ll V$ $\Rightarrow$ $H^2 \sim \frac{V}{M_{\text{Pl}}^2}$ and $H\dot{\phi} \sim V'$

$$V \sim m_\phi^2 \phi^2 \quad \Rightarrow \quad m_\phi \ll H \sim 10^{-33} \text{ eV} \quad \text{Very light field}$$

Displacement in
one Hubble time

$$\Delta\phi \sim \frac{\dot{\phi}}{H} \sim \frac{V'}{H^2}$$

$$\frac{\Delta\phi}{\phi} \ll 1 \quad \Rightarrow \quad \frac{V'}{\phi} \ll H^2$$

Quintessence Dark Energy

Steinhardt, Wang, Zlatev 98

- Tracker solution $V(\phi) = \frac{M^{n+4}}{n\phi^n} \quad n > 2$

Quintessence Dark Energy

Steinhardt, Wang, Zlatev 98

- Tracker solution $V(\phi) = \frac{M^{n+4}}{n\phi^n} \quad n > 2$
- EoM of $\phi(t)$ $\ddot{\phi} + \frac{3\alpha}{t}\dot{\phi} - \frac{M^{n+4}}{\phi^{n+1}} = 0$

Quintessence Dark Energy

Steinhardt, Wang, Zlatev 98

- Tracker solution $V(\phi) = \frac{M^{n+4}}{n\phi^n}$ $n > 2$ $a(t) \propto t^\alpha \Rightarrow H = \frac{\alpha}{t}$
- EoM of $\phi(t)$ $\ddot{\phi} + \frac{3\alpha}{t}\dot{\phi} - \frac{M^{n+4}}{\phi^{n+1}} = 0$

Quintessence Dark Energy

Steinhardt, Wang, Zlatev 98

- **Tracker** solution $V(\phi) = \frac{M^{n+4}}{n\phi^n} \quad n > 2 \quad a(t) \propto t^\alpha \Rightarrow H = \frac{\alpha}{t}$

- **EoM** of $\phi(t)$ $\ddot{\phi} + \frac{3\alpha}{t}\dot{\phi} - \frac{M^{n+4}}{\phi^{n+1}} = 0$

$$\alpha = 2/3 \text{ (MD)}$$

$$\alpha = 1/2 \text{ (RD)}$$

Quintessence Dark Energy

Steinhardt, Wang, Zlatev 98

- **Tracker** solution $V(\phi) = \frac{M^{n+4}}{n\phi^n}$ $n > 2$ $a(t) \propto t^\alpha \Rightarrow H = \frac{\alpha}{t}$

- **EoM** of $\phi(t)$ $\ddot{\phi} + \frac{3\alpha}{t}\dot{\phi} - \frac{M^{n+4}}{\phi^{n+1}} = 0$

$$\phi(t) = Ct^\beta \Rightarrow \beta = \frac{2}{n+2} \quad \text{and} \quad \beta(\beta - 1 + 3\alpha) = \frac{M^{n+4}}{C^{n+2}}$$

$$\alpha = 2/3 \text{ (MD)}$$

$$\alpha = 1/2 \text{ (RD)}$$

Quintessence Dark Energy

Steinhardt, Wang, Zlatev 98

- **Tracker** solution $V(\phi) = \frac{M^{n+4}}{n\phi^n}$ $n > 2$ $a(t) \propto t^\alpha \Rightarrow H = \frac{\alpha}{t}$

- **EoM** of $\phi(t)$ $\ddot{\phi} + \frac{3\alpha}{t}\dot{\phi} - \frac{M^{n+4}}{\phi^{n+1}} = 0$

$$\alpha = 2/3 \text{ (MD)}$$

$$\alpha = 1/2 \text{ (RD)}$$

$$\phi(t) = Ct^\beta \Rightarrow \beta = \frac{2}{n+2} \text{ and } \beta(\beta - 1 + 3\alpha) = \frac{M^{n+4}}{C^{n+2}}$$

Attractor solution $\phi = Ct^\beta + \varphi \Rightarrow \varphi \propto t^\gamma$ with γ is complex

Quintessence Dark Energy

Steinhardt, Wang, Zlatev 98

- **Tracker** solution $V(\phi) = \frac{M^{n+4}}{n\phi^n}$ $n > 2$ $a(t) \propto t^\alpha \Rightarrow H = \frac{\alpha}{t}$

- **EoM** of $\phi(t)$ $\ddot{\phi} + \frac{3\alpha}{t}\dot{\phi} - \frac{M^{n+4}}{\phi^{n+1}} = 0$

$$\alpha = 2/3 \text{ (MD)}$$

$$\alpha = 1/2 \text{ (RD)}$$

$$\phi(t) = Ct^\beta \Rightarrow \beta = \frac{2}{n+2} \text{ and } \beta(\beta - 1 + 3\alpha) = \frac{M^{n+4}}{C^{n+2}}$$

Attractor solution $\phi = Ct^\beta + \varphi \Rightarrow \varphi \propto t^\gamma$ with γ is complex

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \propto t^{-2n/(n+2)}$$

Quintessence Dark Energy

Steinhardt, Wang, Zlatev 98

- **Tracker** solution $V(\phi) = \frac{M^{n+4}}{n\phi^n}$ $n > 2$ $a(t) \propto t^\alpha \Rightarrow H = \frac{\alpha}{t}$

- **EoM** of $\phi(t)$ $\ddot{\phi} + \frac{3\alpha}{t}\dot{\phi} - \frac{M^{n+4}}{\phi^{n+1}} = 0$

$$\alpha = 2/3 \text{ (MD)}$$

$$\alpha = 1/2 \text{ (RD)}$$

$$\phi(t) = Ct^\beta \Rightarrow \beta = \frac{2}{n+2} \text{ and } \beta(\beta - 1 + 3\alpha) = \frac{M^{n+4}}{C^{n+2}}$$

Attractor solution $\phi = Ct^\beta + \varphi \Rightarrow \varphi \propto t^\gamma$ with γ is complex

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \propto t^{-\frac{2n}{n+2}}$$

ρ_ϕ decreases slower than ρ_R and ρ_M

$$\rho_R \propto a^{-4} \propto t^{-2}$$

$$\rho_M \propto a^{-3} \propto t^{-2}$$

Quintessence Dark Energy

- **EoS** parameter $n > 2$
$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} = \frac{n}{n+2}w_i - \frac{2}{n+2}$$
$$\alpha = \frac{2}{3(1+w_i)} \quad \text{and} \quad V(\phi) \sim \dot{\phi}^2 \sim \frac{\phi^2}{t^2}$$

w_i of component that dominates Universe

Not slow roll

Quintessence Dark Energy

- **EoS** parameter $n > 2$

$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} = \frac{n}{n+2}w_i - \frac{2}{n+2}$$

$$\alpha = \frac{2}{3(1+w_i)} \quad \text{and} \quad V(\phi) \sim \dot{\phi}^2 \sim \frac{\phi^2}{t^2}$$

w_i of component that dominates Universe

Not slow roll



Quintessence Dark Energy

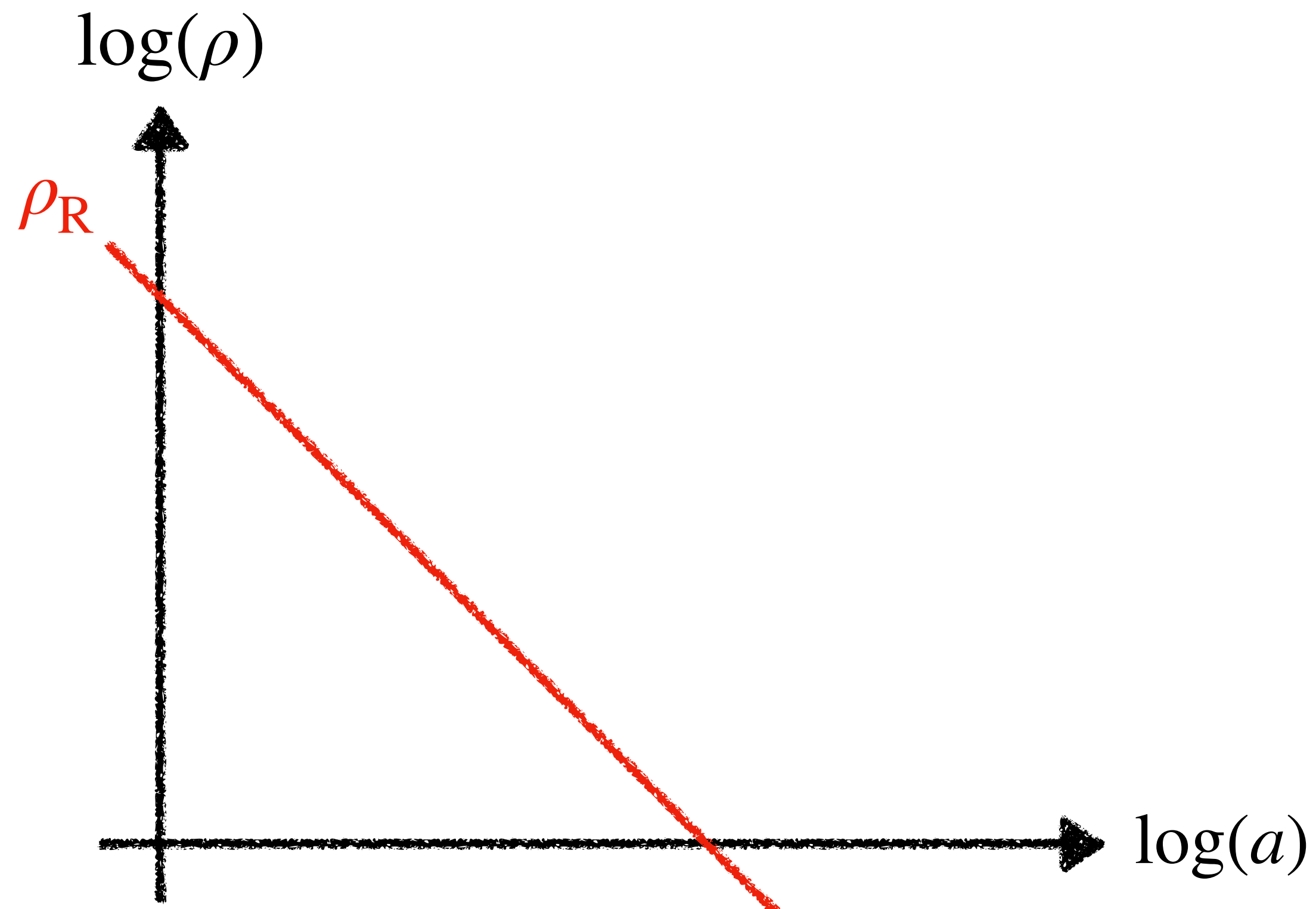
- **EoS** parameter $n > 2$

$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} = \frac{n}{n+2}w_i - \frac{2}{n+2}$$

$$\alpha = \frac{2}{3(1+w_i)} \quad \text{and} \quad V(\phi) \sim \dot{\phi}^2 \sim \frac{\phi^2}{t^2}$$

w_i of component that dominates Universe

Not slow roll



Quintessence Dark Energy

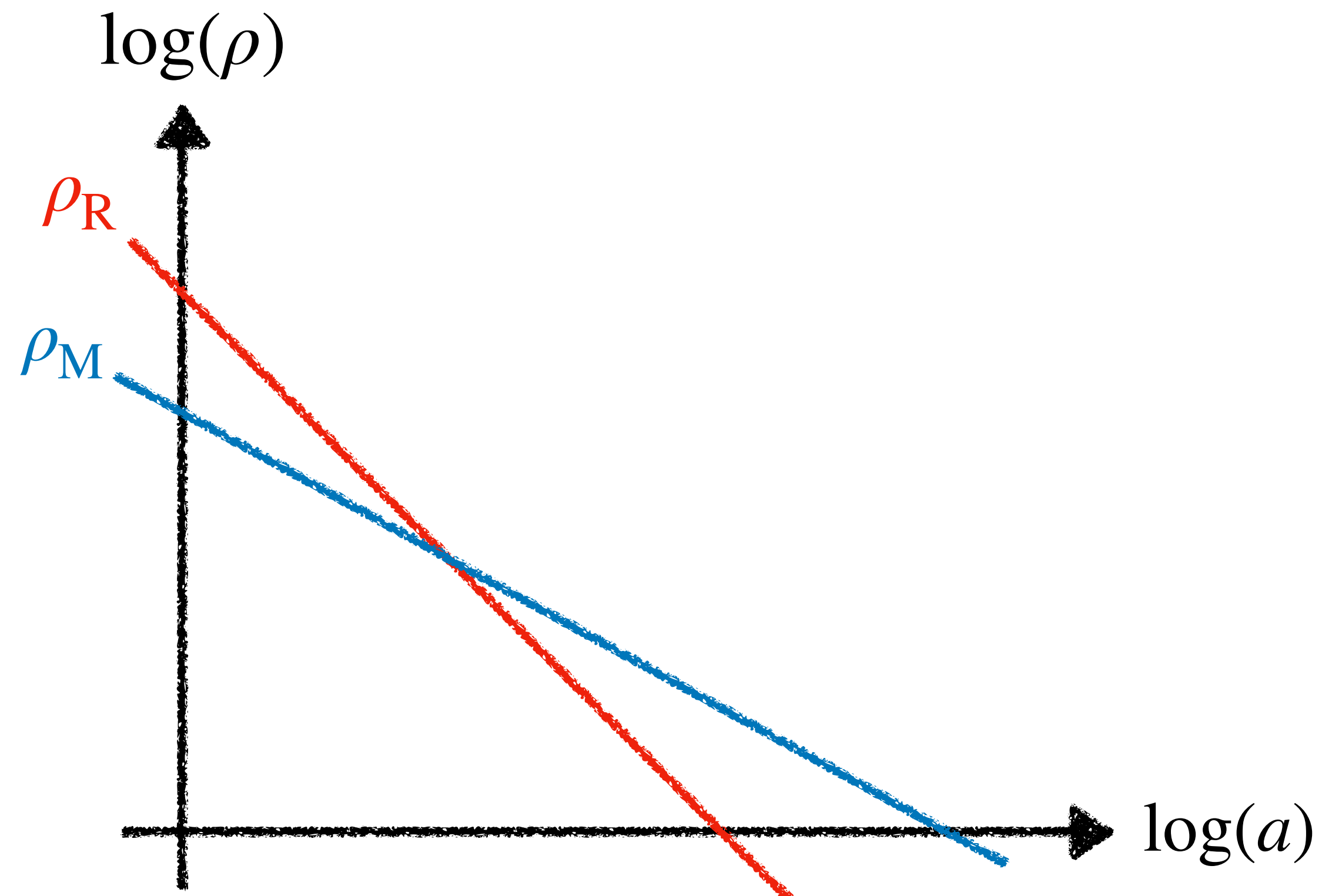
- **EoS** parameter $n > 2$

$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} = \frac{n}{n+2}w_i - \frac{2}{n+2}$$

$$\alpha = \frac{2}{3(1+w_i)} \quad \text{and} \quad V(\phi) \sim \dot{\phi}^2 \sim \frac{\phi^2}{t^2}$$

w_i of component that dominates Universe

Not slow roll



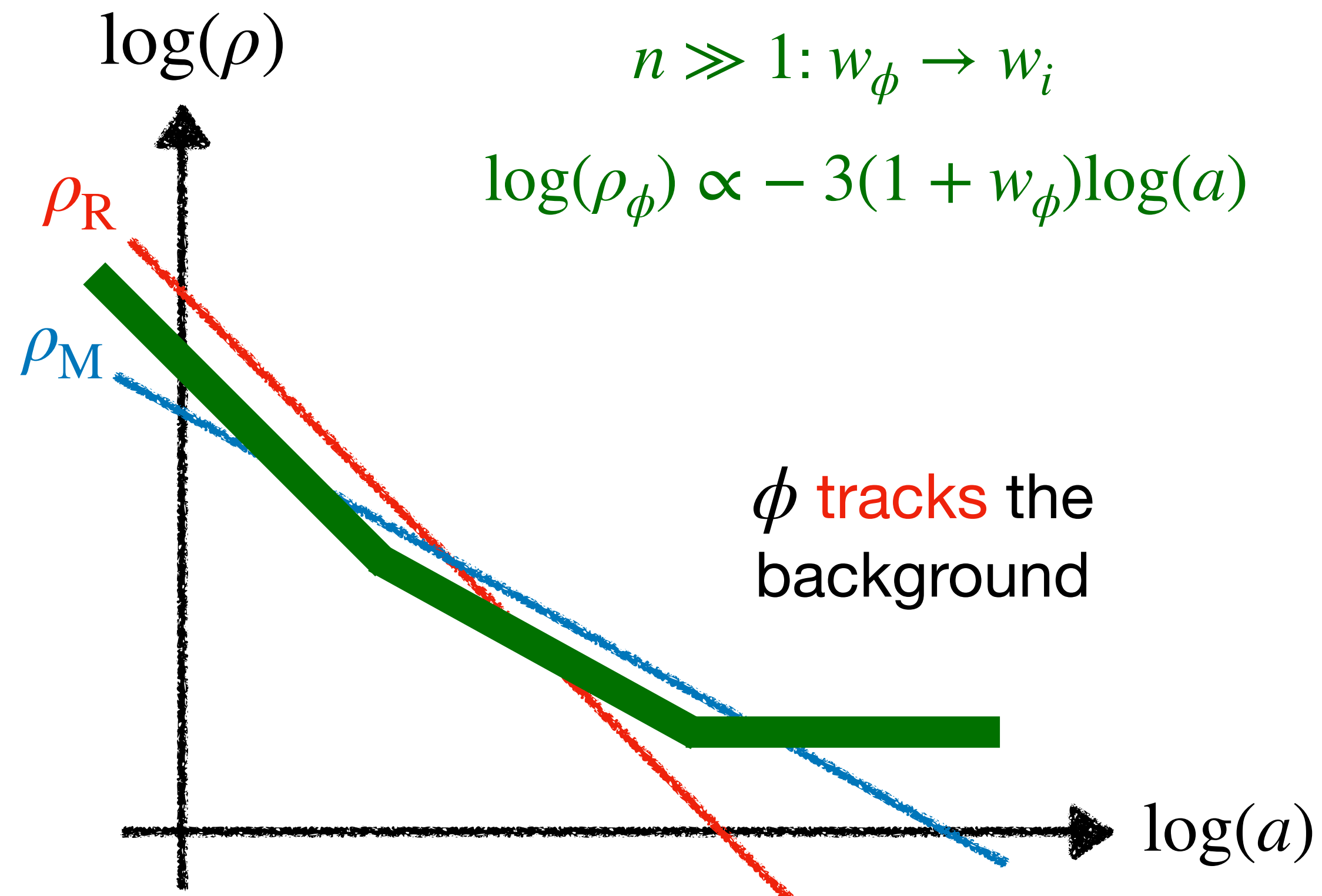
Quintessence Dark Energy

- **EoS** parameter $n > 2$

$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} = \frac{n}{n+2}w_i - \frac{2}{n+2}$$

$$\alpha = \frac{2}{3(1+w_i)} \quad \text{and} \quad V(\phi) \sim \dot{\phi}^2 \sim \frac{\phi^2}{t^2}$$

w_i of component that dominates Universe Not slow roll



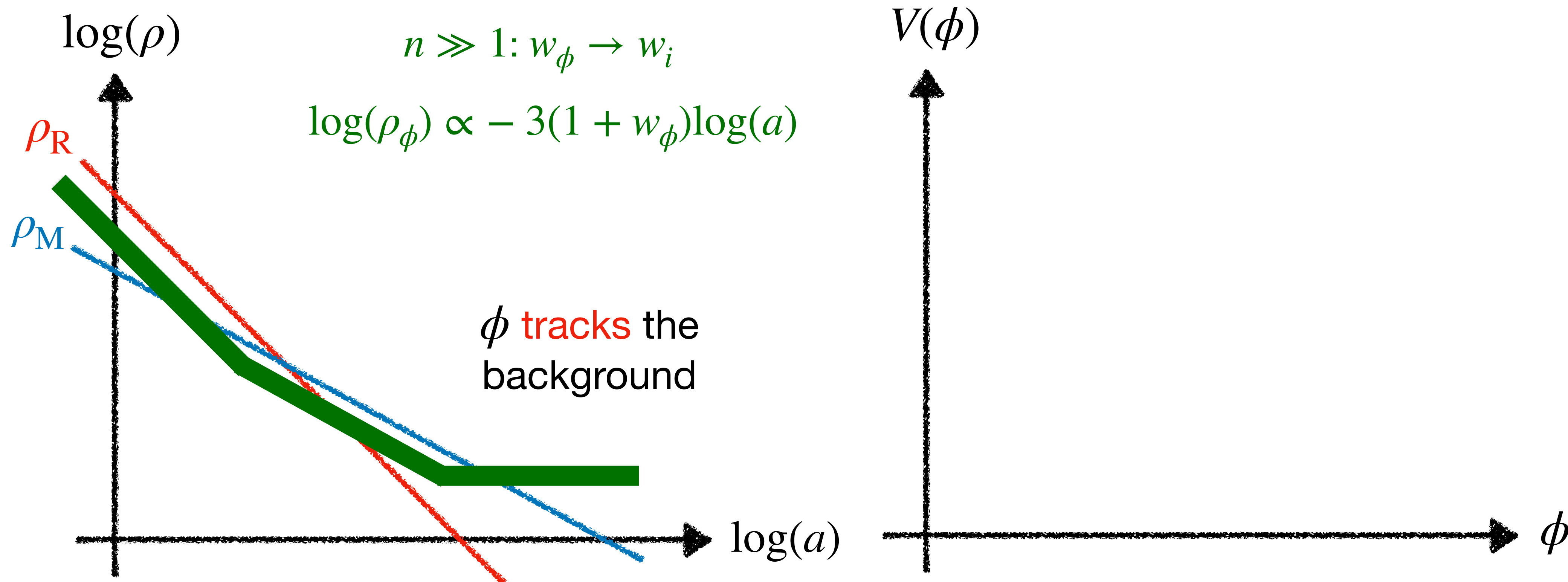
Quintessence Dark Energy

- EoS parameter $n > 2$

$$\alpha = \frac{2}{3(1 + w_i)} \quad \text{and} \quad V(\phi) \sim \dot{\phi}^2 \sim \frac{\phi^2}{t^2} \quad \text{Not slow roll}$$

$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} = \frac{n}{n+2}w_i - \frac{2}{n+2}$$

w_i of component that dominates Universe



Quintessence Dark Energy

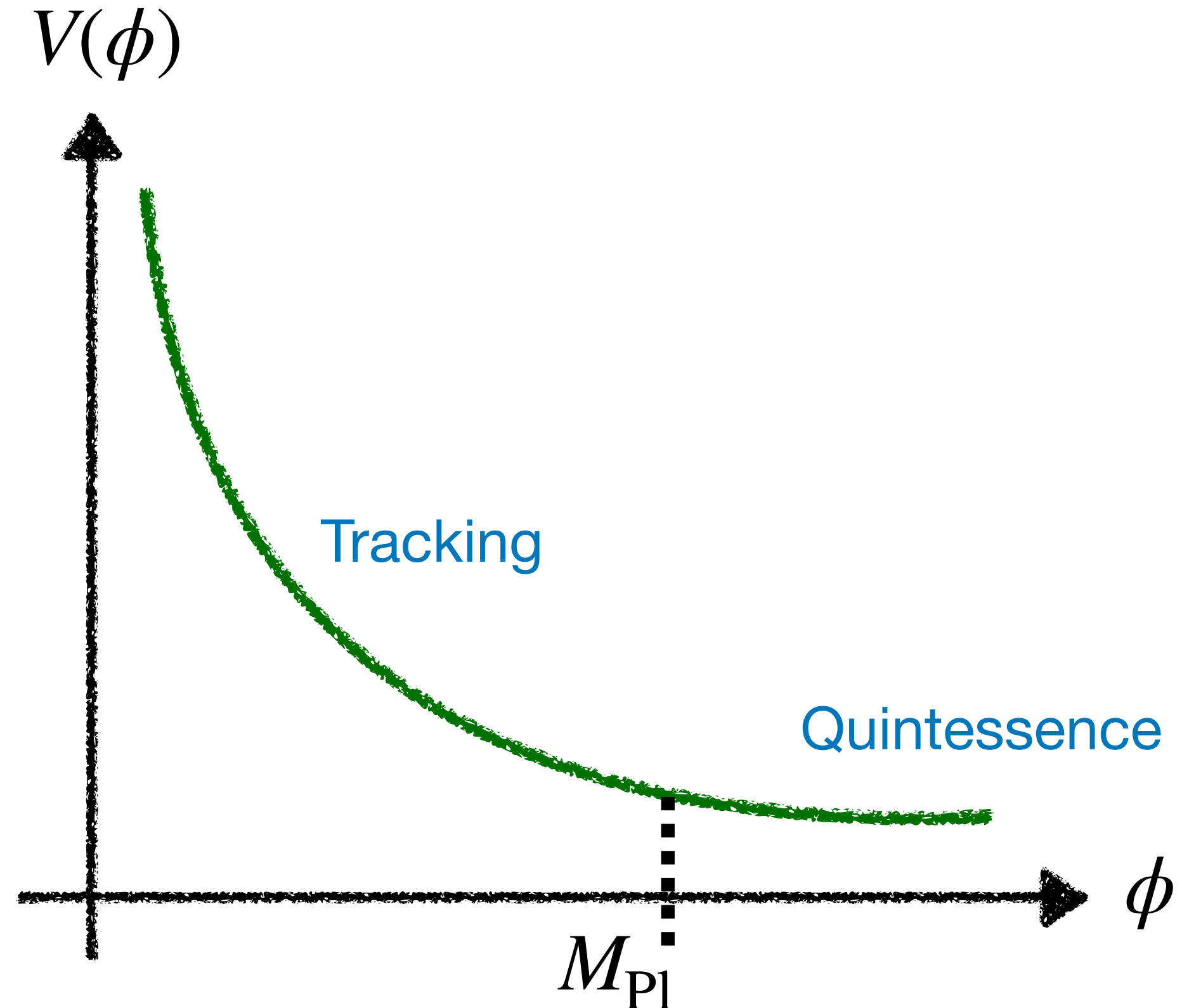
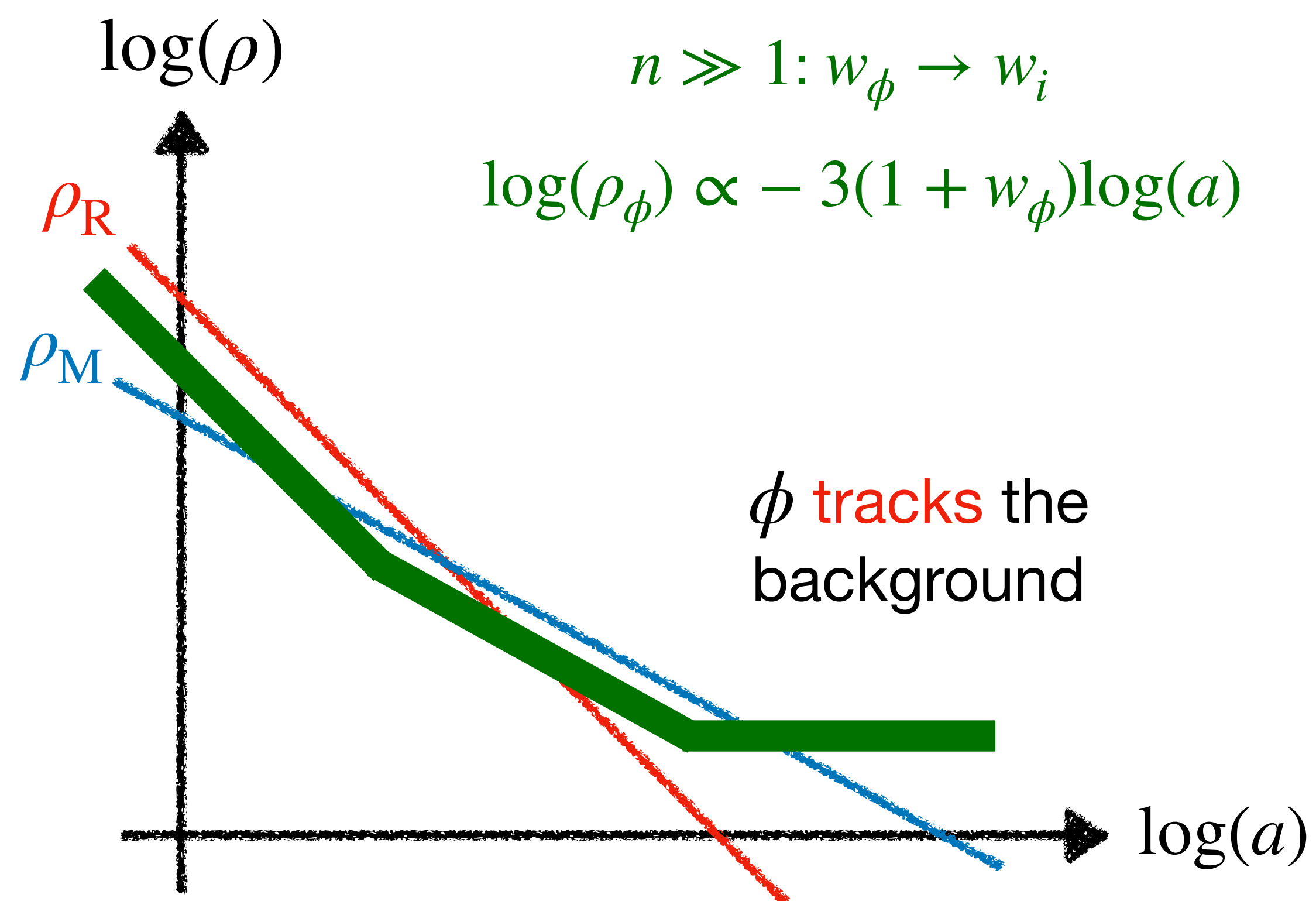
- **EoS** parameter $n > 2$

$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} = \frac{n}{n+2}w_i - \frac{2}{n+2}$$

$$\alpha = \frac{2}{3(1+w_i)} \quad \text{and} \quad V(\phi) \sim \dot{\phi}^2 \sim \frac{\phi^2}{t^2}$$

w_i of component that dominates Universe

Not slow roll



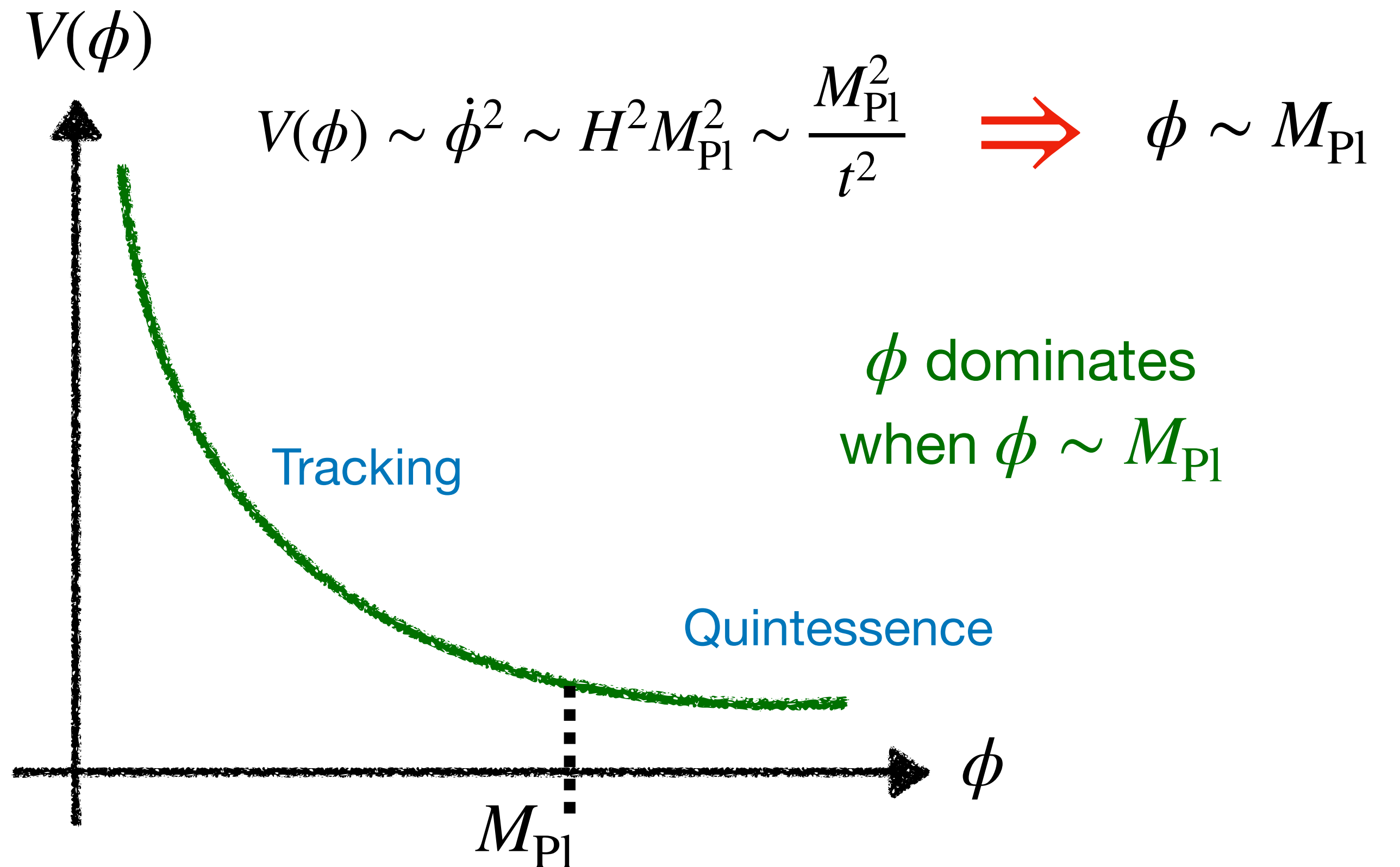
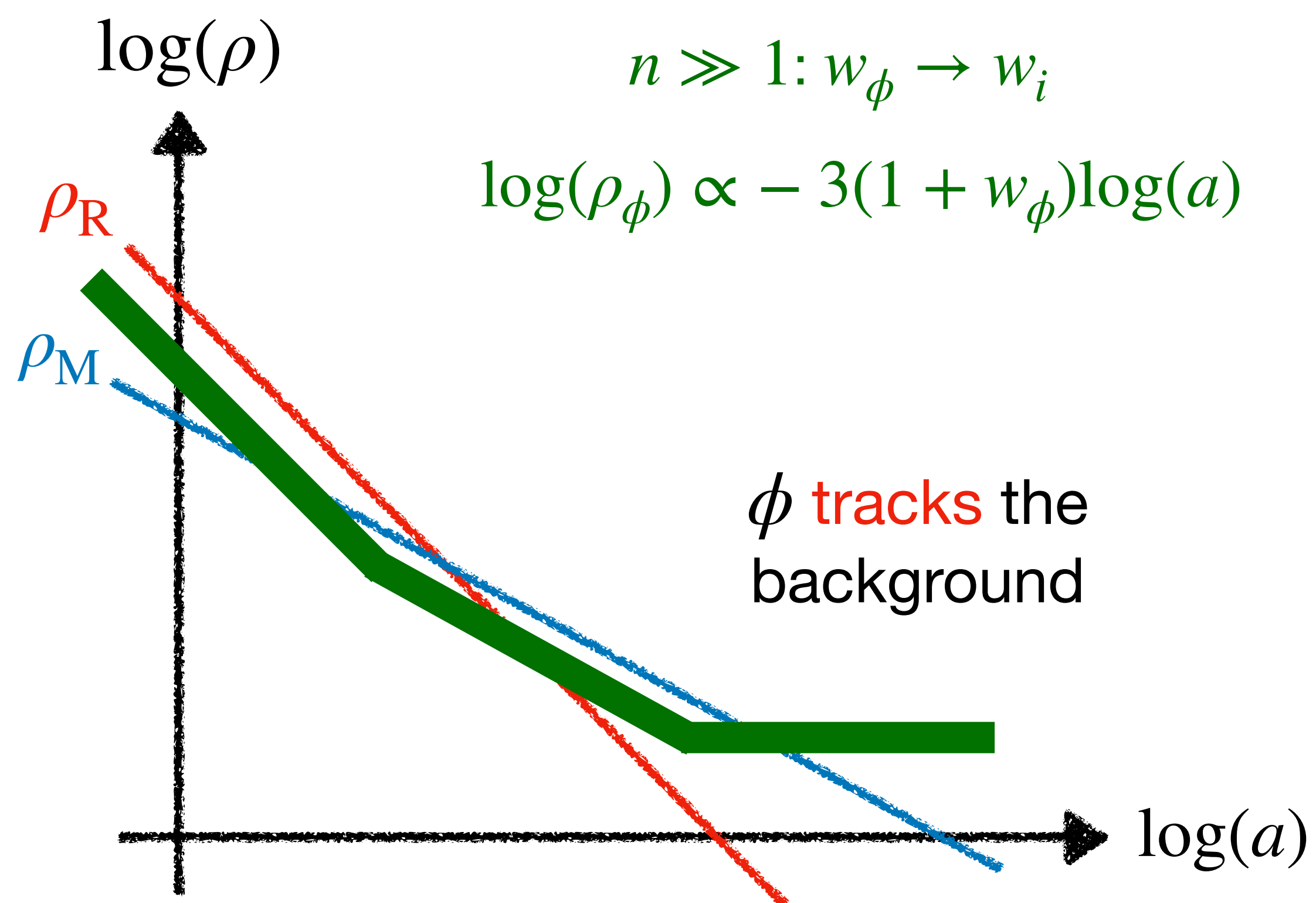
Quintessence Dark Energy

- EoS parameter $n > 2$

$$\alpha = \frac{2}{3(1 + w_i)} \quad \text{and} \quad V(\phi) \sim \dot{\phi}^2 \sim \frac{\phi^2}{t^2} \quad \text{Not slow roll}$$

$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} = \frac{n}{n+2}w_i - \frac{2}{n+2}$$

w_i of component that dominates Universe



Quintessence Dark Energy

- **Perturbations** of Quintessence $\phi(t, \vec{x}) = \phi_0(t) + \varphi(t, \vec{x})$

Quintessence Dark Energy

- **Perturbations** of Quintessence $\phi(t, \vec{x}) = \phi_0(t) + \varphi(t, \vec{x})$

$$\Rightarrow g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi = g^{00} (\dot{\phi}_0 + \dot{\varphi})^2 + 2g^{0i} (\dot{\phi}_0 + \dot{\varphi}) \partial_i \varphi + g^{ij} \partial_i \varphi \partial_j \varphi$$

Quintessence Dark Energy

- **Perturbations** of Quintessence $\phi(t, \vec{x}) = \phi_0(t) + \varphi(t, \vec{x})$

$$\begin{aligned} \Rightarrow g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi &= g^{00} (\dot{\phi}_0 + \dot{\varphi})^2 + 2g^{0i} (\dot{\phi}_0 + \dot{\varphi}) \partial_i \varphi + g^{ij} \partial_i \varphi \partial_j \varphi \\ &= \text{Zero-order} + \text{First-order} - \dot{\varphi}^2 + \frac{(\partial_i \varphi)^2}{a^2} \end{aligned}$$

Quintessence Dark Energy

- **Perturbations** of Quintessence $\phi(t, \vec{x}) = \phi_0(t) + \varphi(t, \vec{x})$

$$\Rightarrow g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi = g^{00} (\dot{\phi}_0 + \dot{\varphi})^2 + 2g^{0i} (\dot{\phi}_0 + \dot{\varphi}) \partial_i \varphi + g^{ij} \partial_i \varphi \partial_j \varphi$$

$$= \text{Zero-order} + \text{First-order} - \dot{\varphi}^2 + \frac{(\partial_i \varphi)^2}{a^2}$$

Background

Quintessence Dark Energy

- **Perturbations** of Quintessence $\phi(t, \vec{x}) = \phi_0(t) + \varphi(t, \vec{x})$

$$\Rightarrow g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi = g^{00} (\dot{\phi}_0 + \dot{\varphi})^2 + 2g^{0i} (\dot{\phi}_0 + \dot{\varphi}) \partial_i \varphi + g^{ij} \partial_i \varphi \partial_j \varphi$$

$$= \text{Zero-order} + \text{First-order} - \dot{\varphi}^2 + \frac{(\partial_i \varphi)^2}{a^2}$$

Background

Tadpole = 0 by
background EoMs

Quintessence Dark Energy

- **Perturbations** of Quintessence $\phi(t, \vec{x}) = \phi_0(t) + \varphi(t, \vec{x})$

$$\Rightarrow g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi = g^{00} (\dot{\phi}_0 + \dot{\varphi})^2 + 2g^{0i} (\dot{\phi}_0 + \dot{\varphi}) \partial_i \varphi + g^{ij} \partial_i \varphi \partial_j \varphi$$

$$= \text{Zero-order} + \text{First-order} - \dot{\varphi}^2 + \frac{(\partial_i \varphi)^2}{a^2}$$

Background

Tadpole = 0 by
background EoMs

$$\Rightarrow V(\phi) = V(\phi_0 + \varphi) = V(\phi_0) + V'(\phi_0)\varphi + \frac{1}{2}V''(\phi_0)\varphi^2 + \dots$$

Quintessence Dark Energy

- **Perturbations** of Quintessence $\phi(t, \vec{x}) = \phi_0(t) + \varphi(t, \vec{x})$

$$\Rightarrow g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi = g^{00} (\dot{\phi}_0 + \dot{\varphi})^2 + 2g^{0i} (\dot{\phi}_0 + \dot{\varphi}) \partial_i \varphi + g^{ij} \partial_i \varphi \partial_j \varphi$$

$$= \text{Zero-order} + \text{First-order} - \dot{\varphi}^2 + \frac{(\partial_i \varphi)^2}{a^2}$$

Background

Tadpole = 0 by
background EoMs

$$\Rightarrow V(\phi) = V(\phi_0 + \varphi) = V(\phi_0) + V'(\phi_0)\varphi + \frac{1}{2}V''(\phi_0)\varphi^2 + \dots$$

Quadratic action for
 $\varphi(t, \vec{x})$:

$$S_\varphi^{(2)} = \int d^3x dt a^3 \left[\frac{1}{2} \dot{\varphi}^2 - \frac{1}{2a^2} (\partial_i \varphi)^2 + \dots \right]$$

Quintessence Dark Energy

- **Perturbations** of Quintessence $\phi(t, \vec{x}) = \phi_0(t) + \varphi(t, \vec{x})$

$$\Rightarrow g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi = g^{00} (\dot{\phi}_0 + \dot{\varphi})^2 + 2g^{0i} (\dot{\phi}_0 + \dot{\varphi}) \partial_i \varphi + g^{ij} \partial_i \varphi \partial_j \varphi$$

$$= \text{Zero-order} + \text{First-order} - \dot{\varphi}^2 + \frac{(\partial_i \varphi)^2}{a^2}$$

Background Tadpole = 0 by
background EoMs

$$\Rightarrow V(\phi) = V(\phi_0 + \varphi) = V(\phi_0) + V'(\phi_0)\varphi + \frac{1}{2}V''(\phi_0)\varphi^2 + \dots$$

Quadratic action for
 $\varphi(t, \vec{x})$:

$$S_\varphi^{(2)} = \int d^3x dt a^3 \left[\frac{1}{2} \dot{\varphi}^2 - \frac{1}{2a^2} (\partial_i \varphi)^2 + \dots \right] \Rightarrow$$

$$\omega^2 = k^2$$

The speed of φ = speed of light

Subhorizon limit: $k \gg aH$

k-essence Dark Energy

- **k-essence** model = clustering of quintessence

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R + P(X, \phi) \right] \quad X \equiv (\partial_\mu \phi)^2$$

k-essence Dark Energy

- **k-essence** model = clustering of quintessence

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R + P(X, \phi) \right] \quad X \equiv (\partial_\mu \phi)^2$$

$$u_\mu = - \frac{\partial_\mu \phi}{\sqrt{-X}}$$

$$T_{\mu\nu} = 2X P_X \partial_\mu \phi \partial_\nu \phi + g_{\mu\nu} P$$

k-essence Dark Energy

- **k-essence** model = clustering of quintessence

$$u_\mu = -\frac{\partial_\mu \phi}{\sqrt{-X}}$$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R + P(X, \phi) \right] \quad X \equiv (\partial_\mu \phi)^2$$

$$T_{\mu\nu} = 2XP_X \partial_\mu \phi \partial_\nu \phi + g_{\mu\nu} P$$

- Match with the perfect fluid $T^\mu_\nu = \text{diag}(-\rho, p, p, p) \Rightarrow \rho = 2XP_X - P \quad p = P(X, \phi)$

k-essence Dark Energy

- **k-essence** model = clustering of quintessence

$$u_\mu = -\frac{\partial_\mu \phi}{\sqrt{-X}}$$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R + P(X, \phi) \right] \quad X \equiv (\partial_\mu \phi)^2$$

$$T_{\mu\nu} = 2XP_X \partial_\mu \phi \partial_\nu \phi + g_{\mu\nu} P$$

- Match with the perfect fluid $T^\mu_\nu = \text{diag}(-\rho, p, p, p) \Rightarrow \rho = 2XP_X - P \quad p = P(X, \phi)$
- Neglect ϕ -dependence: $P = P(X) = \text{Superfluid}$

k-essence Dark Energy

- **k-essence** model = clustering of quintessence

$$u_\mu = -\frac{\partial_\mu \phi}{\sqrt{-X}}$$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R + P(X, \phi) \right] \quad X \equiv (\partial_\mu \phi)^2 \quad T_{\mu\nu} = 2XP_X \partial_\mu \phi \partial_\nu \phi + g_{\mu\nu} P$$

- Match with the perfect fluid $T^\mu_\nu = \text{diag}(-\rho, p, p, p) \Rightarrow \rho = 2XP_X - P \quad p = P(X, \phi)$
- Neglect ϕ -dependence: $P = P(X) = \text{Superfluid}$
- **Expand**: $\phi = ct + \varphi(t, \vec{x})$

$$\mathcal{L}_\varphi^{(2)} = P_X(c^2) [\dot{\varphi}^2 - (\nabla_i \varphi)^2] + 2P_{XX}(c^2) c^2 \dot{\varphi}^2$$

k-essence Dark Energy

- **k-essence** model = clustering of quintessence

$$u_\mu = - \frac{\partial_\mu \phi}{\sqrt{-X}}$$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R + P(X, \phi) \right] \quad X \equiv (\partial_\mu \phi)^2 \quad T_{\mu\nu} = 2XP_X \partial_\mu \phi \partial_\nu \phi + g_{\mu\nu} P$$

- Match with the perfect fluid $T^\mu_\nu = \text{diag}(-\rho, p, p, p) \Rightarrow \rho = 2XP_X - P \quad p = P(X, \phi)$
- Neglect ϕ -dependence: $P = P(X) = \text{Superfluid}$

- **Expand:** $\phi = ct + \varphi(t, \vec{x})$

Clustering allows φ to travel with speed $\neq$ speed of light

$$\mathcal{L}_\varphi^{(2)} = P_X(c^2) [\dot{\varphi}^2 - (\nabla_i \varphi)^2] + 2P_{XX}(c^2) c^2 \dot{\varphi}^2 \Rightarrow$$

$$c_s^2 = \frac{P_X(c^2)}{P_X(c^2) + 2XP_{XX}(c^2)}$$

$c_s^2 \ll 1$, DE actively contributes to **structure formation** as DM

k-essence Dark Energy

- **k-essence** model = clustering of quintessence

$$u_\mu = - \frac{\partial_\mu \phi}{\sqrt{-X}}$$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R + P(X, \phi) \right] \quad X \equiv (\partial_\mu \phi)^2 \quad T_{\mu\nu} = 2XP_X \partial_\mu \phi \partial_\nu \phi + g_{\mu\nu} P$$

- Match with the perfect fluid $T^\mu_\nu = \text{diag}(-\rho, p, p, p) \Rightarrow \rho = 2XP_X - P \quad p = P(X, \phi)$
- Neglect ϕ -dependence: $P = P(X) = \text{Superfluid}$

- **Expand:** $\phi = ct + \varphi(t, \vec{x})$

Clustering allows φ to travel with speed $\neq$ speed of light

For $P(X) = X^2$

$$\mathcal{L}_\varphi^{(2)} = P_X(c^2) [\dot{\varphi}^2 - (\nabla_i \varphi)^2] + 2P_{XX}(c^2) c^2 \dot{\varphi}^2 \Rightarrow$$

$$c_s^2 = \frac{P_X(c^2)}{P_X(c^2) + 2XP_{XX}(c^2)}$$

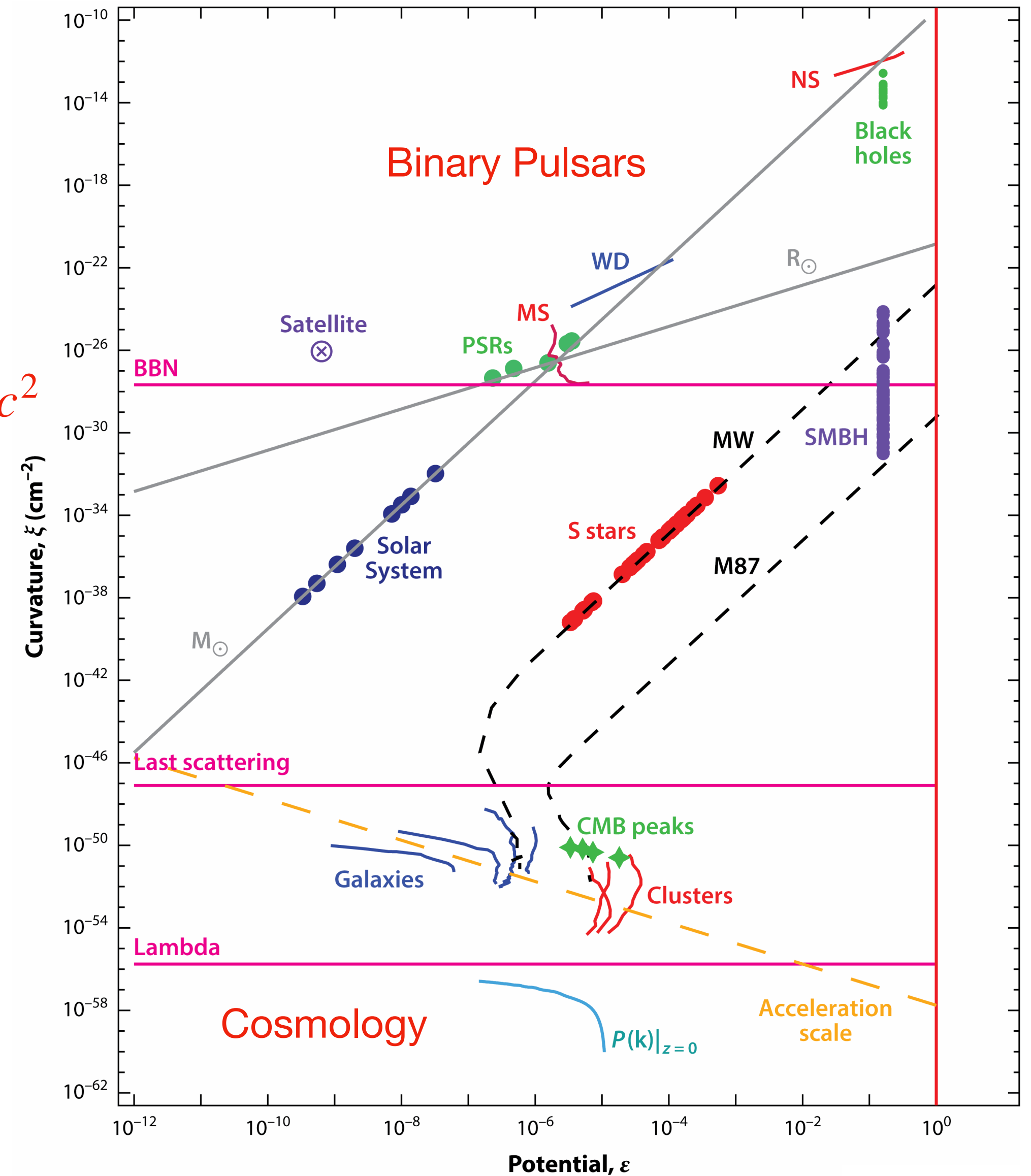
$$c_s^2 = \frac{1}{3} \text{ (Rad fluid)}$$

$c_s^2 \ll 1$, DE actively contributes to **structure formation** as DM

Gravity on large scales

- Accelerated expansion of our Universe today leads to a lot of activities trying to extend gravity and cosmology beyond Λ CDM

$$\xi = GM/r^3 c^2$$



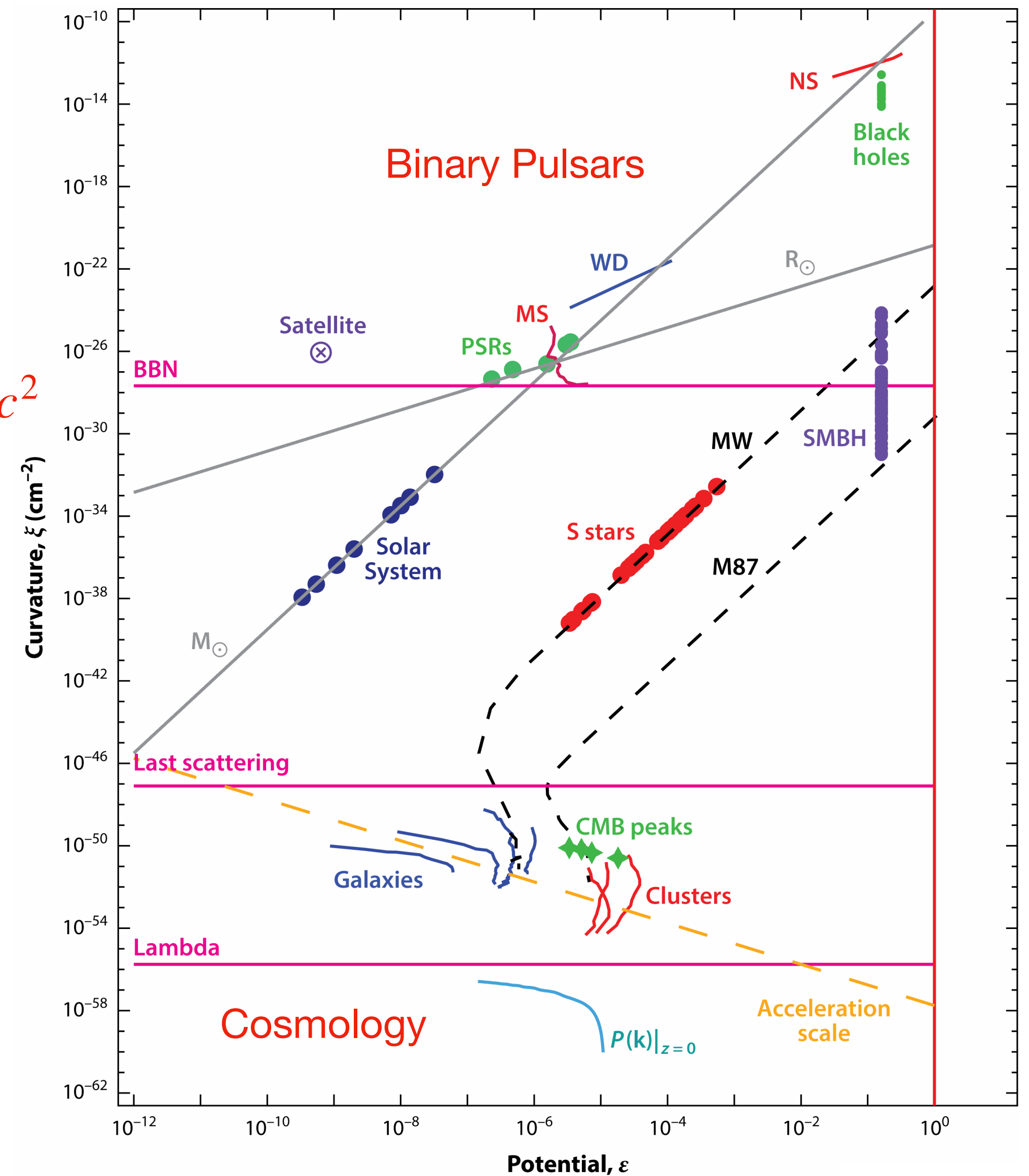
Grav. potential

Ferreira 19

Gravity on large scales

- Accelerated expansion of our Universe today leads to a lot of activities trying to extend gravity and cosmology beyond Λ CDM
- Gravity has been tested on various regimes

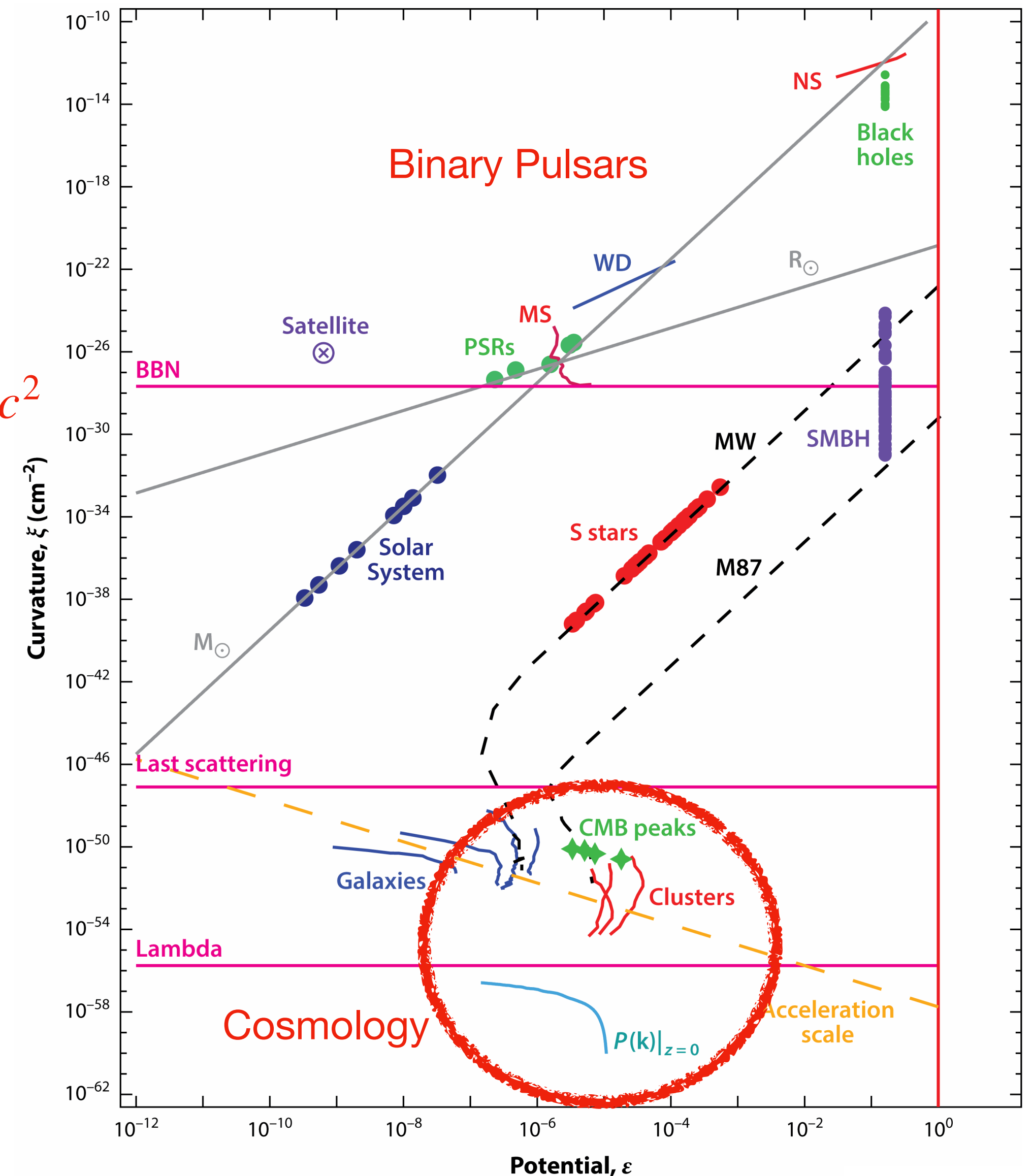
$$\xi = GM/r^3 c^2$$



Gravity on large scales

- Accelerated expansion of our Universe today leads to a lot of activities trying to extend gravity and cosmology beyond Λ CDM
- Gravity has been tested on various regimes

$$\xi = GM/r^3 c^2$$



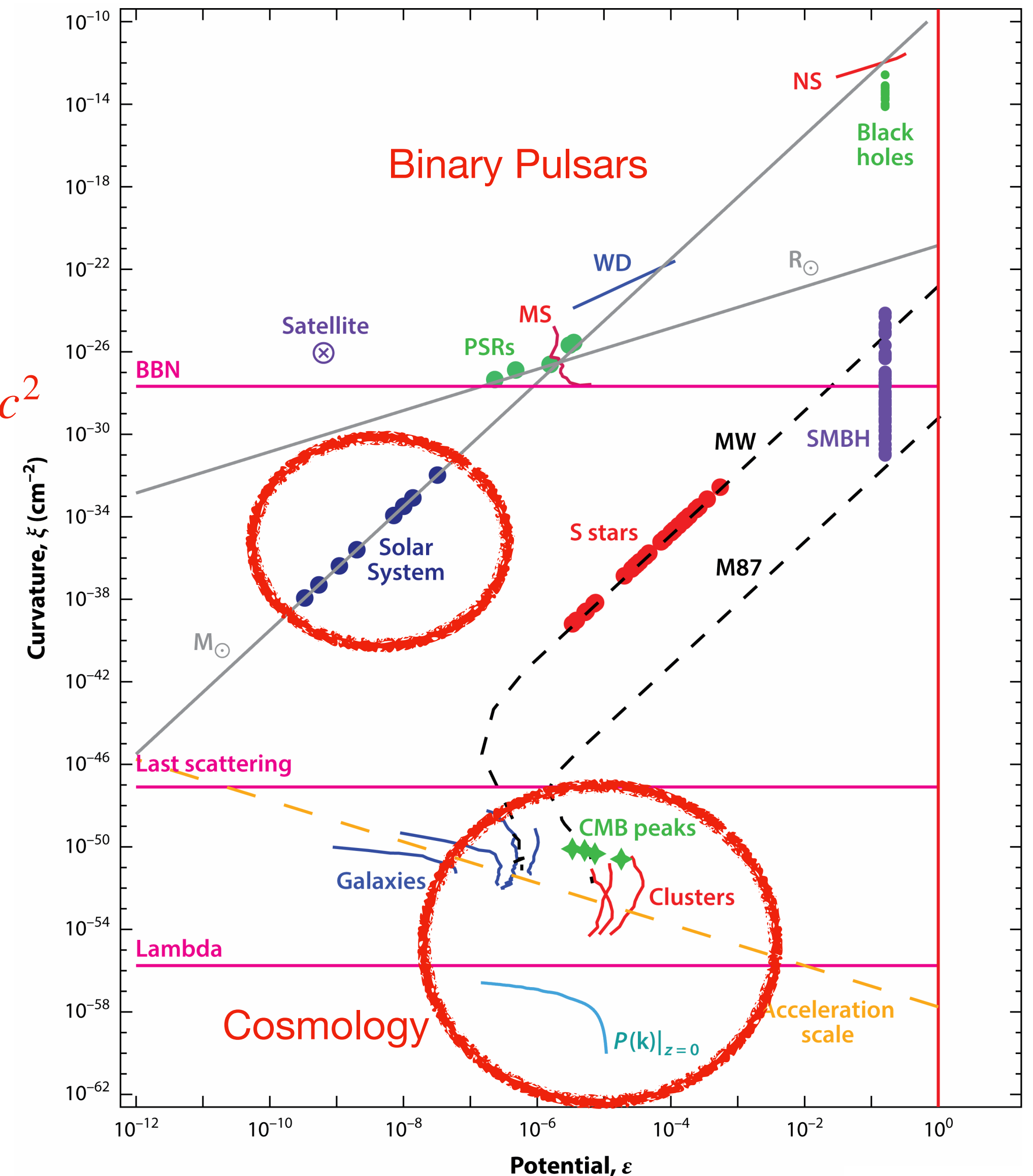
Grav. potential

Gravity on large scales

- Accelerated expansion of our Universe today leads to a lot of activities trying to extend gravity and cosmology beyond Λ CDM
- $\xi = GM$
- Gravity has been tested on various regimes
 - Allow for modification of gravity on large scales

Understand and test gravity better

$$\xi = GM/r^3 c^2$$



Grav. potential

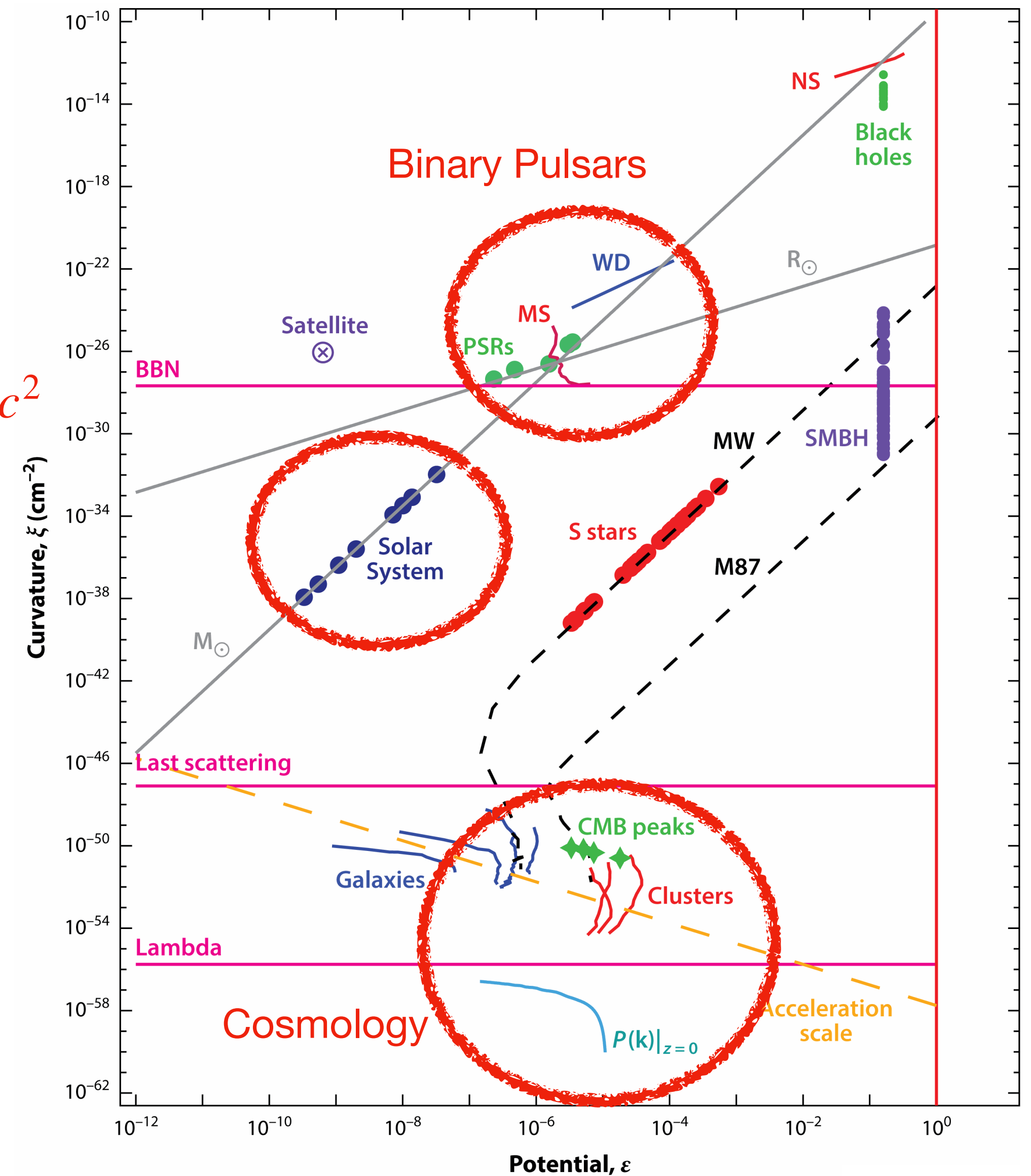
Gravity on large scales

- Accelerated expansion of our Universe today leads to a lot of activities trying to extend gravity and cosmology beyond Λ CDM

$$\xi = GM/r^3 c^2$$

- Gravity has been tested on various regimes
- Allow for modification of gravity on large scales

Understand and test gravity better



Grav. potential

Gravity on large scales

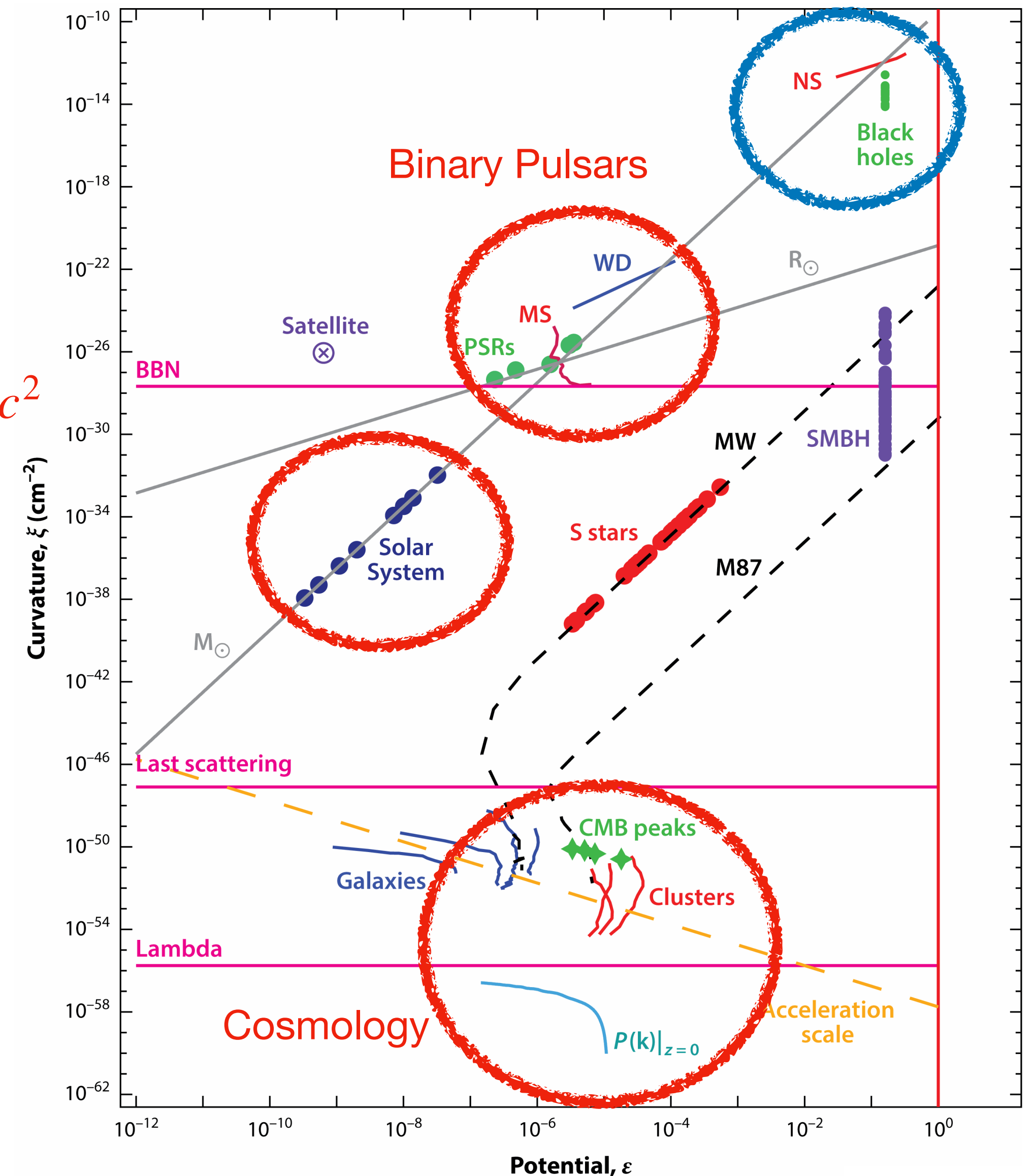
- Accelerated expansion of our Universe today leads to a lot of activities trying to extend gravity and cosmology beyond Λ CDM

- Gravity has been tested on various regimes
- Allow for modification of gravity on large scales

Understand and test gravity better

- Focus on **strong-gravity** regime where deviations from GR are expected: **BH/NS mergers**

$$\xi = GM/r^3 c^2$$



Grav. potential

Scalar-tensor theories

- Scalar-tensor theories: 2 tensor + 1 scalar dofs

Scalar-tensor theories

- **Scalar-tensor theories**: 2 tensor + 1 scalar dofs

Ferreira 19

- Simple examples: Quintessence, Brans-Dicke, k-essence $\mathcal{L} = \mathcal{L}(\phi, \nabla_\mu \phi)$

Scalar-tensor theories

- **Scalar-tensor theories**: 2 tensor + 1 scalar dofs

Ferreira 19

- Simple examples: Quintessence, Brans-Dicke, k-essence $\mathcal{L} = \mathcal{L}(\phi, \nabla_\mu \phi)$

- Generalized theories: Galileons, Horndeski +++ $\mathcal{L} = \mathcal{L}(\phi, \nabla_\mu \phi, \nabla_\mu \nabla_\nu \phi)$

Scalar-tensor theories

- **Scalar-tensor theories**: 2 tensor + 1 scalar dofs Ferreira 19
- Simple examples: Quintessence, Brans-Dicke, k-essence $\mathcal{L} = \mathcal{L}(\phi, \nabla_\mu \phi)$
- Generalized theories: Galileons, Horndeski +++ $\mathcal{L} = \mathcal{L}(\phi, \nabla_\mu \phi, \nabla_\mu \nabla_\nu \phi)$

Can provide **self-acceleration** and **non-linearities** (Vainshtein screening)
with controlled quantum corrections

Scalar-tensor theories

- **Scalar-tensor theories**: 2 tensor + 1 scalar dofs Ferreira 19
- Simple examples: Quintessence, Brans-Dicke, k-essence $\mathcal{L} = \mathcal{L}(\phi, \nabla_\mu \phi)$
- Generalized theories: Galileons, Horndeski +++ $\mathcal{L} = \mathcal{L}(\phi, \nabla_\mu \phi, \nabla_\mu \nabla_\nu \phi)$

Can provide **self-acceleration** and **non-linearities** (Vainshtein screening)
with controlled quantum corrections

Nicolis et al. 09 +++

$$\mathcal{L} = -\frac{1}{2} \left(1 + \frac{\square \phi}{\Lambda_3^3} \right) (\partial_\mu \phi)^2 - V(\phi) + \frac{\alpha}{M_{\text{Pl}}} \phi T_{\text{m}}$$

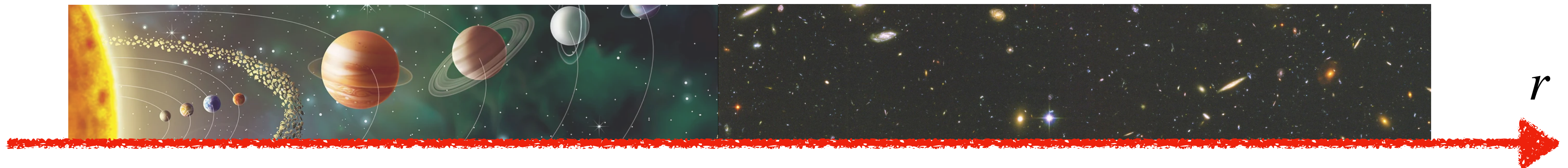
Scalar-tensor theories

- **Scalar-tensor theories**: 2 tensor + 1 scalar dofs Ferreira 19
- Simple examples: Quintessence, Brans-Dicke, k-essence $\mathcal{L} = \mathcal{L}(\phi, \nabla_\mu \phi)$
- Generalized theories: Galileons, Horndeski +++ $\mathcal{L} = \mathcal{L}(\phi, \nabla_\mu \phi, \nabla_\mu \nabla_\nu \phi)$

Can provide **self-acceleration** and **non-linearities** (Vainshtein screening)
with controlled quantum corrections

Nicolis et al. 09 +++

$$\mathcal{L} = -\frac{1}{2} \left(1 + \frac{\square \phi}{\Lambda_3^3} \right) (\partial_\mu \phi)^2 - V(\phi) + \frac{\alpha}{M_{\text{Pl}}} \phi T_{\text{m}}$$



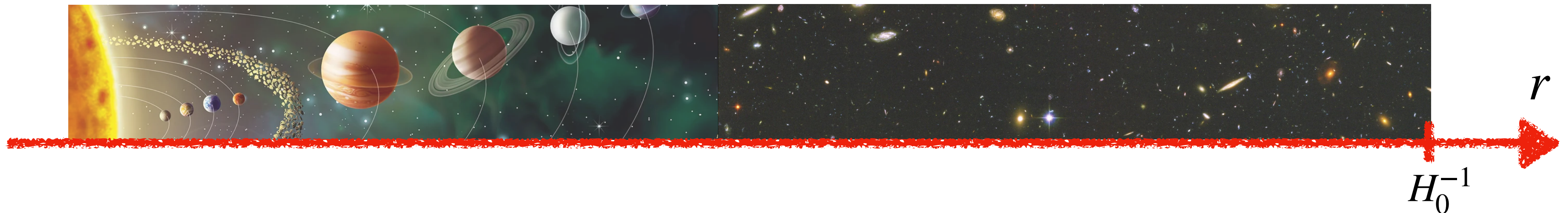
Scalar-tensor theories

- **Scalar-tensor theories**: 2 tensor + 1 scalar dofs Ferreira 19
- Simple examples: Quintessence, Brans-Dicke, k-essence $\mathcal{L} = \mathcal{L}(\phi, \nabla_\mu \phi)$
- Generalized theories: Galileons, Horndeski +++ $\mathcal{L} = \mathcal{L}(\phi, \nabla_\mu \phi, \nabla_\mu \nabla_\nu \phi)$

Can provide **self-acceleration** and **non-linearities** (Vainshtein screening)
with controlled quantum corrections

Nicolis et al. 09 +++

$$\mathcal{L} = -\frac{1}{2} \left(1 + \frac{\square \phi}{\Lambda_3^3} \right) (\partial_\mu \phi)^2 - V(\phi) + \frac{\alpha}{M_{\text{Pl}}} \phi T_{\text{m}}$$



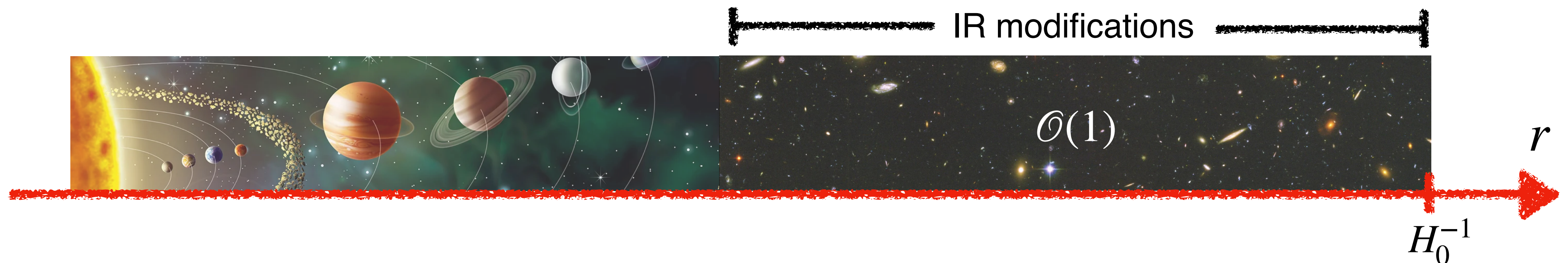
Scalar-tensor theories

- **Scalar-tensor theories**: 2 tensor + 1 scalar dofs Ferreira 19
- Simple examples: Quintessence, Brans-Dicke, k-essence $\mathcal{L} = \mathcal{L}(\phi, \nabla_\mu \phi)$
- Generalized theories: Galileons, Horndeski +++ $\mathcal{L} = \mathcal{L}(\phi, \nabla_\mu \phi, \nabla_\mu \nabla_\nu \phi)$

Can provide **self-acceleration** and **non-linearities** (Vainshtein screening)
with controlled quantum corrections

Nicolis et al. 09 +++

$$\mathcal{L} = -\frac{1}{2} \left(1 + \frac{\square \phi}{\Lambda_3^3} \right) (\partial_\mu \phi)^2 - V(\phi) + \frac{\alpha}{M_{\text{Pl}}} \phi T_{\text{m}}$$



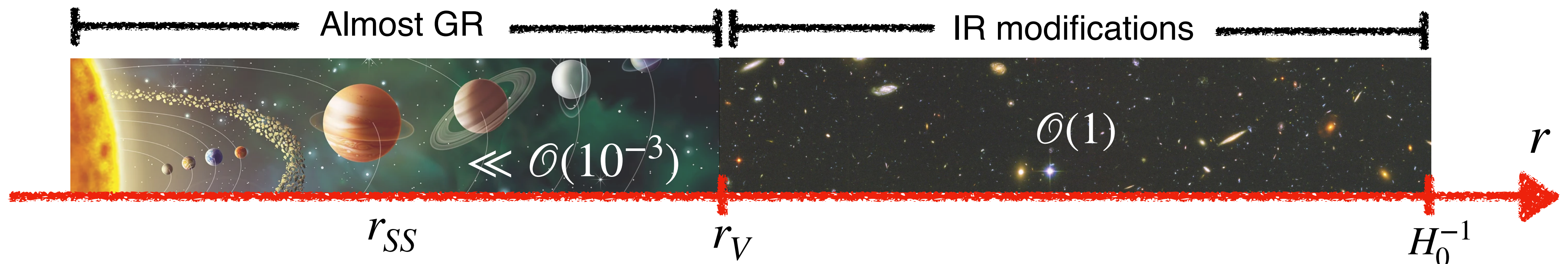
Scalar-tensor theories

- **Scalar-tensor theories**: 2 tensor + 1 scalar dofs Ferreira 19
- Simple examples: Quintessence, Brans-Dicke, k-essence $\mathcal{L} = \mathcal{L}(\phi, \nabla_\mu \phi)$
- Generalized theories: Galileons, Horndeski +++ $\mathcal{L} = \mathcal{L}(\phi, \nabla_\mu \phi, \nabla_\mu \nabla_\nu \phi)$

Can provide **self-acceleration** and **non-linearities** (Vainshtein screening)
with controlled quantum corrections

Nicolis et al. 09 +++

$$\mathcal{L} = -\frac{1}{2} \left(1 + \frac{\square \phi}{\Lambda_3^3} \right) (\partial_\mu \phi)^2 - V(\phi) + \frac{\alpha}{M_{\text{Pl}}} \phi T_{\text{m}}$$



Scalar-tensor theories

- **Scalar-tensor theories**: 2 tensor + 1 scalar dofs

Ferreira 19

- Simple examples: Quintessence, Brans-Dicke, k-essence $\mathcal{L} = \mathcal{L}(\phi, \nabla_\mu \phi)$

- Generalized theories: Galileons, Horndeski +++ $\mathcal{L} = \mathcal{L}(\phi, \nabla_\mu \phi, \nabla_\mu \nabla_\nu \phi)$

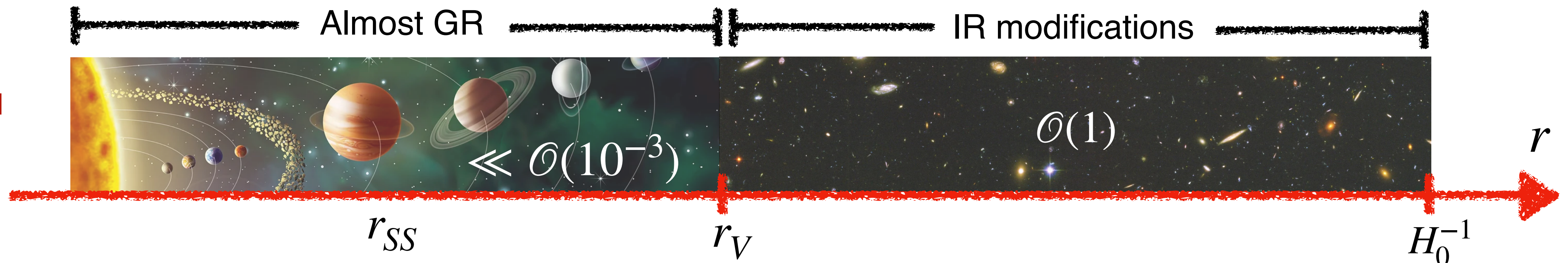
Can provide **self-acceleration** and **non-linearities** (Vainshtein screening)
with controlled quantum corrections

Nicolis et al. 09 +++

$$\mathcal{L} = -\frac{1}{2} \left(1 + \frac{\square \phi}{\Lambda_3^3} \right) (\partial_\mu \phi)^2 - V(\phi) + \frac{\alpha}{M_{\text{Pl}}} \phi T_{\text{m}}$$

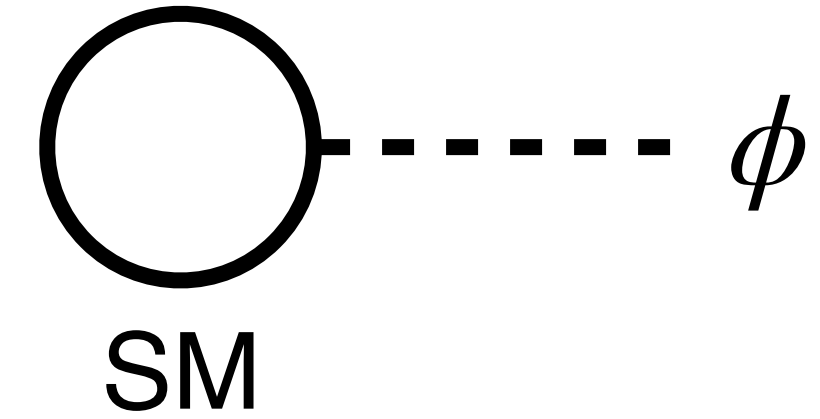
$$\frac{\square \phi}{\Lambda_3^3} \gg 1$$

Large classical
non-linearities



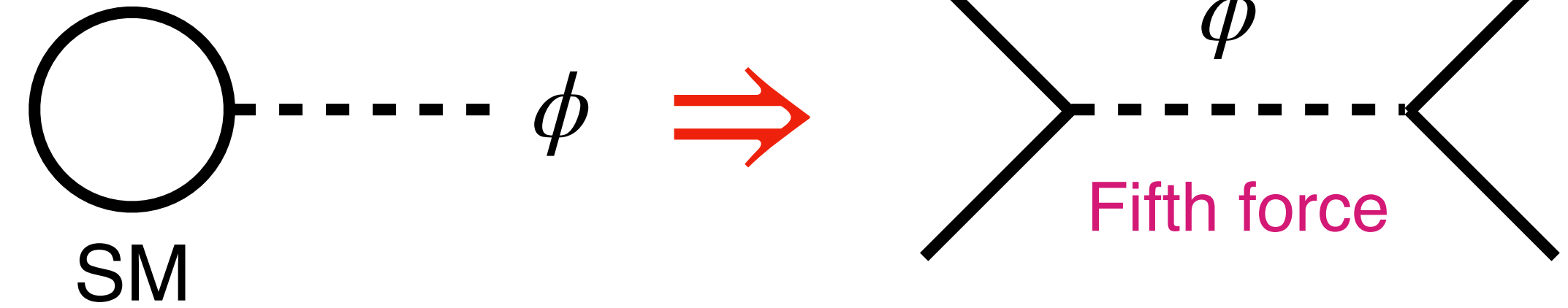
Couplings of DE with matter

- For our **DE** to contribute to the **CC**. problem



Couplings of DE with matter

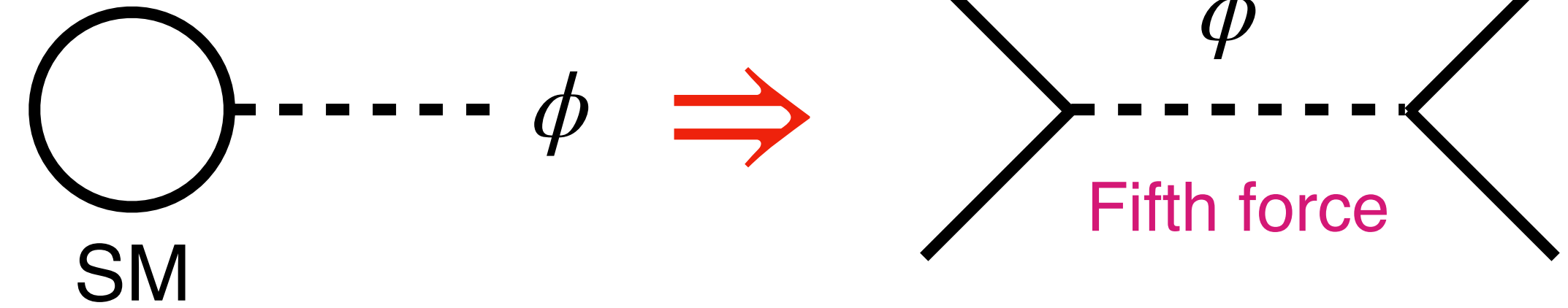
- For our **DE** to contribute to the **CC**. problem



Couplings of DE with matter

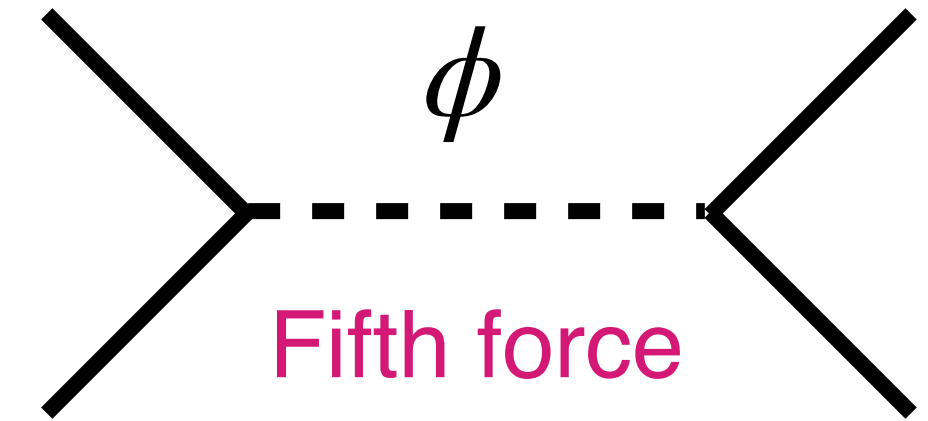
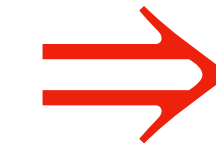
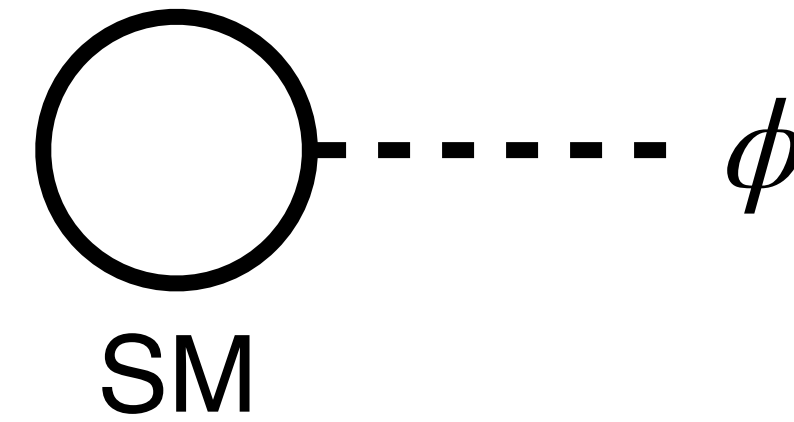
- For our **DE** to contribute to the **CC**. problem
- Traditional scalar-tensor theories

$$S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \left[f(\phi) R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]$$



Couplings of DE with matter

- For our **DE** to contribute to the **CC**. problem
- Traditional scalar-tensor theories

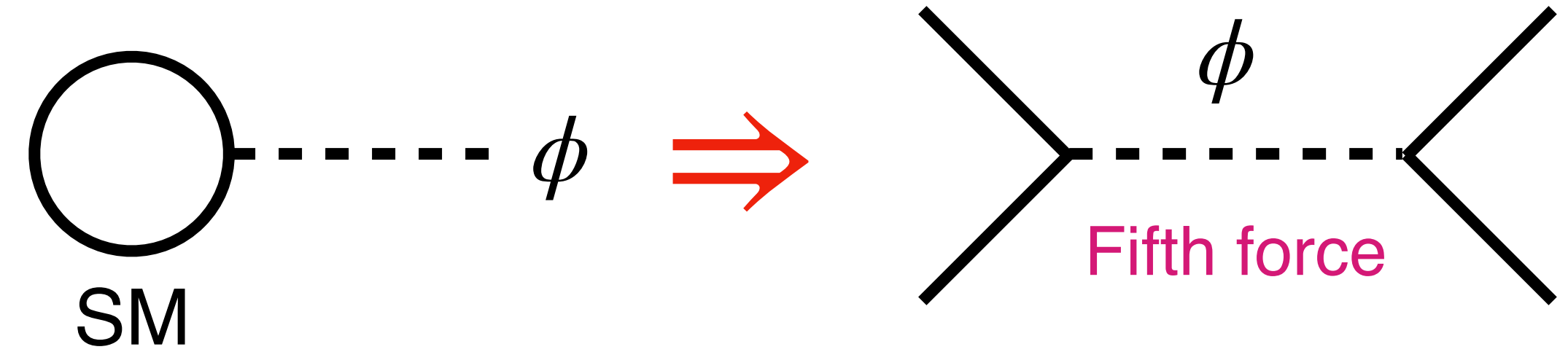


$$S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \left[\underline{f(\phi)R} - \frac{1}{2}(\partial\phi)^2 - V(\phi) \right]$$

Non-minimally coupling btw. ϕ and $g_{\mu\nu}$

Couplings of DE with matter

- For our **DE** to contribute to the **CC**. problem
- Traditional scalar-tensor theories

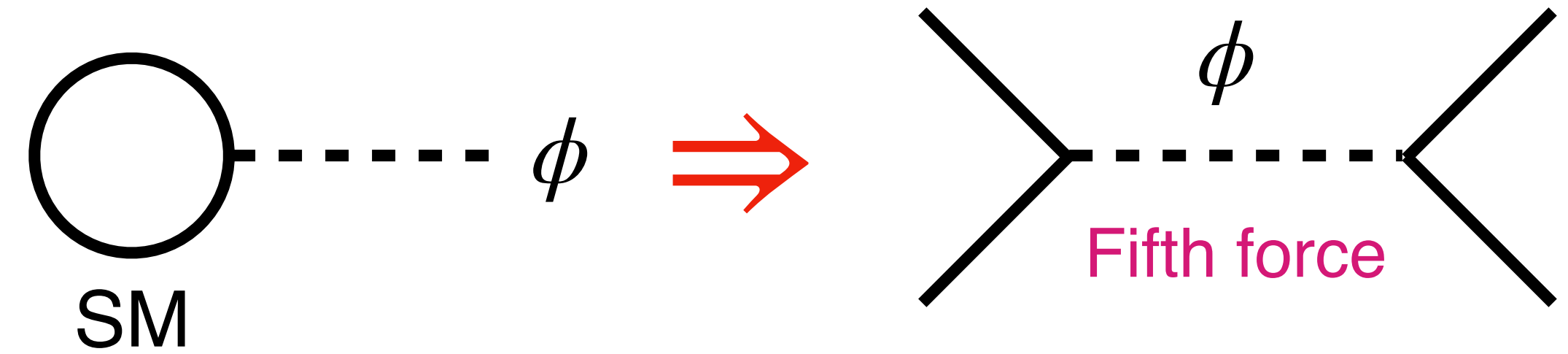


$$S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \left[\underline{f(\phi)R} - \frac{1}{2}(\partial\phi)^2 - V(\phi) \right] \quad \Leftrightarrow \quad \chi = f(\phi) \quad f'(\phi)^2 = \chi/w(\chi)$$

Non-minimally coupling btw. ϕ and $g_{\mu\nu}$ Jordan-Brans-Dicke theory

Couplings of DE with matter

- For our **DE** to contribute to the **CC**. problem
- Traditional scalar-tensor theories



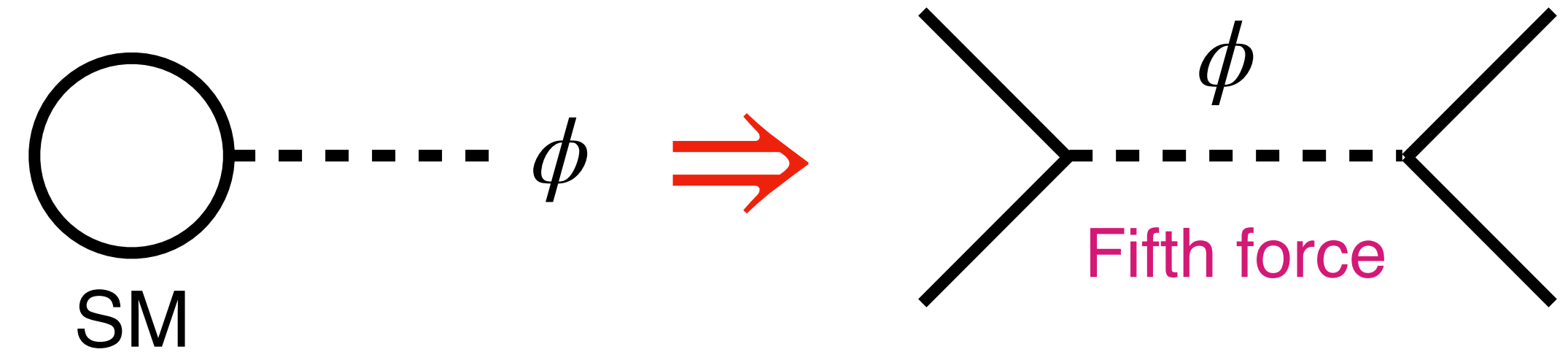
$$S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \left[\underline{f(\phi)R} - \frac{1}{2}(\partial\phi)^2 - V(\phi) \right] \iff \chi = f(\phi) \quad f'(\phi)^2 = \chi/w(\chi)$$

Non-minimally coupling btw. ϕ and $g_{\mu\nu}$ Jordan-Brans-Dicke theory

- Add **matter** couplings $S_J = \int d^4x \sqrt{-g} \left[f(\phi)R - \frac{1}{2}(\partial\phi)^2 - V(\phi) \right] + S[\psi_m; g_{\mu\nu}]$

Couplings of DE with matter

- For our **DE** to contribute to the **CC**. problem
- Traditional scalar-tensor theories



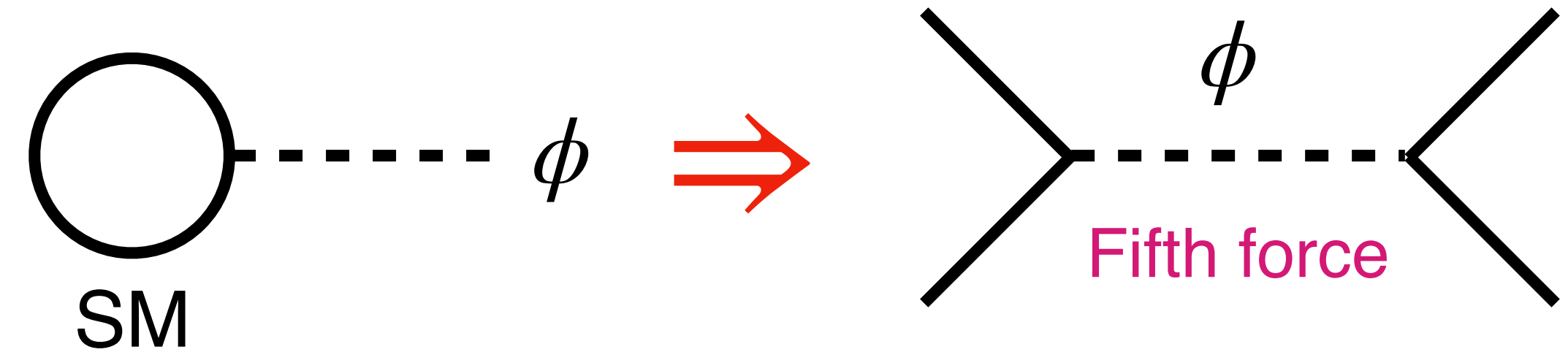
$$S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \left[\underline{f(\phi)R} - \frac{1}{2}(\partial\phi)^2 - V(\phi) \right] \iff \chi = f(\phi) \quad f'(\phi)^2 = \chi/w(\chi)$$

Non-minimally coupling btw. ϕ and $g_{\mu\nu}$ Jordan-Brans-Dicke theory

- Add **matter** couplings $S_J = \int d^4x \sqrt{-g} \left[f(\phi)R - \frac{1}{2}(\partial\phi)^2 - V(\phi) \right] + \underline{S[\psi_m; g_{\mu\nu}]}$ **Jordan frame**
- Minimally coupled Einstein eq. is modified

Couplings of DE with matter

- For our **DE** to contribute to the **CC**. problem
- Traditional scalar-tensor theories



$$S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \left[\underline{f(\phi)R} - \frac{1}{2}(\partial\phi)^2 - V(\phi) \right] \iff \chi = f(\phi) \quad f'(\phi)^2 = \chi/w(\chi)$$

Non-minimally coupling btw. ϕ and $g_{\mu\nu}$ Jordan-Brans-Dicke theory

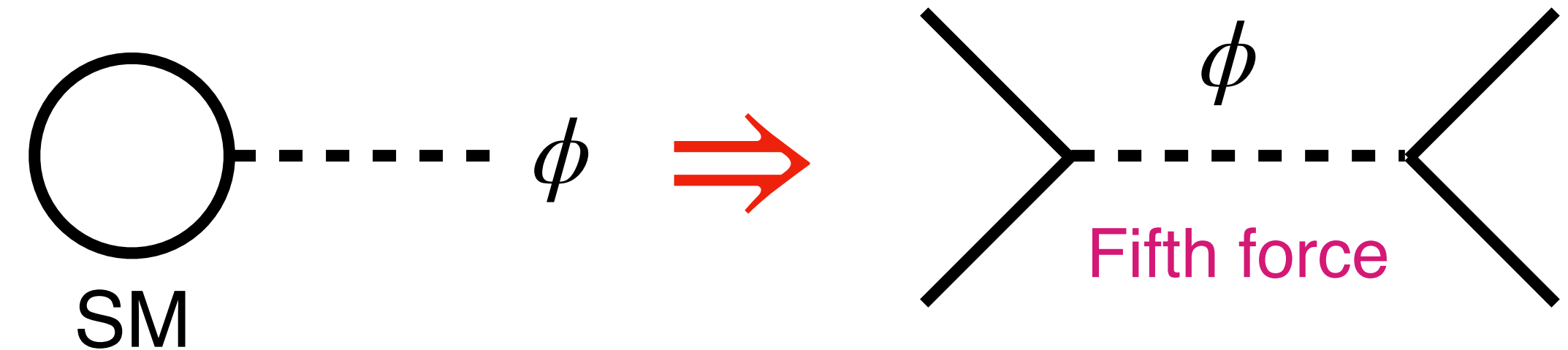
- Add **matter** couplings $S_J = \int d^4x \sqrt{-g} \left[f(\phi)R - \frac{1}{2}(\partial\phi)^2 - V(\phi) \right] + \underline{S[\psi_m; g_{\mu\nu}]}$ **Jordan frame**

$g_{\mu\nu}^E = f(\phi)g_{\mu\nu}$ Minimally coupled Einstein eq. is modified

$$\Rightarrow S_E = \int d^4x \sqrt{-g_E} \left[R_E - Z(\phi) g_E^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V_E(\phi) \right] + S \left[\psi_m; f^{-1}(\phi) g_{\mu\nu}^E \right]$$

Couplings of DE with matter

- For our **DE** to contribute to the **CC**. problem
- Traditional scalar-tensor theories



$$S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \left[\underline{f(\phi)R} - \frac{1}{2}(\partial\phi)^2 - V(\phi) \right] \iff \chi = f(\phi) \quad f'(\phi)^2 = \chi/w(\chi)$$

Non-minimally coupling btw. ϕ and $g_{\mu\nu}$ Jordan-Brans-Dicke theory

- Add **matter** couplings $S_J = \int d^4x \sqrt{-g} \left[f(\phi)R - \frac{1}{2}(\partial\phi)^2 - V(\phi) \right] + \underline{S[\psi_m; g_{\mu\nu}]}$ **Jordan frame**

$g_{\mu\nu}^E = f(\phi)g_{\mu\nu}$ $V(\phi)/f^2$ Minimally coupled Einstein eq. is modified

$$\Rightarrow S_E = \int d^4x \sqrt{-g_E} \left[R_E - Z(\phi)g_E^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \underline{V_E(\phi)} \right] + S \left[\psi_m; f^{-1}(\phi)g_{\mu\nu}^E \right] \quad \text{Einstein frame}$$

Gravity is standard, but matter is coupled with ϕ via ϕT_m

Quintessence coupling with matter

- In **Einstein** frame: gravity is standard $S_m^E \left[\psi_m; f^{-1}(\phi) g_{\mu\nu}^E \right]$

Quintessence coupling with matter

- In **Einstein** frame: gravity is standard $S_m^E \left[\psi_m; \cancel{f^{-1}(\phi)} g_{\mu\nu}^E \right]$
 $A(\phi)^2 g_{\mu\nu}^E$

Quintessence coupling with matter

- In **Einstein** frame: gravity is standard $S_m^E \left[\psi_m; \underbrace{f^{-1}(\phi)}_{A(\phi)^2} g_{\mu\nu}^E \right]$

$$A(\phi)^2 g_{\mu\nu}^E$$

$$T_E^{\mu\nu} = \frac{2}{\sqrt{-g_E}} \frac{\delta S_m^E}{\delta g_{\mu\nu}^E} \Rightarrow \nabla_\mu^E T_E^\mu{}_\nu = \frac{A'(\phi)}{A(\phi)} T_E \nabla_\nu \phi \quad \text{with} \quad T_E = g_E^{\mu\nu} T_{\mu\nu}^E$$

Quintessence coupling with matter

- In **Einstein** frame: gravity is standard $S_m^E \left[\psi_m; \underbrace{f^{-1}(\phi)}_{A(\phi)^2} g_{\mu\nu}^E \right]$

$$T_E^{\mu\nu} = \frac{2}{\sqrt{-g_E}} \frac{\delta S_m^E}{\delta g_{\mu\nu}^E} \Rightarrow \nabla_\mu^E T_E^\mu{}_\nu = \frac{A'(\phi)}{A(\phi)} T_E \nabla_\nu \phi \quad \text{with} \quad T_E = g_E^{\mu\nu} T_{\mu\nu}^E$$

Exercise: Show that $\square_E \phi - V_E'(\phi) = -\frac{A'(\phi)}{A(\phi)} T_E$ ϕ is sourced by T_E

Quintessence coupling with matter

- In **Einstein** frame: gravity is standard $S_m^E \left[\psi_m; \underbrace{f^{-1}(\phi)}_{A(\phi)^2} g_{\mu\nu}^E \right]$

$$A(\phi)^2 g_{\mu\nu}^E$$

$$T_E^{\mu\nu} = \frac{2}{\sqrt{-g_E}} \frac{\delta S_m^E}{\delta g_{\mu\nu}^E} \Rightarrow \nabla_\mu^E T_E^\mu{}_\nu = \frac{A'(\phi)}{A(\phi)} T_E \nabla_\nu \phi \quad \text{with} \quad T_E = g_E^{\mu\nu} T_{\mu\nu}^E$$

Exercise: Show that $\square_E \phi - V'_E(\phi) = -\frac{A'(\phi)}{A(\phi)} T_E$ ϕ is sourced by T_E

Solution: $g_{\mu\nu}^J = A(\phi)^2 g_{\mu\nu}^E \quad S[g_{\mu\nu}^J] = S_{g\phi}[g_{\mu\nu}^J, \phi] + S_m[g_{\mu\nu}^J, \psi_m]$

$$\Rightarrow \delta S_m[g_{\mu\nu}^J, \psi_m] = \frac{1}{2} \int d^4x \sqrt{-g_J} T_J^{\mu\nu} \delta g_{\mu\nu}^J$$

To Einstein frame

$$\delta S_m[A^2 g_{\mu\nu}^E, \psi_m] = \int d^4x \sqrt{-g_E} \frac{A'}{A} T_E \delta \phi$$

$$\delta g_{\mu\nu}^J = \delta(A^2 g_{\mu\nu}^E) = \underline{2AA' g_{\mu\nu}^E \delta \phi} + A^2 \delta g_{\mu\nu}^E$$

$$T_J^{\mu\nu} = A^{-6} T_E^{\mu\nu}$$

Matter-DE coupling

Quintessence coupling with matter

- Geodesic in **Einstein** frame $ds^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Psi)d\vec{x}^2$

$$\ddot{\vec{x}} = -\vec{\nabla}\Phi_E - \beta \vec{\nabla}\phi \quad \beta \equiv \frac{A'}{A} \quad \text{and} \quad \nabla^2\Phi_E = 4\pi GM\delta^{(3)}(\vec{x}) \quad \Rightarrow \quad \Phi_E = -\frac{GM}{r}$$

~~5th force~~
Fifth force

Quintessence coupling with matter

- Geodesic in **Einstein** frame $ds^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Psi)d\vec{x}^2$

$$\ddot{\vec{x}} = -\vec{\nabla}\Phi_E - \beta \vec{\nabla}\phi \quad \beta \equiv \frac{A'}{A} \quad \text{and} \quad \nabla^2\Phi_E = 4\pi GM\delta^{(3)}(\vec{x}) \quad \Rightarrow \quad \Phi_E = -\frac{GM}{r}$$

Fifth force

- For ϕ , in **quasi-static** limit $\nabla^2\phi - m_\phi^2\phi = \beta M\delta^{(3)}(\vec{x}) \Rightarrow \phi = -\frac{\beta M}{4\pi r}e^{-m_\phi r}$

Quintessence coupling with matter

- Geodesic in **Einstein** frame $ds^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Psi)d\vec{x}^2$

$$\ddot{\vec{x}} = -\vec{\nabla}\Phi_E - \beta \vec{\nabla}\phi \quad \beta \equiv \frac{A'}{A} \quad \text{and} \quad \nabla^2\Phi_E = 4\pi GM\delta^{(3)}(\vec{x}) \quad \Rightarrow \quad \Phi_E = -\frac{GM}{r}$$

Fifth force

- For ϕ , in **quasi-static** limit $\nabla^2\phi - m_\phi^2\phi = \beta M\delta^{(3)}(\vec{x}) \Rightarrow \phi = -\frac{\beta M}{4\pi r}e^{-m_\phi r}$

- From $g_{\mu\nu} = A^2 g_{\mu\nu}^E \Rightarrow \delta g_{\mu\nu} = 2AA'\bar{g}_{\mu\nu}^E\phi + A^2\delta g_{\mu\nu}^E$

Quintessence coupling with matter

- Geodesic in **Einstein** frame $ds^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Psi)d\vec{x}^2$

$$\ddot{\vec{x}} = -\vec{\nabla}\Phi_E - \beta \vec{\nabla}\phi \quad \beta \equiv \frac{A'}{A} \quad \text{and} \quad \nabla^2\Phi_E = 4\pi GM\delta^{(3)}(\vec{x}) \quad \Rightarrow \quad \Phi_E = -\frac{GM}{r}$$

Fifth force

- For ϕ , in **quasi-static** limit $\nabla^2\phi - m_\phi^2\phi = \beta M\delta^{(3)}(\vec{x}) \Rightarrow \phi = -\frac{\beta M}{4\pi r}e^{-m_\phi r}$

- From $g_{\mu\nu} = A^2 g_{\mu\nu}^E \Rightarrow \delta g_{\mu\nu} = 2AA'\bar{g}_{\mu\nu}^E\phi + A^2\delta g_{\mu\nu}^E$

$$\Phi = \Phi_E + \beta\phi \quad \text{and} \quad \Psi = \Psi_E - \beta\phi \quad \text{with} \quad \Phi_E = \Psi_E$$

Quintessence coupling with matter

- Geodesic in **Einstein** frame $ds^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Psi)d\vec{x}^2$

$$\ddot{\vec{x}} = -\vec{\nabla}\Phi_E - \beta \vec{\nabla}\phi \quad \beta \equiv \frac{A'}{A} \quad \text{and} \quad \nabla^2\Phi_E = 4\pi GM\delta^{(3)}(\vec{x}) \quad \Rightarrow \quad \Phi_E = -\frac{GM}{r}$$

Fifth force

- For ϕ , in **quasi-static** limit $\nabla^2\phi - m_\phi^2\phi = \beta M\delta^{(3)}(\vec{x}) \Rightarrow \phi = -\frac{\beta M}{4\pi r}e^{-m_\phi r}$

- From $g_{\mu\nu} = A^2 g_{\mu\nu}^E \Rightarrow \delta g_{\mu\nu} = 2AA'\bar{g}_{\mu\nu}^E\phi + A^2\delta g_{\mu\nu}^E$

$$\Phi = \Phi_E + \beta\phi \quad \text{and} \quad \Psi = \Psi_E - \beta\phi \quad \text{with} \quad \Phi_E = \Psi_E$$

$$\Phi = -\frac{GM}{r}\left(1 + \frac{\beta^2}{4\pi G}\right)$$

$$\Psi = -\frac{GM}{r}\left(1 - \frac{\beta^2}{4\pi G}\right)$$

Quintessence coupling with matter

- Geodesic in **Einstein** frame $ds^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Psi)d\vec{x}^2$

$$\ddot{\vec{x}} = -\vec{\nabla}\Phi_E - \beta \vec{\nabla}\phi \quad \beta \equiv \frac{A'}{A} \quad \text{and} \quad \nabla^2\Phi_E = 4\pi GM\delta^{(3)}(\vec{x}) \quad \Rightarrow \quad \Phi_E = -\frac{GM}{r}$$

Fifth force

- For ϕ , in **quasi-static** limit $\nabla^2\phi - m_\phi^2\phi = \beta M\delta^{(3)}(\vec{x}) \Rightarrow \phi = -\frac{\beta M}{4\pi r}e^{-m_\phi r}$

- From $g_{\mu\nu} = A^2 g_{\mu\nu}^E \Rightarrow \delta g_{\mu\nu} = 2AA'\bar{g}_{\mu\nu}^E\phi + A^2\delta g_{\mu\nu}^E$

$$\Phi = \Phi_E + \beta\phi \quad \text{and} \quad \Psi = \Psi_E - \beta\phi \quad \text{with} \quad \Phi_E = \Psi_E$$

$$\Phi = -\frac{GM}{r}\left(1 + \frac{\beta^2}{4\pi G}\right)$$

$$\Psi = -\frac{GM}{r}\left(1 - \frac{\beta^2}{4\pi G}\right)$$

$$\Rightarrow \gamma = \frac{\Psi}{\Phi} \neq 1$$

Quintessence coupling with matter

- Geodesic in **Einstein** frame $ds^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Psi)d\vec{x}^2$

$$\ddot{\vec{x}} = -\vec{\nabla}\Phi_E - \beta \vec{\nabla}\phi \quad \beta \equiv \frac{A'}{A} \quad \text{and} \quad \nabla^2\Phi_E = 4\pi GM\delta^{(3)}(\vec{x}) \quad \Rightarrow \quad \Phi_E = -\frac{GM}{r}$$

Fifth force

- For ϕ , in **quasi-static** limit $\nabla^2\phi - m_\phi^2\phi = \beta M\delta^{(3)}(\vec{x}) \Rightarrow \phi = -\frac{\beta M}{4\pi r}e^{-m_\phi r}$

- From $g_{\mu\nu} = A^2 g_{\mu\nu}^E \Rightarrow \delta g_{\mu\nu} = 2AA'\bar{g}_{\mu\nu}^E\phi + A^2\delta g_{\mu\nu}^E$

$$\Phi = \Phi_E + \beta\phi \quad \text{and} \quad \Psi = \Psi_E - \beta\phi \quad \text{with} \quad \Phi_E = \Psi_E$$

$$\Phi = -\frac{GM}{r}\left(1 + \frac{\beta^2}{4\pi G}\right)$$

$$\Psi = -\frac{GM}{r}\left(1 - \frac{\beta^2}{4\pi G}\right)$$

$$\Rightarrow \gamma = \frac{\Psi}{\Phi} \neq 1$$

Gravitational slip

$$|\gamma - 1| \lesssim 10^{-5}$$

Shapiro time delay

Screening mechanism

- **Solar system** tests impose severe bounds on modifications of gravity
- Modifications of gravity are **screened** in high-density regime (solar system)

Screening mechanism

- **Solar system** tests impose severe bounds on modifications of gravity
- Modifications of gravity are **screened** in high-density regime (solar system)
- In Einstein frame: $\mathcal{L} = -\frac{1}{2}Z^{\mu\nu}(\phi, \partial\phi, \dots)\partial_\mu\phi\partial_\nu\phi - V(\phi) + g(\phi)T_{\text{m}}$

Screening mechanism

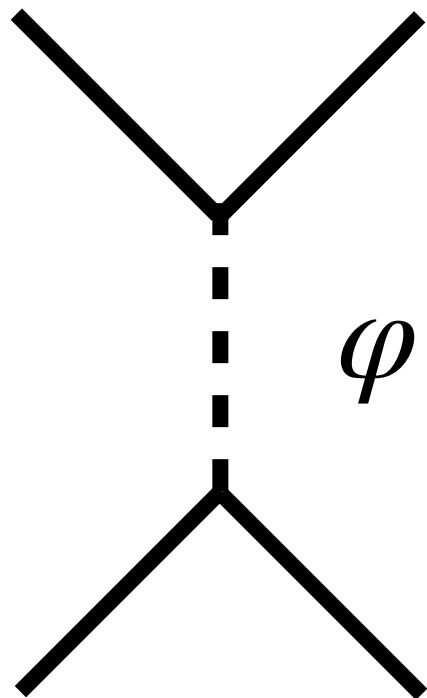
- **Solar system** tests impose severe bounds on modifications of gravity
- Modifications of gravity are **screened** in high-density regime (solar system)
- In Einstein frame: $\mathcal{L} = -\frac{1}{2}Z^{\mu\nu}(\phi, \partial\phi, \dots)\partial_\mu\phi\partial_\nu\phi - V(\phi) + g(\phi)T_{\text{m}}$
- **Static** source of mass M and $\phi = \bar{\phi}(r) + \varphi$

$$Z(\bar{\phi})(\ddot{\phi} - c_s^2 \nabla^2 \varphi) + m_\phi^2(\bar{\phi})\varphi = g(\bar{\phi})M\delta^{(3)}(\vec{x})$$

Screening mechanism

- **Solar system** tests impose severe bounds on modifications of gravity
- Modifications of gravity are **screened** in high-density regime (solar system)
- In Einstein frame: $\mathcal{L} = -\frac{1}{2}Z^{\mu\nu}(\phi, \partial\phi, \dots)\partial_\mu\phi\partial_\nu\phi - V(\phi) + g(\phi)T_{\text{m}}$
- **Static** source of mass M and $\phi = \bar{\phi}(r) + \varphi$

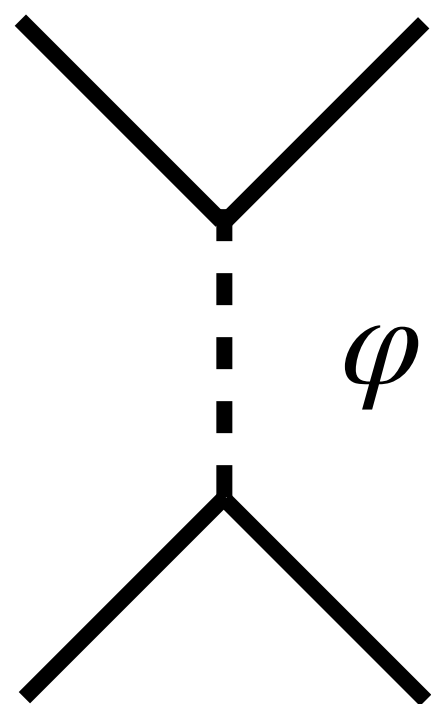
$$Z(\bar{\phi})(\ddot{\varphi} - c_s^2 \nabla^2 \varphi) + m_\phi^2(\bar{\phi})\varphi = g(\bar{\phi})M\delta^{(3)}(\vec{x})$$



Screening mechanism

- **Solar system** tests impose severe bounds on modifications of gravity
- Modifications of gravity are **screened** in high-density regime (solar system)
- In Einstein frame: $\mathcal{L} = -\frac{1}{2}Z^{\mu\nu}(\phi, \partial\phi, \dots)\partial_\mu\phi\partial_\nu\phi - V(\phi) + g(\phi)T_{\text{m}}$
- **Static** source of mass M and $\phi = \bar{\phi}(r) + \varphi$

$$Z(\bar{\phi})(\ddot{\varphi} - c_s^2 \nabla^2 \varphi) + m_\phi^2(\bar{\phi})\varphi = g(\bar{\phi})M\delta^{(3)}(\vec{x})$$

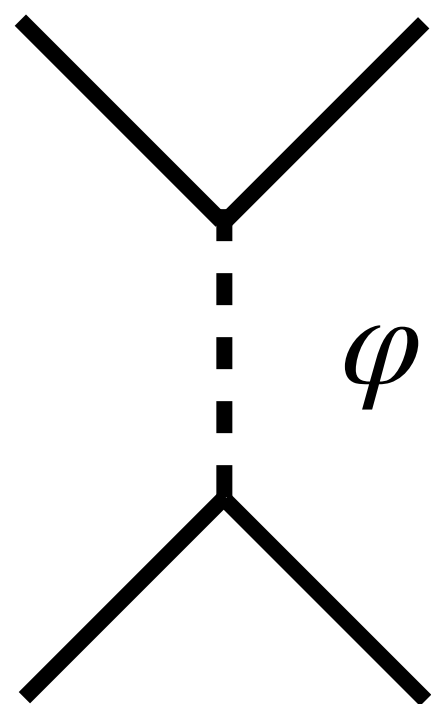


$$\Rightarrow V_\phi(r) = -\frac{g(\bar{\phi})^2}{Z(\bar{\phi})c_s^2(\bar{\phi})} \frac{M}{4\pi r} \exp\left[-\frac{m_\phi(\bar{\phi})}{Z(\bar{\phi})^{1/2}c_s(\bar{\phi})}\right]$$

Screening mechanism

- **Solar system** tests impose severe bounds on modifications of gravity
- Modifications of gravity are **screened** in high-density regime (solar system)
- In Einstein frame: $\mathcal{L} = -\frac{1}{2}Z^{\mu\nu}(\phi, \partial\phi, \dots)\partial_\mu\phi\partial_\nu\phi - V(\phi) + g(\phi)T_m$
- **Static** source of mass M and $\phi = \bar{\phi}(r) + \varphi$

$$Z(\bar{\phi})(\ddot{\varphi} - c_s^2 \nabla^2 \varphi) + m_\phi^2(\bar{\phi})\varphi = g(\bar{\phi})M\delta^{(3)}(\vec{x})$$



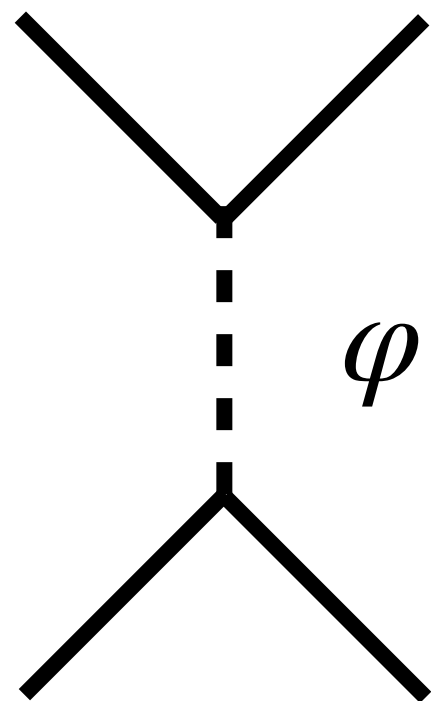
$$V_\phi(r) = -\frac{g(\bar{\phi})^2}{Z(\bar{\phi})c_s^2(\bar{\phi})} \frac{M}{4\pi r} \exp\left[-\frac{m_\phi(\bar{\phi})}{Z(\bar{\phi})^{1/2}c_s(\bar{\phi})}\right]$$

To suppress this V_ϕ

Screening mechanism

- **Solar system** tests impose severe bounds on modifications of gravity
- Modifications of gravity are **screened** in high-density regime (solar system)
- In Einstein frame: $\mathcal{L} = -\frac{1}{2}Z^{\mu\nu}(\phi, \partial\phi, \dots)\partial_\mu\phi\partial_\nu\phi - V(\phi) + g(\phi)T_m$
- **Static** source of mass M and $\phi = \bar{\phi}(r) + \varphi$

$$Z(\bar{\phi})(\ddot{\varphi} - c_s^2 \nabla^2 \varphi) + m_\phi^2(\bar{\phi})\varphi = g(\bar{\phi})M\delta^{(3)}(\vec{x})$$



$$\Rightarrow V_\phi(r) = -\frac{g(\bar{\phi})^2}{Z(\bar{\phi})c_s^2(\bar{\phi})} \frac{M}{4\pi r} \exp\left[-\frac{m_\phi(\bar{\phi})}{Z(\bar{\phi})^{1/2}c_s(\bar{\phi})}\right]$$

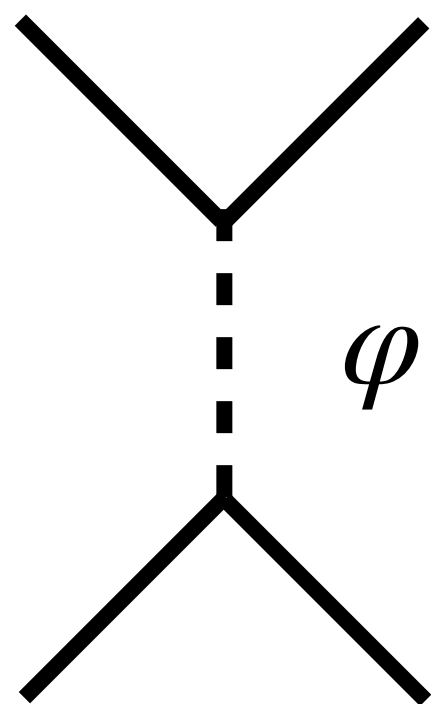
To suppress this V_ϕ

1. $g(\bar{\phi}) \ll 1$: **weak** coupling

Screening mechanism

- **Solar system** tests impose severe bounds on modifications of gravity
- Modifications of gravity are **screened** in high-density regime (solar system)
- In Einstein frame: $\mathcal{L} = -\frac{1}{2}Z^{\mu\nu}(\phi, \partial\phi, \dots)\partial_\mu\phi\partial_\nu\phi - V(\phi) + g(\phi)T_m$
- **Static** source of mass M and $\phi = \bar{\phi}(r) + \varphi$

$$Z(\bar{\phi})(\ddot{\varphi} - c_s^2 \nabla^2 \varphi) + m_\phi^2(\bar{\phi})\varphi = g(\bar{\phi})M\delta^{(3)}(\vec{x})$$



$$\Rightarrow V_\phi(r) = -\frac{g(\bar{\phi})^2}{Z(\bar{\phi})c_s^2(\bar{\phi})} \frac{M}{4\pi r} \exp\left[-\frac{m_\phi(\bar{\phi})}{Z(\bar{\phi})^{1/2}c_s(\bar{\phi})}\right]$$

To suppress this V_ϕ

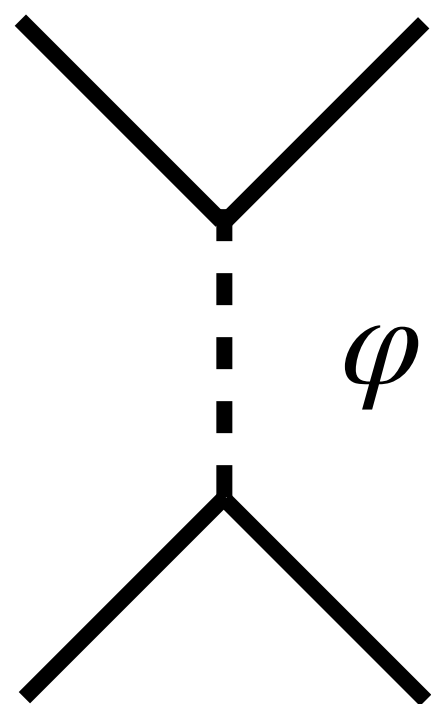
1. $g(\bar{\phi}) \ll 1$: **weak** coupling
2. $m(\bar{\phi}) \gg 1$: **large** mass

Chameleon mechanism

Screening mechanism

- **Solar system** tests impose severe bounds on modifications of gravity
- Modifications of gravity are **screened** in high-density regime (solar system)
- In Einstein frame: $\mathcal{L} = -\frac{1}{2}Z^{\mu\nu}(\phi, \partial\phi, \dots)\partial_\mu\phi\partial_\nu\phi - V(\phi) + g(\phi)T_m$
- **Static** source of mass M and $\phi = \bar{\phi}(r) + \varphi$

$$Z(\bar{\phi})(\ddot{\varphi} - c_s^2 \nabla^2 \varphi) + m_\phi^2(\bar{\phi})\varphi = g(\bar{\phi})M\delta^{(3)}(\vec{x})$$



$$V_\phi(r) = -\frac{g(\bar{\phi})^2}{Z(\bar{\phi})c_s^2(\bar{\phi})} \frac{M}{4\pi r} \exp\left[-\frac{m_\phi(\bar{\phi})}{Z(\bar{\phi})^{1/2}c_s(\bar{\phi})}\right]$$

To suppress this V_ϕ

1. $g(\bar{\phi}) \ll 1$: **weak** coupling
2. $m(\bar{\phi}) \gg 1$: **large** mass
3. $Z(\bar{\phi}) \gg 1$: **large** nonlinearities

Chameleon mechanism

Kinetic screening

e.g. Vainshtein & k-mouflage

Vainshtein screening

- Consider the **cubic** Galileon theory

Vainshtein 72, Babichev & Deffayet 13, +++

$$\mathcal{L} = -\frac{c_2}{2}(\partial_\mu\phi)^2 - \frac{c_3}{\Lambda^3}\square\phi(\partial_\mu\phi)^2 + \frac{g}{M_{\text{Pl}}}\phi T_{\text{m}}$$

Vainshtein screening

- Consider the **cubic** Galileon theory

Vainshtein 72, Babichev & Deffayet 13, +++

$$\mathcal{L} = -\frac{c_2}{2}(\partial_\mu\phi)^2 - \frac{c_3}{\Lambda^3}\square\phi(\partial_\mu\phi)^2 + \frac{g}{M_{\text{Pl}}}\phi T_{\text{m}}$$

- EoM** of ϕ (neglect gravity): Galileon symmetry: $\phi \rightarrow \phi + c + b_\mu x^\mu$

Vainshtein screening

- Consider the **cubic** Galileon theory

Vainshtein 72, Babichev & Deffayet 13, +++

$$\mathcal{L} = -\frac{c_2}{2}(\partial_\mu\phi)^2 - \frac{c_3}{\Lambda^3}\Box\phi(\partial_\mu\phi)^2 + \frac{g}{M_{\text{Pl}}}\phi T_{\text{m}}$$

- EoM** of ϕ (neglect gravity): Galileon symmetry: $\phi \rightarrow \phi + c + b_\mu x^\mu$

$$c_2\Box\phi + \frac{2c_2}{\Lambda^3}\left[(\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2\right] = -\frac{g}{M_{\text{Pl}}}T_{\text{m}}$$

Non-linear EoM, but
up to 2nd derivatives

Vainshtein screening

- Consider the **cubic** Galileon theory

Vainshtein 72, Babichev & Deffayet 13, +++

$$\mathcal{L} = -\frac{c_2}{2}(\partial_\mu\phi)^2 - \frac{c_3}{\Lambda^3}\Box\phi(\partial_\mu\phi)^2 + \frac{g}{M_{\text{Pl}}}\phi T_{\text{m}}$$

- EoM** of ϕ (neglect gravity): **Galileon symmetry:** $\phi \rightarrow \phi + c + b_\mu x^\mu$

$$c_2\Box\phi + \frac{2c_2}{\Lambda^3}\left[(\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2\right] = -\frac{g}{M_{\text{Pl}}}T_{\text{m}}$$

Non-linear EoM, but
up to 2nd derivatives

- Spherical symmetric $\phi = \phi(r)$ around point source $T_{\text{m}} = -M\delta^{(3)}(\vec{x})$

Point-particle:
 $T_{\mu\nu} \propto Mu_\mu u_\nu \delta^{(3)}(\vec{x})$

Vainshtein screening

- Consider the **cubic** Galileon theory

Vainshtein 72, Babichev & Deffayet 13, +++

$$\mathcal{L} = -\frac{c_2}{2}(\partial_\mu\phi)^2 - \frac{c_3}{\Lambda^3}\Box\phi(\partial_\mu\phi)^2 + \frac{g}{M_{\text{Pl}}}\phi T_{\text{m}}$$

- EoM** of ϕ (neglect gravity): **Galileon symmetry:** $\phi \rightarrow \phi + c + b_\mu x^\mu$

$$c_2\Box\phi + \frac{2c_2}{\Lambda^3}\left[(\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2\right] = -\frac{g}{M_{\text{Pl}}}T_{\text{m}}$$

Non-linear EoM, but
up to 2nd derivatives

- Spherical symmetric $\phi = \phi(r)$ around point source $T_{\text{m}} = -M\delta^{(3)}(\vec{x})$

Point-particle:
 $T_{\mu\nu} \propto Mu_\mu u_\nu \delta^{(3)}(\vec{x})$

Exercise:

Use divergence thm.

$$c_2\phi' + \frac{4c_3}{\Lambda^3}\frac{(\phi')^2}{r} = \frac{gM}{4\pi r^2 M_{\text{Pl}}}$$

Vainshtein screening

- Consider the **cubic** Galileon theory

Vainshtein 72, Babichev & Deffayet 13, +++

$$\mathcal{L} = -\frac{c_2}{2}(\partial_\mu\phi)^2 - \frac{c_3}{\Lambda^3}\Box\phi(\partial_\mu\phi)^2 + \frac{g}{M_{\text{Pl}}}\phi T_{\text{m}}$$

- EoM** of ϕ (neglect gravity): **Galileon symmetry:** $\phi \rightarrow \phi + c + b_\mu x^\mu$

$$c_2\Box\phi + \frac{2c_2}{\Lambda^3}\left[(\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2\right] = -\frac{g}{M_{\text{Pl}}}T_{\text{m}}$$

Non-linear EoM, but
up to 2nd derivatives

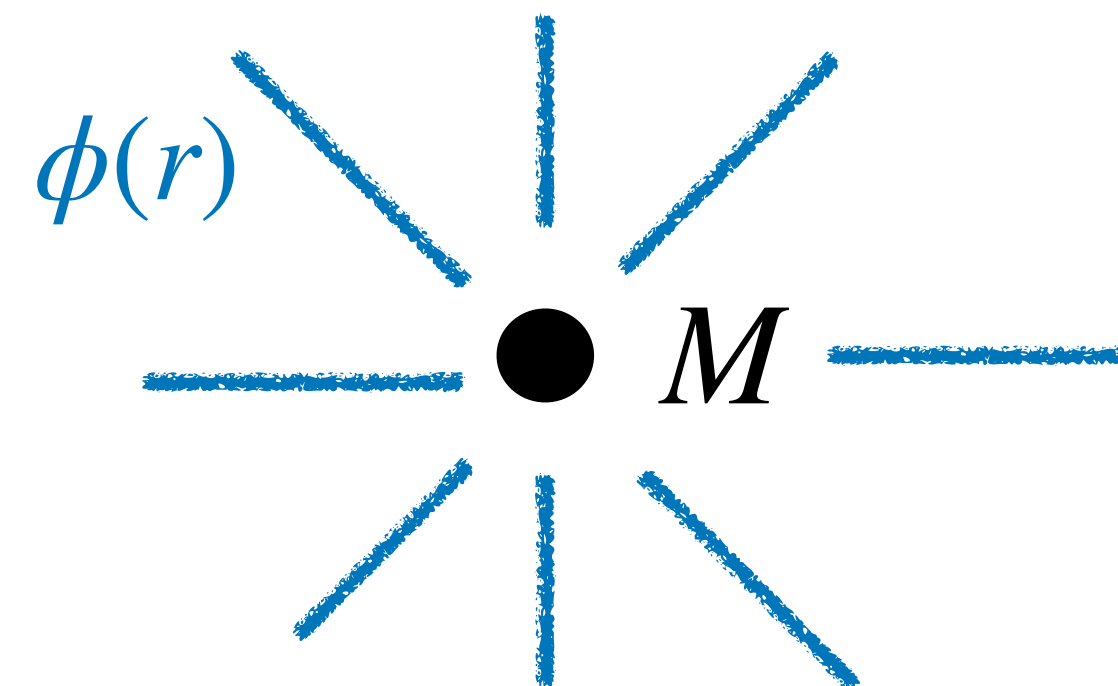
- Spherical symmetric $\phi = \phi(r)$ around point source $T_{\text{m}} = -M\delta^{(3)}(\vec{x})$

Point-particle:
 $T_{\mu\nu} \propto Mu_\mu u_\nu \delta^{(3)}(\vec{x})$

Exercise:

Use divergence thm.

$$c_2\phi' + \frac{4c_3}{\Lambda^3}\frac{(\phi')^2}{r} = \frac{gM}{4\pi r^2 M_{\text{Pl}}}$$



+ Φ
Newton
potential

Vainshtein screening

- **Solution** for $\phi'(r)$

$$\phi'(r) = -\frac{\Lambda^3 c_2 r}{8c_3} \left[1 - \sqrt{1 + \left(\frac{r_V}{r} \right)^3} \right]$$

Vainshtein screening

- **Solution** for $\phi'(r)$

$$\phi'(r) = -\frac{\Lambda^3 c_2 r}{8c_3} \left[1 - \sqrt{1 + \left(\frac{r_V}{r} \right)^3} \right]$$

Vainshtein radius

$$r_V \equiv \frac{1}{\Lambda} \left(\frac{4c_3 g M}{c_2 \pi M_{\text{Pl}}} \right)^{1/3} \sim 100 \text{ pc} \left(\frac{M}{M_\odot} \right)^{1/3}$$

Vainshtein screening

- **Solution** for $\phi'(r)$

$$\phi'(r) = -\frac{\Lambda^3 c_2 r}{8c_3} \left[1 - \sqrt{1 + \left(\frac{r_V}{r} \right)^3} \right]$$

$\phi(r)$



Vainshtein radius

$$r_V \equiv \frac{1}{\Lambda} \left(\frac{4c_3 g M}{c_2 \pi M_{\text{Pl}}} \right)^{1/3} \sim 100 \text{ pc} \left(\frac{M}{M_\odot} \right)^{1/3}$$

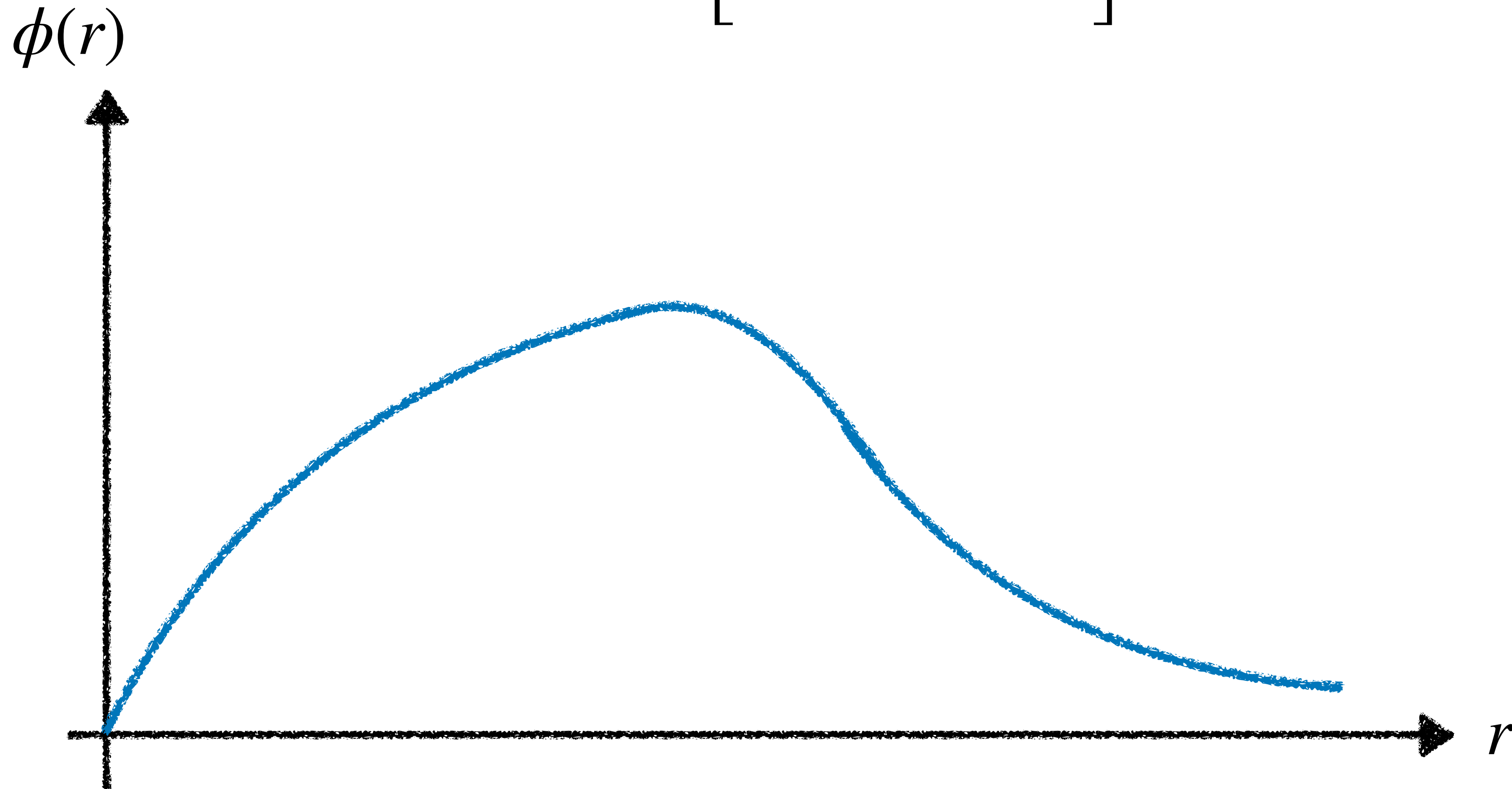
Vainshtein screening

- **Solution** for $\phi'(r)$

$$\phi'(r) = -\frac{\Lambda^3 c_2 r}{8c_3} \left[1 - \sqrt{1 + \left(\frac{r_V}{r} \right)^3} \right]$$

Vainshtein radius

$$r_V \equiv \frac{1}{\Lambda} \left(\frac{4c_3 g M}{c_2 \pi M_{\text{Pl}}} \right)^{1/3} \sim 100 \text{ pc} \left(\frac{M}{M_\odot} \right)^{1/3}$$



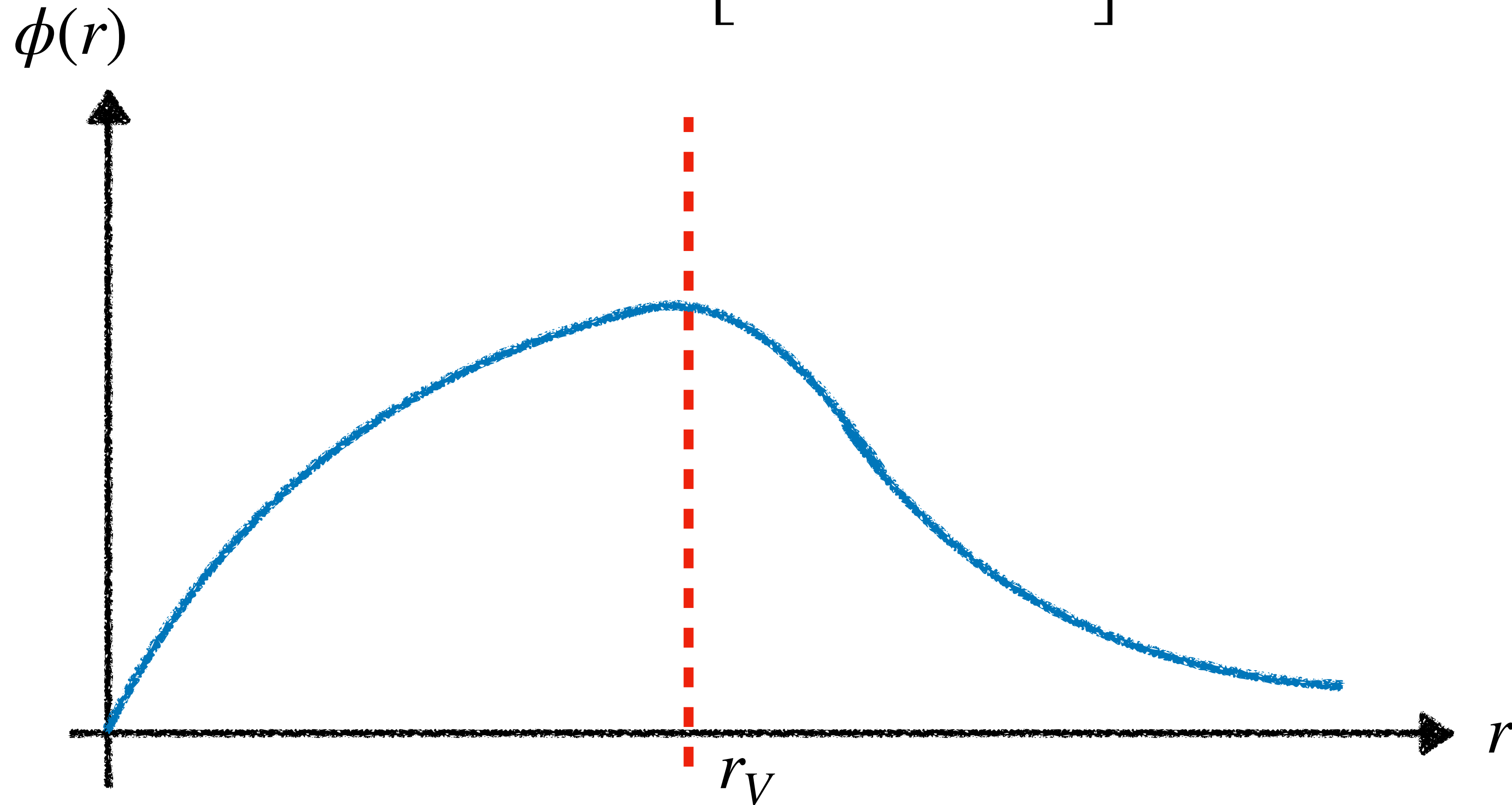
Vainshtein screening

- **Solution** for $\phi'(r)$

$$\phi'(r) = -\frac{\Lambda^3 c_2 r}{8c_3} \left[1 - \sqrt{1 + \left(\frac{r_V}{r} \right)^3} \right]$$

Vainshtein radius

$$r_V \equiv \frac{1}{\Lambda} \left(\frac{4c_3 g M}{c_2 \pi M_{\text{Pl}}} \right)^{1/3} \sim 100 \text{ pc} \left(\frac{M}{M_\odot} \right)^{1/3}$$



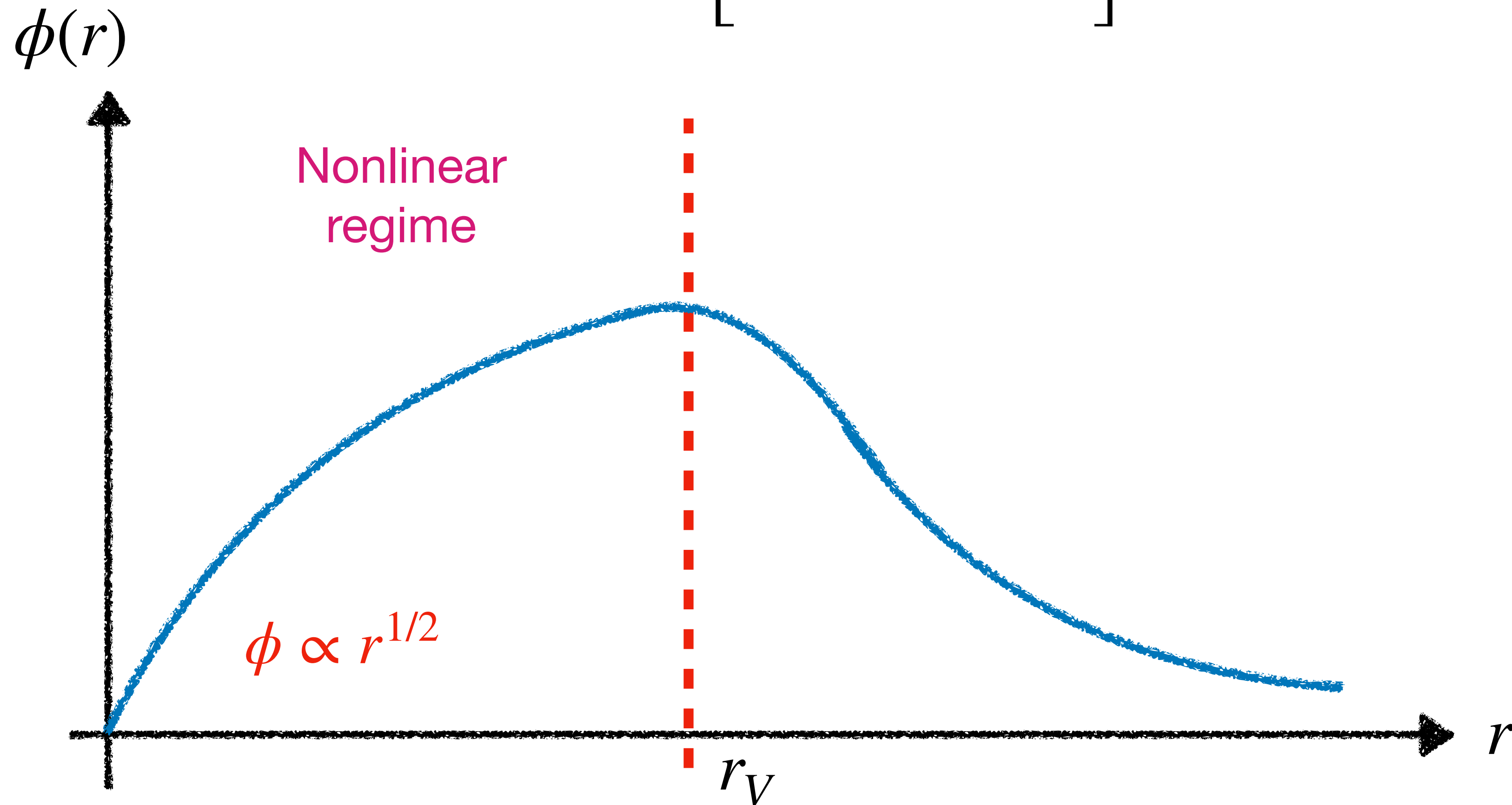
Vainshtein screening

- **Solution** for $\phi'(r)$

$$\phi'(r) = -\frac{\Lambda^3 c_2 r}{8c_3} \left[1 - \sqrt{1 + \left(\frac{r_V}{r} \right)^3} \right]$$

Vainshtein radius

$$r_V \equiv \frac{1}{\Lambda} \left(\frac{4c_3 g M}{c_2 \pi M_{\text{Pl}}} \right)^{1/3} \sim 100 \text{ pc} \left(\frac{M}{M_\odot} \right)^{1/3}$$



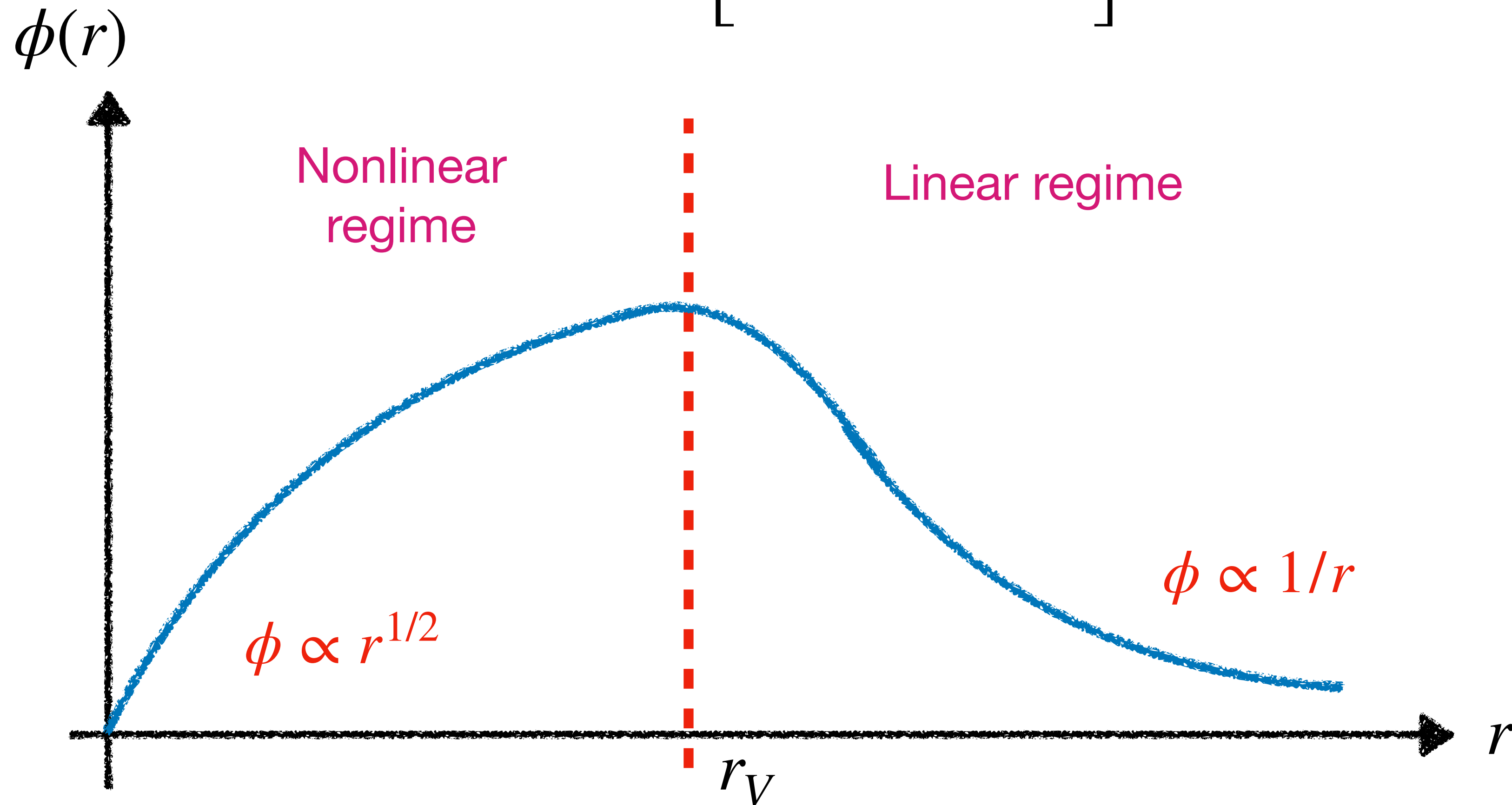
Vainshtein screening

- **Solution** for $\phi'(r)$

$$\phi'(r) = -\frac{\Lambda^3 c_2 r}{8c_3} \left[1 - \sqrt{1 + \left(\frac{r_V}{r} \right)^3} \right]$$

Vainshtein radius

$$r_V \equiv \frac{1}{\Lambda} \left(\frac{4c_3 g M}{c_2 \pi M_{\text{Pl}}} \right)^{1/3} \sim 100 \text{ pc} \left(\frac{M}{M_\odot} \right)^{1/3}$$



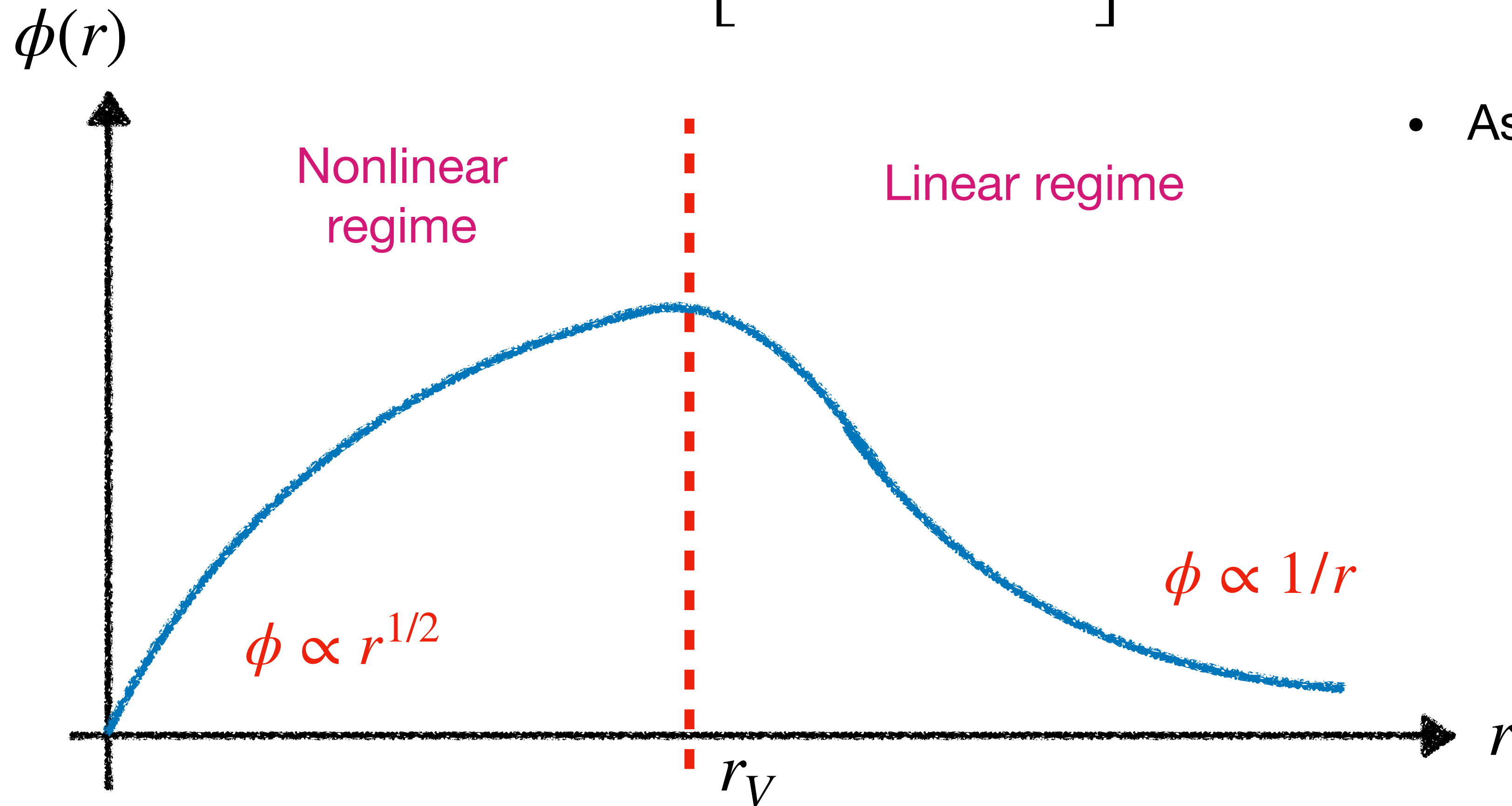
Vainshtein screening

- **Solution** for $\phi'(r)$

$$\phi'(r) = -\frac{\Lambda^3 c_2 r}{8c_3} \left[1 - \sqrt{1 + \left(\frac{r_V}{r} \right)^3} \right]$$

Vainshtein radius

$$r_V \equiv \frac{1}{\Lambda} \left(\frac{4c_3 g M}{c_2 \pi M_{\text{Pl}}} \right)^{1/3} \sim 100 \text{ pc} \left(\frac{M}{M_\odot} \right)^{1/3}$$



- As $r \gg r_V$ $\phi \simeq -\frac{gM}{4\pi r M_{\text{Pl}}}$

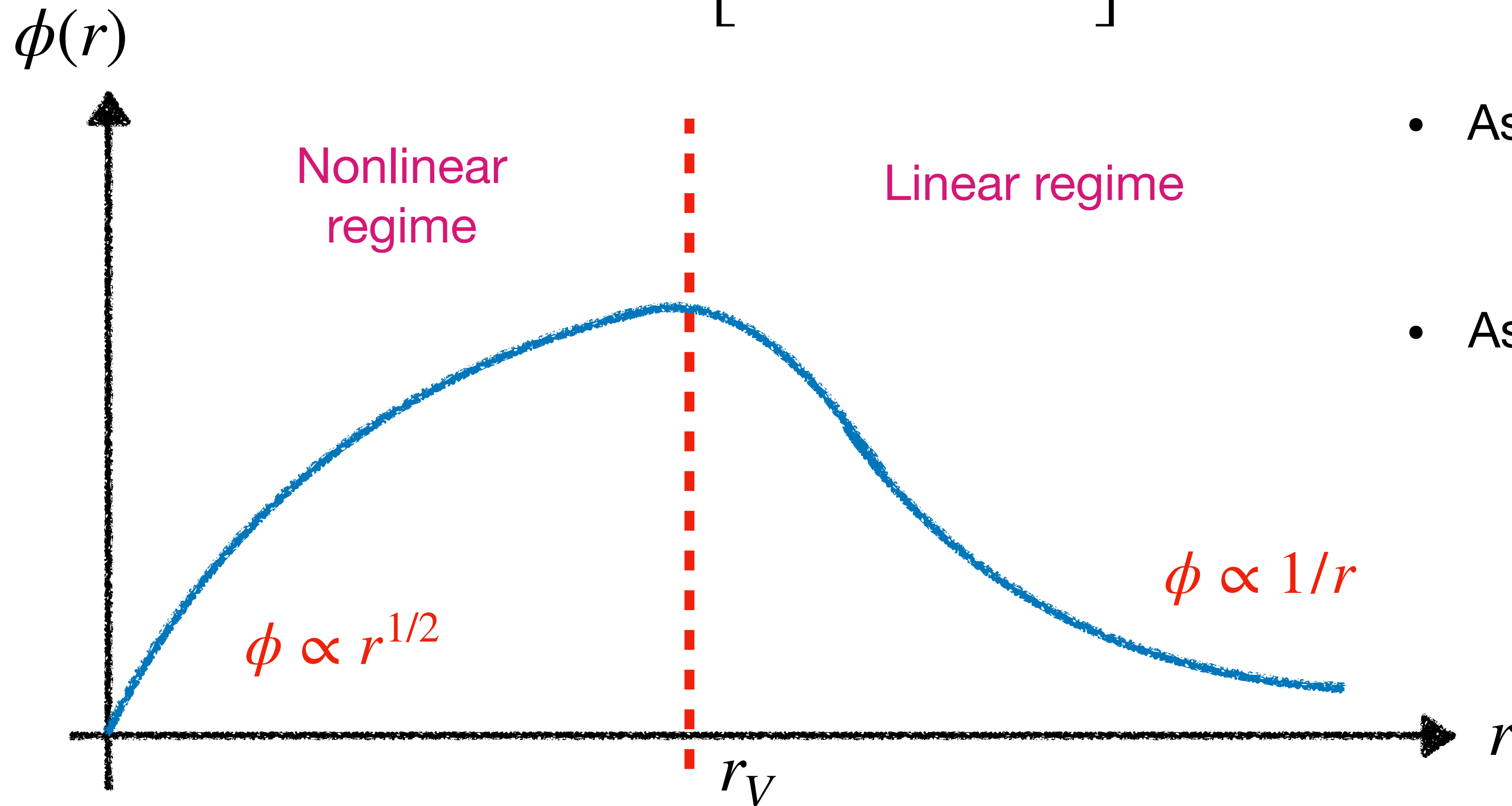
Vainshtein screening

- **Solution** for $\phi'(r)$

$$\phi'(r) = -\frac{\Lambda^3 c_2 r}{8c_3} \left[1 - \sqrt{1 + \left(\frac{r_V}{r} \right)^3} \right]$$

Vainshtein radius

$$r_V \equiv \frac{1}{\Lambda} \left(\frac{4c_3 g M}{c_2 \pi M_{\text{Pl}}} \right)^{1/3} \sim 100 \text{ pc} \left(\frac{M}{M_\odot} \right)^{1/3}$$



- As $r \gg r_V$ $\phi \simeq -\frac{gM}{4\pi r M_{\text{Pl}}}$
- As $r \ll r_V$ $\phi \simeq \frac{gMr^{1/2}}{\pi r_V^{3/2} M_{\text{Pl}}} + \text{const.}$

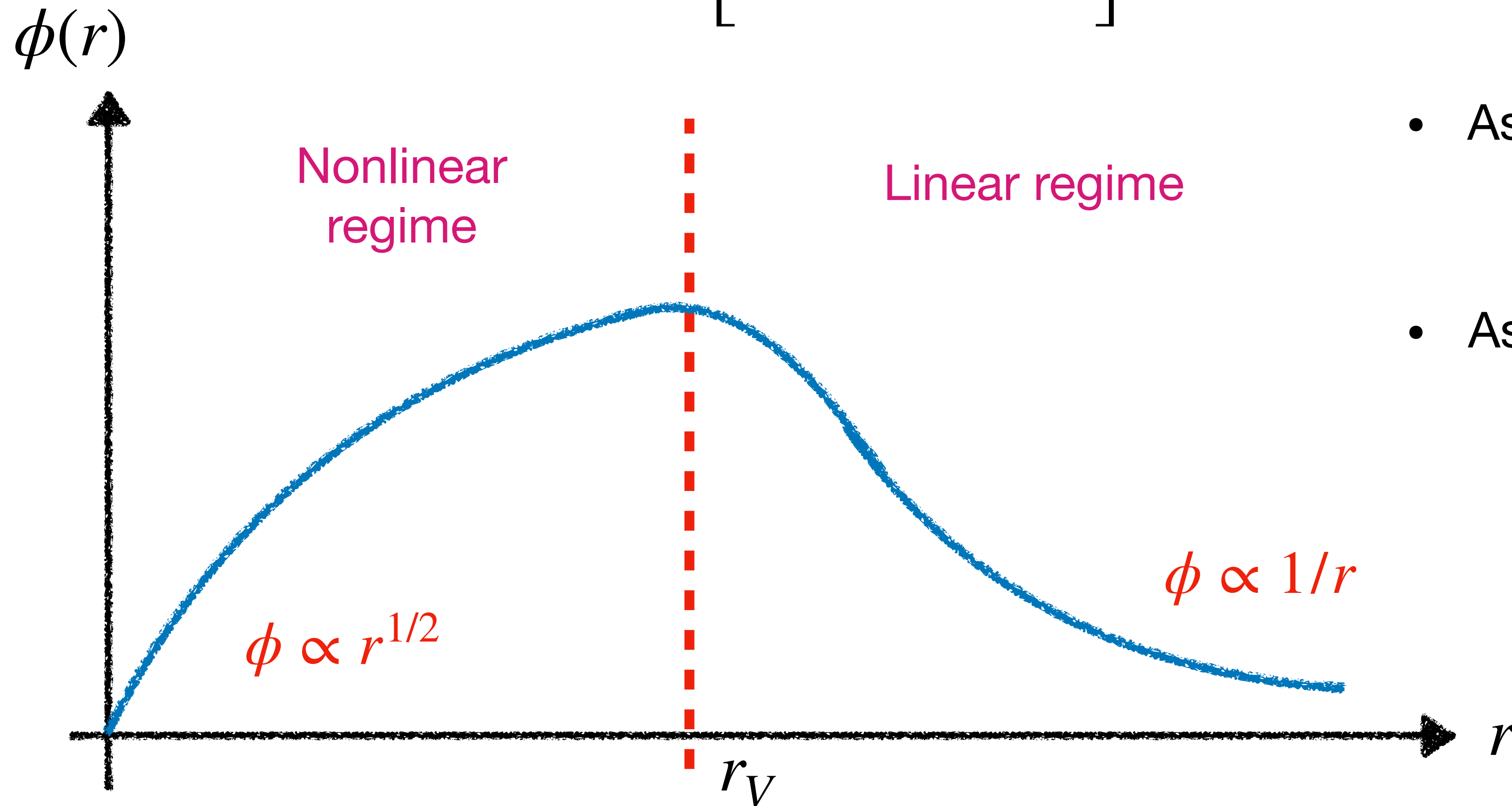
Vainshtein screening

- **Solution** for $\phi'(r)$

$$\phi'(r) = -\frac{\Lambda^3 c_2 r}{8c_3} \left[1 - \sqrt{1 + \left(\frac{r_V}{r} \right)^3} \right]$$

Vainshtein radius

$$r_V \equiv \frac{1}{\Lambda} \left(\frac{4c_3 g M}{c_2 \pi M_{\text{Pl}}} \right)^{1/3} \sim 100 \text{ pc} \left(\frac{M}{M_\odot} \right)^{1/3}$$



- As $r \gg r_V$ $\phi \simeq -\frac{gM}{4\pi r M_{\text{Pl}}}$
- As $r \ll r_V$ $\phi \simeq \frac{gM r^{1/2}}{\pi r_V^{3/2} M_{\text{Pl}}} + \text{const.}$

$$\frac{F_\phi}{F_{\text{gravity}}} \sim \left(\frac{r}{r_V} \right)^{3/2} \ll 1$$

Vainshtein screening

General scalar-tensor theories

- The most general scalar-tensor theories without the Ostrogradski ghost
: [\(Beyond\) Horndeski Theories](#)

$$L_2 = G_2(\Phi, X) \quad L_3 = G_3(\Phi, X) \square \Phi$$

$$L_4 = G_4(\Phi, X)R - 2G_{4X}(\Phi, X)[(\square\Phi)^2 - \Phi_{\mu\nu}\Phi^{\mu\nu}] - F_4(X, \Phi)\epsilon^{\mu\nu\rho}{}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\Phi_{\mu}\Phi_{\mu'}\Phi_{\nu\nu'}\Phi_{\rho\rho'}$$

$$L_5 = G_5(\Phi, X)G_{\mu\nu}\Phi^{\mu\nu} + \frac{1}{3}G_{5X}(\Phi, X)[(\square\Phi)^3 - 3(\square\Phi)\Phi_{\mu\nu}\Phi^{\mu\nu} + 2\Phi_{\mu\nu}\Phi^{\sigma\mu}\Phi^{\nu}_{\sigma}]$$

$$-F_5(\Phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\Phi_{\mu}\Phi_{\mu'}\Phi_{\nu\nu'}\Phi_{\rho\rho'}\Phi_{\sigma\sigma'}$$

$$\Phi_{\mu} \equiv \nabla_{\mu}\Phi \quad X = g^{\mu\nu}\partial_{\mu}\Phi\partial_{\nu}\Phi$$

[Horndeski 74](#), [Deffayet et al. 11](#), [Zumalacárregui and García-Bellido 14](#), [Gleyzes et al. 14](#) +++

General scalar-tensor theories

- The most general scalar-tensor theories without the Ostrogradski ghost
: (Beyond) Horndeski Theories

$$L_2 = G_2(\Phi, X) \quad L_3 = G_3(\Phi, X) \square \Phi \quad \text{Modifies speed of GW on } \Phi(t)$$

$$L_4 = G_4(\Phi, X)R - 2G_{4X}(\Phi, X)[(\square\Phi)^2 - \Phi_{\mu\nu}\Phi^{\mu\nu}] - F_4(X, \Phi)\epsilon^{\mu\nu\rho}{}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\Phi_{\mu}\Phi_{\mu'}\Phi_{\nu\nu'}\Phi_{\rho\rho'}$$

$$L_5 = G_5(\Phi, X)G_{\mu\nu}\Phi^{\mu\nu} + \frac{1}{3}G_{5X}(\Phi, X)[(\square\Phi)^3 - 3(\square\Phi)\Phi_{\mu\nu}\Phi^{\mu\nu} + 2\Phi_{\mu\nu}\Phi^{\sigma\mu}\Phi^{\nu}_{\sigma}]$$

$$-F_5(\Phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\Phi_{\mu}\Phi_{\mu'}\Phi_{\nu\nu'}\Phi_{\rho\rho'}\Phi_{\sigma\sigma'}$$

$$\Phi_{\mu} \equiv \nabla_{\mu}\Phi \quad X = g^{\mu\nu}\partial_{\mu}\Phi\partial_{\nu}\Phi$$

Horndeski 74, Deffayet et al. 11, Zumalacárregui and García-Bellido 14, Gleyzes et al. 14 +++

General scalar-tensor theories

- The most general scalar-tensor theories without the Ostrogradski ghost
: (Beyond) Horndeski Theories

$$L_2 = G_2(\Phi, X) \quad L_3 = G_3(\Phi, X) \square \Phi \quad \text{Modifies speed of GW on } \Phi(t)$$

$$L_4 = G_4(\Phi, X)R - 2G_{4X}(\Phi, X)[(\square\Phi)^2 - \Phi_{\mu\nu}\Phi^{\mu\nu}] - F_4(X, \Phi)\epsilon^{\mu\nu\rho}{}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\Phi_{\mu}\Phi_{\mu'}\Phi_{\nu\nu'}\Phi_{\rho\rho'}$$

$$L_5 = G_5(\Phi, X)G_{\mu\nu}\Phi^{\mu\nu} + \frac{1}{3}G_{5X}(\Phi, X)[(\square\Phi)^3 - 3(\square\Phi)\Phi_{\mu\nu}\Phi^{\mu\nu} + 2\Phi_{\mu\nu}\Phi^{\sigma\mu}\Phi^{\nu}_{\sigma}]$$

$$-F_5(\Phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\Phi_{\mu}\Phi_{\mu'}\Phi_{\nu\nu'}\Phi_{\rho\rho'}\Phi_{\sigma\sigma'}$$

$$\Phi_{\mu} \equiv \nabla_{\mu}\Phi \quad X = g^{\mu\nu}\partial_{\mu}\Phi\partial_{\nu}\Phi$$

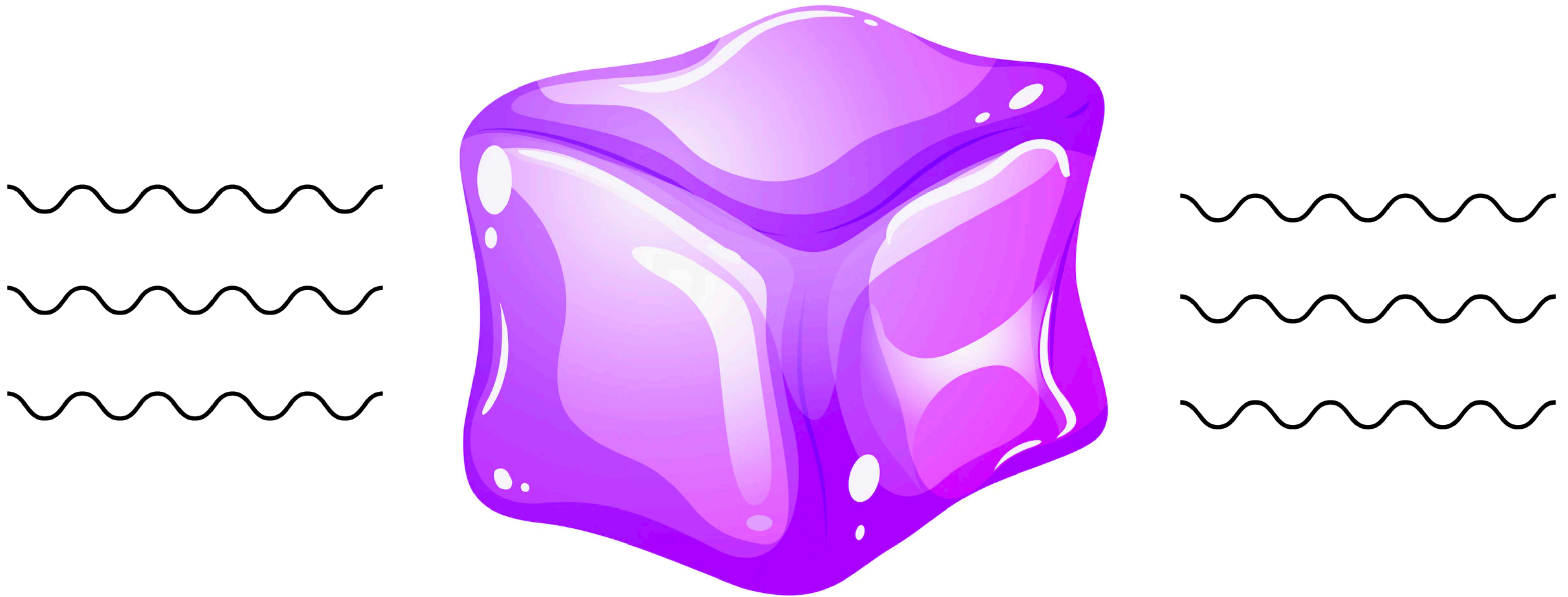
Horndeski 74, Deffayet et al. 11, Zumalacárregui and García-Bellido 14, Gleyzes et al. 14 +++

- (D)HOST Theories: Lagrangian contains second-order derivatives of a scalar field

Langlois and Noui 15, Langlois 17 +++

GW is traveling through a Medium of Dark Energy $\Phi(t)$

(Lorentz violating medium)



In general $c_{\text{GW}}^2 \neq c_{\text{Light}}^2$

GWs in Modified Gravity

GR: $\ddot{h}_{ij} + 3H(t)\dot{h}_{ij} - \frac{1}{a^2(t)}\nabla_k^2 h_{ij} = 0$

GWs in Modified Gravity

GR: $\ddot{h}_{ij} + 3H(t)\dot{h}_{ij} - \frac{1}{a^2(t)}\nabla_k^2 h_{ij} = 0$

Beyond GR: generic effects on GW propagation

GWs in Modified Gravity

GR: $\ddot{h}_{ij} + 3H(t)\dot{h}_{ij} - \frac{1}{a^2(t)}\nabla_k^2 h_{ij} = 0$

Beyond GR: generic effects on GW propagation

$$\ddot{h}_{ij} + (3 + \nu)H(t)\dot{h}_{ij} - \frac{1}{a^2(t)}c_{\text{GW}}^2\nabla_k^2 h_{ij} + m_g^2 h_{ij} = S_{ij}(\Psi_M)$$

GWs in Modified Gravity

GR: $\ddot{h}_{ij} + 3H(t)\dot{h}_{ij} - \frac{1}{a^2(t)}\nabla_k^2 h_{ij} = 0$

Beyond GR: generic effects on GW propagation

$$\ddot{h}_{ij} + (3 + \nu)H(t)\dot{h}_{ij} - \frac{1}{a^2(t)}c_{\text{GW}}^2\nabla_k^2 h_{ij} + m_g^2 h_{ij} = S_{ij}(\Psi_M)$$

Effective friction term



GWs in Modified Gravity

GR: $\ddot{h}_{ij} + 3H(t)\dot{h}_{ij} - \frac{1}{a^2(t)}\nabla_k^2 h_{ij} = 0$

Beyond GR: generic effects on GW propagation

$$\ddot{h}_{ij} + (3 + \nu)H(t)\dot{h}_{ij} - \frac{1}{a^2(t)}c_{\text{GW}}^2\nabla_k^2 h_{ij} + m_g^2 h_{ij} = S_{ij}(\Psi_M)$$

Effective friction term

Speed of GW

GWs in Modified Gravity

GR: $\ddot{h}_{ij} + 3H(t)\dot{h}_{ij} - \frac{1}{a^2(t)}\nabla_k^2 h_{ij} = 0$

Beyond GR: generic effects on GW propagation

$$\ddot{h}_{ij} + (3 + \nu)H(t)\dot{h}_{ij} - \frac{1}{a^2(t)}c_{\text{GW}}^2\nabla_k^2 h_{ij} + m_g^2 h_{ij} = S_{ij}(\Psi_M)$$

Effective friction term

Speed of GW

Effective mass term

GWs in Modified Gravity

GR: $\ddot{h}_{ij} + 3H(t)\dot{h}_{ij} - \frac{1}{a^2(t)}\nabla_k^2 h_{ij} = 0$

Beyond GR: generic effects on GW propagation

$$\ddot{h}_{ij} + (3 + \nu)H(t)\dot{h}_{ij} - \frac{1}{a^2(t)}c_{\text{GW}}^2\nabla_k^2 h_{ij} + m_g^2 h_{ij} = S_{ij}(\Psi_M)$$

Effective friction term Speed of GW Effective mass term Source term e.g. scalars

GWs in Modified Gravity

GR: $\ddot{h}_{ij} + 3H(t)\dot{h}_{ij} - \frac{1}{a^2(t)}\nabla_k^2 h_{ij} = 0$ $\nu = 0$, $c_{\text{GW}}^2 = 1$, $m_g^2 = 0$

Beyond GR: generic effects on GW propagation

$$\ddot{h}_{ij} + \underbrace{(3 + \nu)}_{\text{Effective friction term}} H(t)\dot{h}_{ij} - \frac{1}{a^2(t)} \underbrace{c_{\text{GW}}^2}_{\text{Speed of GW}} \nabla_k^2 h_{ij} + \underbrace{m_g^2}_{\text{Effective mass term}} h_{ij} = \underbrace{S_{ij}(\Psi_M)}_{\text{Source term e.g. scalars}}$$

GWs in Modified Gravity

GR: $\ddot{h}_{ij} + 3H(t)\dot{h}_{ij} - \frac{1}{a^2(t)}\nabla_k^2 h_{ij} = 0$ $\nu = 0$, $c_{\text{GW}}^2 = 1$, $m_g^2 = 0$

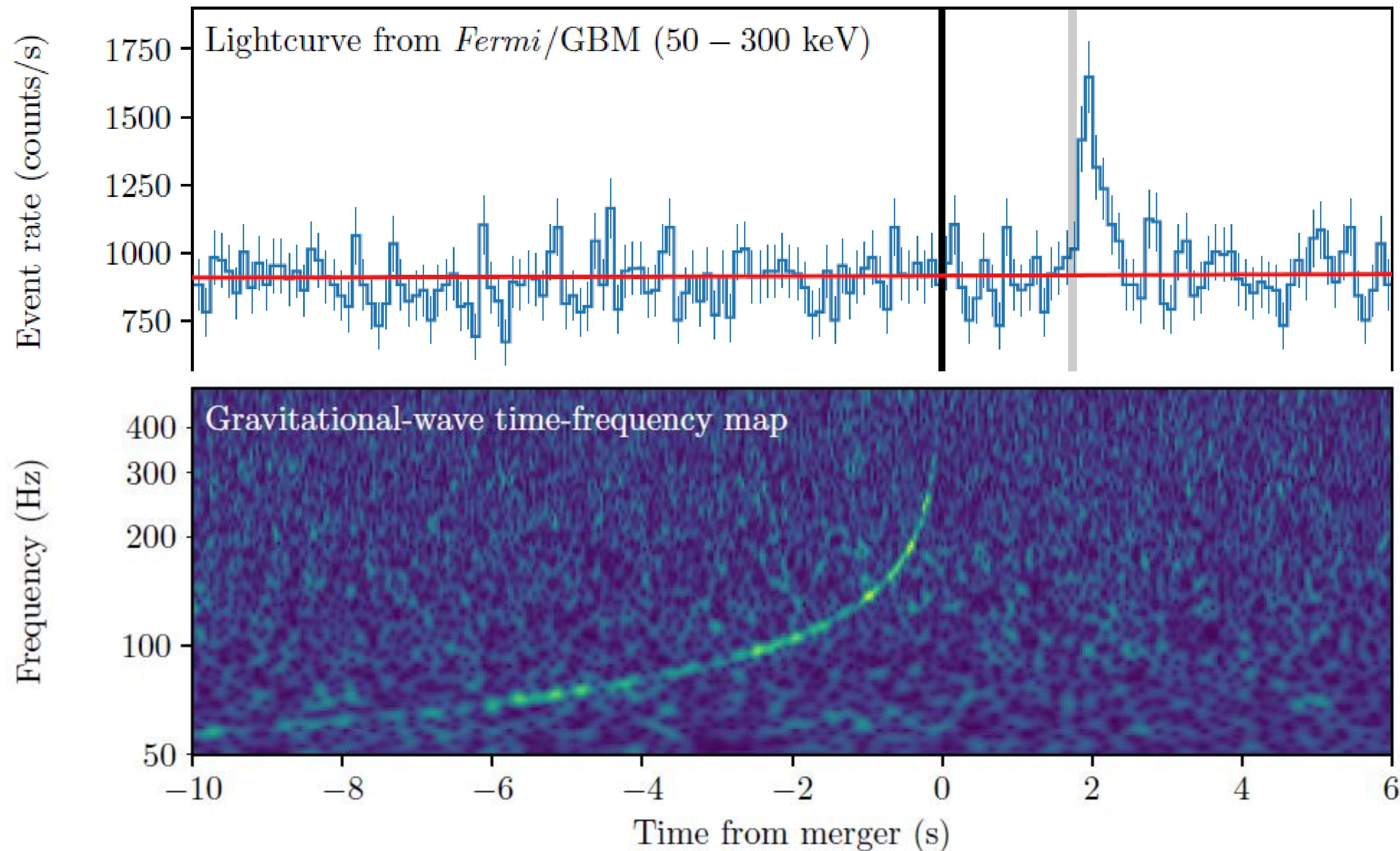
Beyond GR: generic effects on GW propagation

$$\ddot{h}_{ij} + \underbrace{(3 + \nu)}_{\text{Effective friction term}} H(t)\dot{h}_{ij} - \frac{1}{a^2(t)} \underbrace{c_{\text{GW}}^2}_{\text{Speed of GW}} \nabla_k^2 h_{ij} + \underbrace{m_g^2}_{\text{Effective mass term}} h_{ij} = \underbrace{S_{ij}(\Psi_M)}_{\text{Source term e.g. scalars}}$$

A Test of GR and Modified Gravity

GW170817 @ LIGO = GRB170817A @ Fermi

LIGO/Virgo + Fermi/GBM + INTEGRAL 17



$$|c_{\text{GW}}^2 - 1| \lesssim 10^{-15}$$

$$f_{\text{LIGO}} \sim 100 \text{ Hz} \sim 10^{-13} \text{ eV}$$

$$r \sim 40 \text{ Mpc}$$

Chirp mass

$$M_c \simeq 1.18 M_{\odot}$$

Consequence of $c_{\text{GW}}^2 = 1$

- Expand the (beyond) Horndeski action on $\Phi(t)$:
$$S[h_{ij}] = \frac{M^2}{8} \int d^4x \, a^3(t) \left[\dot{h}_{ij}^2 - \frac{c_{\text{GW}}^2}{a^2(t)} (\partial_k h_{ij})^2 \right]$$

Consequence of $c_{\text{GW}}^2 = 1$

- Expand the (beyond) Horndeski action on $\Phi(t)$: $S[h_{ij}] = \frac{M^2}{8} \int d^4x \, a^3(t) \left[\dot{h}_{ij}^2 - \frac{c_{\text{GW}}^2}{a^2(t)} (\partial_k h_{ij})^2 \right]$
- Impose $c_{\text{GW}}^2 = 1$ at **LIGO** scales

$$|c_{\text{GW}}^2 - 1| \propto -[2XG_{4X} + XG_{5\Phi} + (H\dot{\Phi} - \ddot{\Phi})XG_{5X}] + X^2F_4 - 3H\dot{\Phi}X^2F_5$$

Consequence of $c_{\text{GW}}^2 = 1$

- Expand the (beyond) Horndeski action on $\Phi(t)$: $S[h_{ij}] = \frac{M^2}{8} \int d^4x \, a^3(t) \left[\dot{h}_{ij}^2 - \frac{c_{\text{GW}}^2}{a^2(t)} (\partial_k h_{ij})^2 \right]$
- Impose $c_{\text{GW}}^2 = 1$ at **LIGO** scales

$$|c_{\text{GW}}^2 - 1| \propto -[2XG_{4X} + XG_{5\Phi} + (H\dot{\Phi} - \ddot{\Phi})XG_{5X}] + X^2F_4 - 3H\dot{\Phi}X^2F_5$$

$$L_2 = G_2(\Phi, X) \quad L_3 = G_3(\Phi, X) \square \Phi$$

$$L_4 = G_4(\Phi, X)R - 2G_{4X}(\Phi, X)[(\square\Phi)^2 - \Phi_{\mu\nu}\Phi^{\mu\nu}] - F_4(X, \Phi)\epsilon^{\mu\nu\rho}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\Phi_{\mu}\Phi_{\mu'}\Phi_{\nu\nu'}\Phi_{\rho\rho'}$$

$$L_5 = G_5(\Phi, X)G_{\mu\nu}\Phi^{\mu\nu} + \frac{1}{3}G_{5X}(\Phi, X)[(\square\Phi)^3 - 3(\square\Phi)\Phi_{\mu\nu}\Phi^{\mu\nu} + 2\Phi_{\mu\nu}\Phi^{\sigma\mu}\Phi^{\nu}_{\sigma}]$$

$$-F_5(\Phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\Phi_{\mu}\Phi_{\mu'}\Phi_{\nu\nu'}\Phi_{\rho\rho'}\Phi_{\sigma\sigma'}$$

Consequence of $c_{\text{GW}}^2 = 1$

- Expand the (beyond) Horndeski action on $\Phi(t)$: $S[h_{ij}] = \frac{M^2}{8} \int d^4x \, a^3(t) \left[\dot{h}_{ij}^2 - \frac{c_{\text{GW}}^2}{a^2(t)} (\partial_k h_{ij})^2 \right]$
- Impose $c_{\text{GW}}^2 = 1$ at **LIGO** scales

$$|c_{\text{GW}}^2 - 1| \propto -[2XG_{4X} + XG_{5\Phi} + (H\dot{\Phi} - \ddot{\Phi})XG_{5X}] + X^2F_4 - 3H\dot{\Phi}X^2F_5$$

$$L_2 = G_2(\Phi, X) \quad L_3 = G_3(\Phi, X) \square \Phi$$

$$L_4 = G_4(\Phi, X)R - 2G_{4X}(\Phi, X)[(\square\Phi)^2 - \Phi_{\mu\nu}\Phi^{\mu\nu}] - F_4(X, \Phi)\epsilon^{\mu\nu\rho}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\Phi_{\mu}\Phi_{\mu'}\Phi_{\nu\nu'}\Phi_{\rho\rho'}$$

$$L_5 = G_5(\Phi, X)G_{\mu\nu}\Phi^{\mu\nu} + \frac{1}{3}G_{5X}(\Phi, X)[(\square\Phi)^3 - 3(\square\Phi)\Phi_{\mu\nu}\Phi^{\mu\nu} + 2\Phi_{\mu\nu}\Phi^{\sigma\mu}\Phi^{\nu}_{\sigma}]$$

$$-F_5(\Phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\Phi_{\mu}\Phi_{\mu'}\Phi_{\nu\nu'}\Phi_{\rho\rho'}\Phi_{\sigma\sigma'}$$

Example of using the constraint $c_{\text{GW}}^2 = 1$

- Beyond Horndeski Theories with $c_{\text{GW}}^2 = 1$

Example of using the constraint $c_{\text{GW}}^2 = 1$

- Beyond Horndeski Theories with $c_{\text{GW}}^2 = 1$

$$L_{c_{\text{GW}}^2=1} = G_2(\Phi, X) + G_3(\Phi, X) \square \Phi + B_4(\Phi, X) R - \frac{4}{X} B_{4X} (\Phi^\mu \Phi^\nu \Phi_{\mu\nu} \square \Phi - \Phi^\mu \Phi_{\mu\nu} \Phi_\lambda \Phi^{\lambda\nu})$$

Example of using the constraint $c_{\text{GW}}^2 = 1$

- Beyond Horndeski Theories with $c_{\text{GW}}^2 = 1$

$$L_{c_{\text{GW}}^2=1} = G_2(\Phi, X) + G_3(\Phi, X) \square \Phi + B_4(\Phi, X) R - \frac{4}{X} B_{4X} (\Phi^\mu \Phi^\nu \Phi_{\mu\nu} \square \Phi - \Phi^\mu \Phi_{\mu\nu} \Phi_\lambda \Phi^{\lambda\nu})$$

$$B_4 = G_4 + \frac{X}{2} G_{5\Phi}$$

Example of using the constraint $c_{\text{GW}}^2 = 1$

- Beyond Horndeski Theories with $c_{\text{GW}}^2 = 1$

$$L_{c_{\text{GW}}^2=1} = G_2(\Phi, X) + G_3(\Phi, X) \square \Phi + B_4(\Phi, X) R - \frac{4}{X} B_{4X} (\Phi^\mu \Phi^\nu \Phi_{\mu\nu} \square \Phi - \Phi^\mu \Phi_{\mu\nu} \Phi_\lambda \Phi^{\lambda\nu})$$

$$B_4 = G_4 + \frac{X}{2} G_{5\Phi}$$

Caveats: Validity of the theories — the cut-off scale of the theories

de Rham and Melville 18

Example of using the constraint $c_{\text{GW}}^2 = 1$

- Beyond Horndeski Theories with $c_{\text{GW}}^2 = 1$

$$L_{c_{\text{GW}}^2=1} = G_2(\Phi, X) + G_3(\Phi, X) \square \Phi + B_4(\Phi, X) R - \frac{4}{X} B_{4X} (\Phi^\mu \Phi^\nu \Phi_{\mu\nu} \square \Phi - \Phi^\mu \Phi_{\mu\nu} \Phi_\lambda \Phi^{\lambda\nu})$$

$$B_4 = G_4 + \frac{X}{2} G_{5\Phi}$$

Caveats: Validity of the theories — the cut-off scale of the theories

de Rham and Melville 18

- Beyond Horndeski Theories with $c_{\text{GW}}^2 = 1$, no GW decay, and no instabilities of dark energy

Example of using the constraint $c_{\text{GW}}^2 = 1$

- Beyond Horndeski Theories with $c_{\text{GW}}^2 = 1$

$$L_{c_{\text{GW}}^2=1} = G_2(\Phi, X) + G_3(\Phi, X) \square \Phi + B_4(\Phi, X)R - \frac{4}{X}B_{4X}(\Phi^\mu \Phi^\nu \Phi_{\mu\nu} \square \Phi - \Phi^\mu \Phi_{\mu\nu} \Phi_\lambda \Phi^{\lambda\nu})$$

$$B_4 = G_4 + \frac{X}{2}G_{5\Phi}$$

Caveats: Validity of the theories — the cut-off scale of the theories

de Rham and Melville 18

- Beyond Horndeski Theories with $c_{\text{GW}}^2 = 1$, no GW decay, and no instabilities of dark energy

$$L_{c_{\text{GW}}^2=1, \text{no-decay}} = G_2(\Phi, X) + C(\Phi, X)R + \frac{6C_X(\Phi, X)^2}{C(\Phi, X)}\Phi^\mu \Phi_{\mu\nu} \Phi_\lambda \Phi^{\lambda\nu}$$

After performing $g_{\mu\nu} \rightarrow C(\Phi, X)g_{\mu\nu}$

Creminelli et al. 18, Creminelli, Tambalo, Vernizzi and VY 19 + 20

Effective Field Theory approach

- Efficient way to study a perturbation around fixed background
- Effective Field Theory of [Inflation/Dark Energy](#)

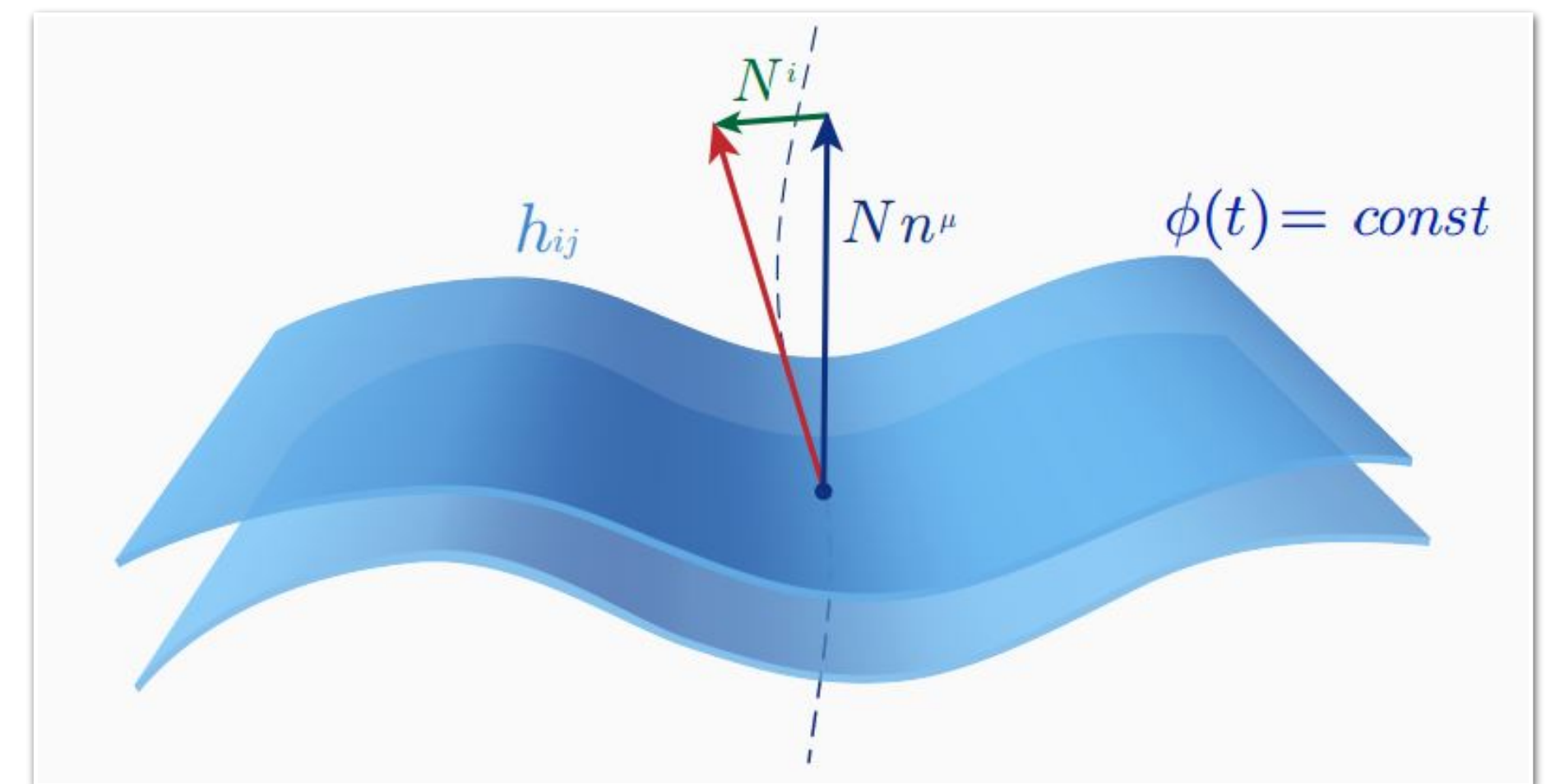
Cheung et al. 08, Gubitosi et al. 12, +++

$$ds^2 = -dt^2 + a(t)^2 dx_i dx^i$$

$\Phi(t)$ spontaneously breaks time diffeomorphism

$$S = \int d^4x \sqrt{-g} \, L[t; N, K_j^i, {}^{(3)}R, \dots]$$

The action is invariant under [3d](#) diff.



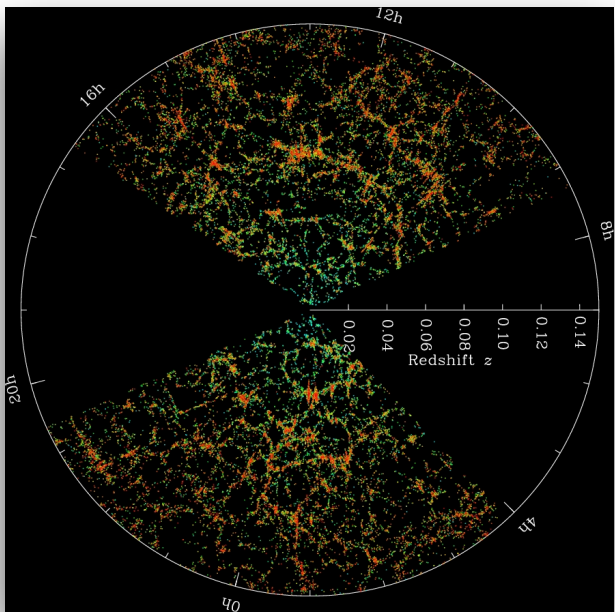
$$ds^2 = -N^2 dt^2 + h_{ij}(N^i dt + dx^i)(N^j dt + dx^j)$$

$$S^{(2)} = \int d^4x \sqrt{h} \frac{M^2}{2} \left[\delta K_j^i \delta K_i^j - \delta K^2 + {}^{(3)}R \delta N {}^{(3)}R + \sum_i \alpha_i(t) \mathcal{O}_i^{(2)}(\delta N, \delta K, \dots) \right]$$

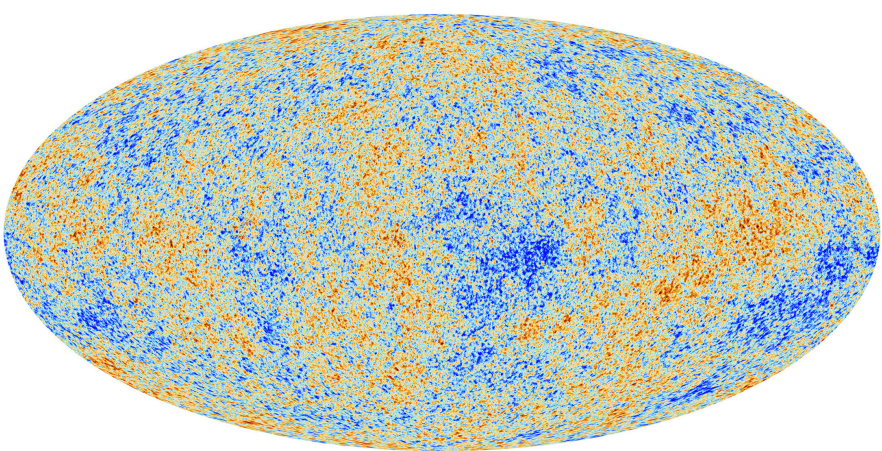
$\delta K_j^i = K_j^i - H \delta_j^i$

Deviations from GR

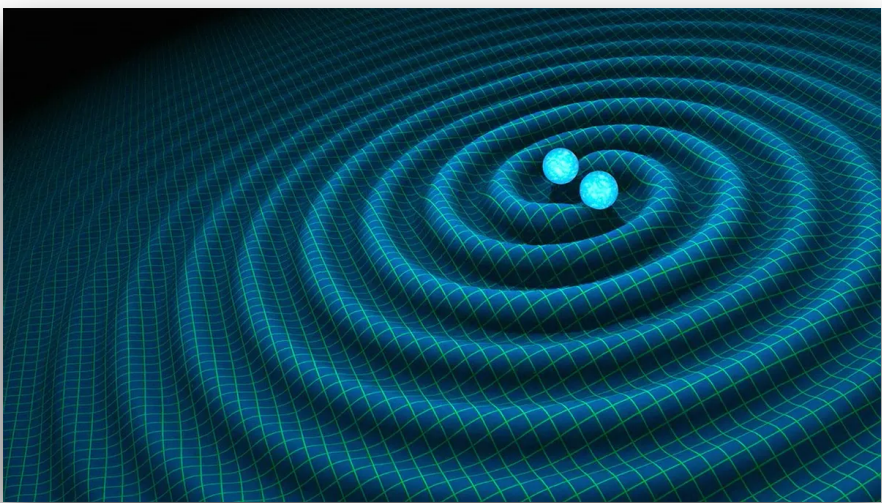
Observations



Galaxy clusters — SDSS

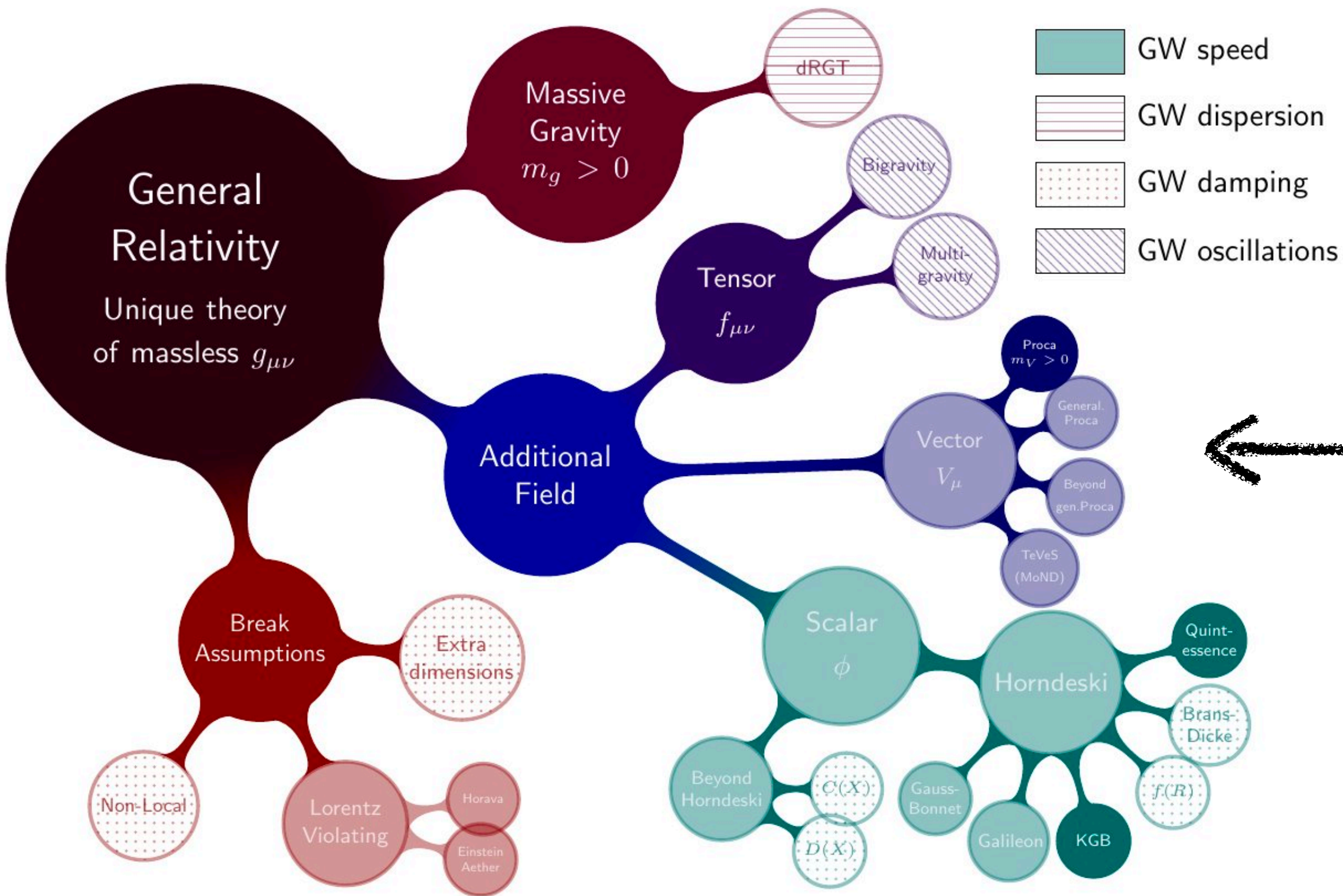


CMB — Planck



Grav. Waves

Modified gravity roadmap



$\alpha_K, \alpha_B, \alpha_H,$
 $\alpha_M, \alpha_T, \text{etc.}$

EFT of DE

Bridges the models and observations in a minimal and systematic way

EFT of DE after GW170817

- EFT of Dark Energy in **unitary gauge**: $\delta\Phi = 0$

$$S^{\text{EFT}} = \int d^4x \sqrt{-g} \left[\frac{M_\star^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} + \frac{m_2^4(t)}{2} (\delta g^{00})^2 - \frac{m_3^3(t)}{2} \delta g^{00} \delta K - m_4^2(t) \delta \mathcal{K}_2 \right. \\ \left. + \frac{\tilde{m}_4^2(t)}{2} \delta g^{00(3)} R - \frac{m_5^2(t)}{2} \delta g^{00} \delta \mathcal{K}_2 - \frac{m_6(t)}{3} \delta \mathcal{K}_3 - \tilde{m}_6(t) \delta g^{00} \delta \mathcal{G}_2 - \frac{m_7(t)}{3} \delta g^{00} \delta \mathcal{K}_3 \right]$$

$$\delta g^{00} = 1 + g^{00} \quad \delta K_\nu^\mu = K_\nu^\mu - H \delta_\nu^\mu$$

$$\delta \mathcal{K}_2 \equiv \delta K^2 - \delta K_\nu^\mu \delta K_\mu^\nu$$

$$\delta \mathcal{G}_2 \equiv \delta K_\nu^{\mu(3)} R_\mu^\nu - \delta K^{(3)} R / 2$$

$$\delta \mathcal{K}_3 \equiv \delta K^3 - 3 \delta K \delta K_\nu^\mu \delta K_\mu^\nu + 2 \delta K_\mu^\nu \delta K_\rho^\mu \delta K_\nu^\rho$$

$$m_4^2 \delta \mathcal{K}_2 \supset m_4^2 \dot{\gamma}_{ij}^2$$

EFT of DE after GW170817

- EFT of Dark Energy in **unitary gauge**: $\delta\Phi = 0$

$$S^{\text{EFT}} = \int d^4x \sqrt{-g} \left[\frac{M_\star^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} + \frac{m_2^4(t)}{2} (\delta g^{00})^2 - \frac{m_3^3(t)}{2} \delta g^{00} \delta K - m_4^2(t) \delta \mathcal{K}_2 \right. \\ \left. + \frac{\tilde{m}_4^2(t)}{2} \delta g^{00(3)} R - \frac{m_5^2(t)}{2} \delta g^{00} \delta \mathcal{K}_2 - \frac{m_6(t)}{3} \delta \mathcal{K}_3 - \tilde{m}_6(t) \delta g^{00} \delta \mathcal{G}_2 - \frac{m_7(t)}{3} \delta g^{00} \delta \mathcal{K}_3 \right]$$

$$\delta g^{00} = 1 + g^{00} \quad \delta K_\nu^\mu = K_\nu^\mu - H \delta_\nu^\mu$$

$$\delta \mathcal{K}_2 \equiv \delta K^2 - \delta K_\nu^\mu \delta K_\mu^\nu$$

$$\delta \mathcal{G}_2 \equiv \delta K_\nu^{\mu(3)} R_\mu^\nu - \delta K^{(3)} R / 2$$

$$\delta \mathcal{K}_3 \equiv \delta K^3 - 3 \delta K \delta K_\nu^\mu \delta K_\mu^\nu + 2 \delta K_\mu^\nu \delta K_\rho^\mu \delta K_\nu^\rho$$

$$c_T^2 = 1$$

Creminelli and Vernizzi 17, Ezquiaga
and Zumalacárregui 17, Baker et al.
17, Sakstein and Jain 17

$$m_4^2 \delta \mathcal{K}_2 \supset m_4^2 \dot{\gamma}_{ij}^2$$

EFT of DE after GW170817

- EFT of Dark Energy in **unitary gauge**: $\delta\Phi = 0$

$$S^{\text{EFT}} = \int d^4x \sqrt{-g} \left[\frac{M_\star^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} + \frac{m_2^4(t)}{2} (\delta g^{00})^2 - \frac{m_3^3(t)}{2} \delta g^{00} \delta K - \cancel{m_4^2(t)} \delta \mathcal{K}_2 \right. \\ \left. + \overset{\tilde{m}_4^2 = m_5^2}{\frac{\tilde{m}_4^2(t)}{2}} \delta g^{00(3)} R - \frac{m_5^2(t)}{2} \delta g^{00} \delta \mathcal{K}_2 - \cancel{\frac{m_6(t)}{3}} \delta \mathcal{K}_3 - \cancel{\tilde{m}_6(t)} \delta g^{00} \delta \mathcal{G}_2 - \cancel{\frac{m_7(t)}{3}} \delta g^{00} \delta \mathcal{K}_3 \right]$$

$$\delta g^{00} = 1 + g^{00} \quad \delta K_\nu^\mu = K_\nu^\mu - H \delta_\nu^\mu$$

$$\delta \mathcal{K}_2 \equiv \delta K^2 - \delta K_\nu^\mu \delta K_\mu^\nu$$

$$\delta \mathcal{G}_2 \equiv \delta K_\nu^{\mu(3)} R_\mu^\nu - \delta K^{(3)} R / 2$$

$$\delta \mathcal{K}_3 \equiv \delta K^3 - 3 \delta K \delta K_\nu^\mu \delta K_\mu^\nu + 2 \delta K_\mu^\nu \delta K_\rho^\mu \delta K_\nu^\rho$$

$$c_T^2 = 1$$

Creminelli and Vernizzi 17, Ezquiaga
and Zumalacárregui 17, Baker et al.
17, Sakstein and Jain 17

$$m_4^2 \delta \mathcal{K}_2 \supset m_4^2 \dot{\gamma}_{ij}^2$$

EFT of DE after GW170817

- EFT of Dark Energy in **unitary gauge**: $\delta\Phi = 0$

$$S^{\text{EFT}} = \int d^4x \sqrt{-g} \left[\frac{M_\star^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} + \frac{m_2^4(t)}{2} (\delta g^{00})^2 - \frac{m_3^3(t)}{2} \delta g^{00} \delta K - \cancel{m_4^2(t) \delta \mathcal{K}_2} \right. \\ \left. + \overset{\tilde{m}_4^2 = m_5^2}{\frac{\tilde{m}_4^2(t)}{2} \delta g^{00(3)} R} - \frac{m_5^2(t)}{2} \delta g^{00} \delta \mathcal{K}_2 - \cancel{\frac{m_6(t)}{3} \delta \mathcal{K}_3} - \cancel{\tilde{m}_6(t) \delta g^{00} \delta \mathcal{G}_2} - \cancel{\frac{m_7(t)}{3} \delta g^{00} \delta \mathcal{K}_3} \right]$$

$$\delta g^{00} = 1 + g^{00} \quad \delta K_\nu^\mu = K_\nu^\mu - H \delta_\nu^\mu$$

$$\delta \mathcal{K}_2 \equiv \delta K^2 - \delta K_\nu^\mu \delta K_\mu^\nu$$

$$\delta \mathcal{G}_2 \equiv \delta K_\nu^{\mu(3)} R_\mu^\nu - \delta K^{(3)} R / 2$$

$$\delta \mathcal{K}_3 \equiv \delta K^3 - 3 \delta K \delta K_\nu^\mu \delta K_\mu^\nu + 2 \delta K_\mu^\nu \delta K_\rho^\mu \delta K_\nu^\rho$$

$$c_T^2 = 1$$

Creminelli and Vernizzi 17, Ezquiaga and Zumalacárregui 17, Baker et al. 17, Sakstein and Jain 17

$$m_4^2 \delta \mathcal{K}_2 \supset m_4^2 \dot{\gamma}_{ij}^2 \Rightarrow 1 - c_{\text{GW}}^2 = \frac{2m_4^2}{M_\star^2 f}$$

GW decay $\gamma \rightarrow \pi\pi$

- EFT of DE after **GW170817**

Creminelli et al. 18

$$S_{c_T^2=1}^{\text{EFT}} = \int d^4x \sqrt{-g} \left[\frac{M_*^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} + \frac{m_2^4(t)}{2} (\delta g^{00})^2 - \frac{m_3^3(t)}{2} \delta g^{00} \delta K + \frac{\tilde{m}_4^2(t)}{2} (\delta g^{00(3)} R - \delta g^{00} \delta \mathcal{K}_2) \right]$$

GW decay $\gamma \rightarrow \pi\pi$

- EFT of DE after **GW170817**

Creminelli et al. 18

$$S_{c_T^2=1}^{\text{EFT}} = \int d^4x \sqrt{-g} \left[\frac{M_*^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} + \frac{m_2^4(t)}{2} (\delta g^{00})^2 - \frac{m_3^3(t)}{2} \delta g^{00} \delta K + \frac{\tilde{m}_4^2(t)}{2} \left(\delta g^{00(3)} R - \delta g^{00} \delta \mathcal{K}_2 \right) \right]$$

Most relevant terms

GW decay $\gamma \rightarrow \pi\pi$

- EFT of DE after **GW170817**

Creminelli et al. 18

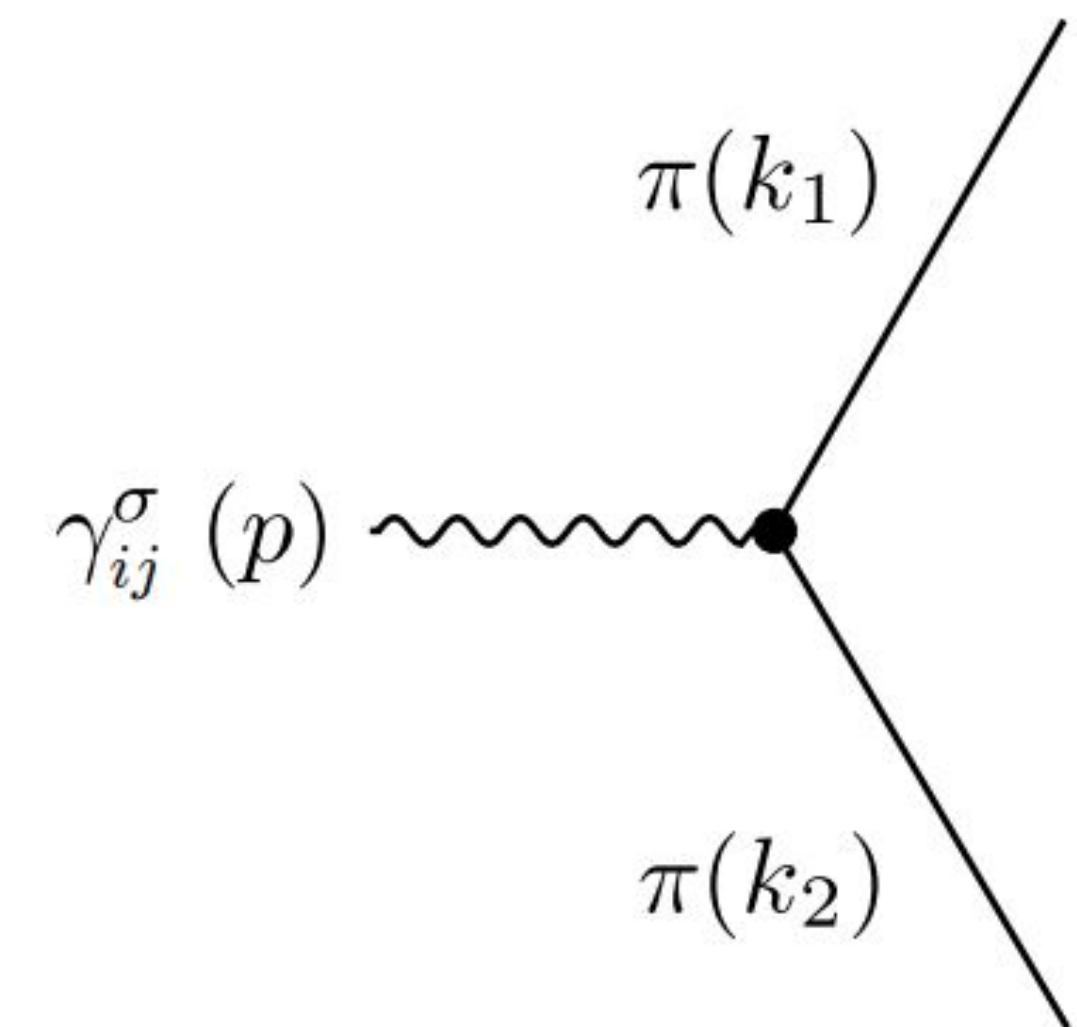
$$S_{c_T^2=1}^{\text{EFT}} = \int d^4x \sqrt{-g} \left[\frac{M_*^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} + \frac{m_2^4(t)}{2} (\delta g^{00})^2 - \frac{m_3^3(t)}{2} \delta g^{00} \delta K + \frac{\tilde{m}_4^2(t)}{2} \left(\delta g^{00(3)} R - \delta g^{00} \delta \mathcal{K}_2 \right) \right]$$

Most relevant terms

$$\mathcal{L}_{\gamma\pi\pi} = \frac{\alpha_H}{\Lambda_3^3} \dot{\gamma}_{ij} \partial_i \pi \partial_j \pi$$

$$\alpha_H = \frac{2\tilde{m}_4^2}{M_{\text{Pl}}^2}$$

$$\Lambda_3 = 10^{-13} \text{ eV}$$



GW decay $\gamma \rightarrow \pi\pi$

- EFT of DE after **GW170817** Creminelli et al. 18

$$S_{c_T^2=1}^{\text{EFT}} = \int d^4x \sqrt{-g} \left[\frac{M_*^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} + \frac{m_2^4(t)}{2} (\delta g^{00})^2 - \frac{m_3^3(t)}{2} \delta g^{00} \delta K + \frac{\tilde{m}_4^2(t)}{2} \left(\delta g^{00(3)} R - \delta g^{00} \delta \mathcal{K}_2 \right) \right]$$

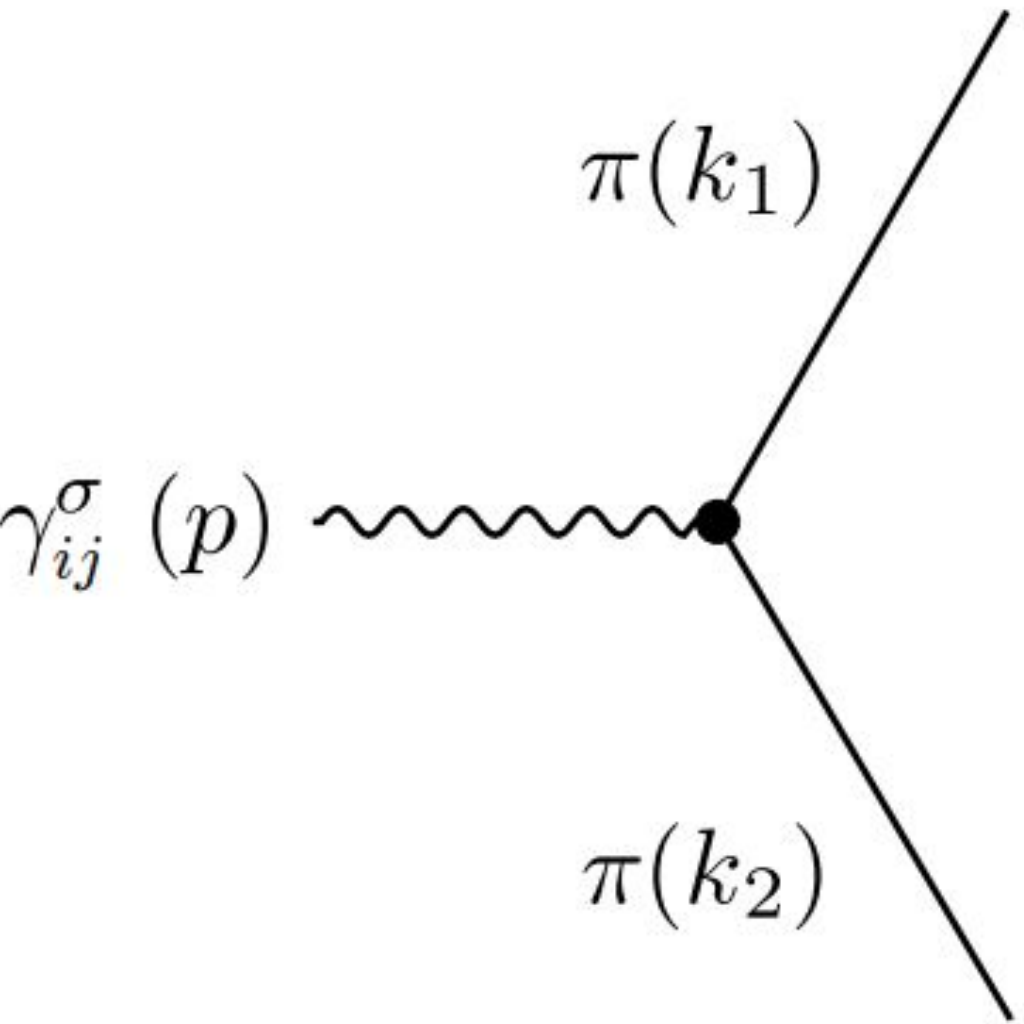
Most relevant terms

$$\mathcal{L}_{\gamma\pi\pi} = \frac{\alpha_H}{\Lambda_3^3} \dot{\gamma}_{ij} \partial_i \pi \partial_j \pi$$

$$\alpha_H = \frac{2\tilde{m}_4^2}{M_{\text{Pl}}^2}$$

$\Lambda_3 = 10^{-13} \text{ eV}$

$$\frac{\Gamma_{\gamma \rightarrow \pi\pi}}{H_0} \sim 10^{20} \alpha_H^2 \frac{(1 - c_s^2)^2}{480 \pi c_s^7} \lesssim 1$$



GW decay $\gamma \rightarrow \pi\pi$

- EFT of DE after **GW170817**

Creminelli et al. 18

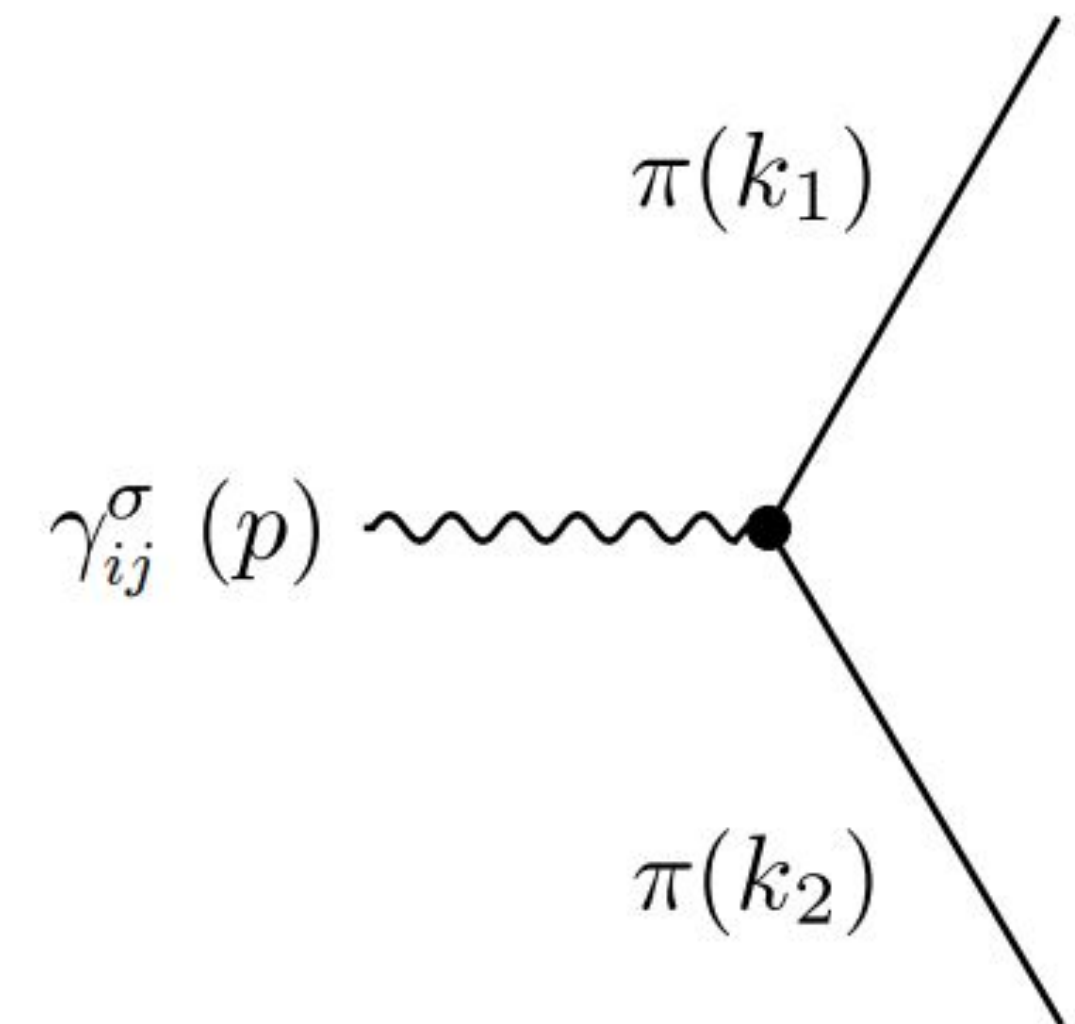
$$S_{c_T^2=1}^{\text{EFT}} = \int d^4x \sqrt{-g} \left[\frac{M_*^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} + \frac{m_2^4(t)}{2} (\delta g^{00})^2 - \frac{m_3^3(t)}{2} \delta g^{00} \delta K + \frac{\tilde{m}_4^2(t)}{2} \left(\delta g^{00(3)} R - \delta g^{00} \delta \mathcal{K}_2 \right) \right]$$

Most relevant terms

$$\mathcal{L}_{\gamma\pi\pi} = \frac{\alpha_H}{\Lambda_3^3} \dot{\gamma}_{ij} \partial_i \pi \partial_j \pi \quad \alpha_H = \frac{2\tilde{m}_4^2}{M_{\text{Pl}}^2} \quad \Lambda_3 = 10^{-13} \text{ eV}$$

$$\frac{\Gamma_{\gamma \rightarrow \pi\pi}}{H_0} \sim 10^{20} \alpha_H^2 \frac{(1 - c_s^2)^2}{480 \pi c_s^7} \lesssim 1 \quad \Rightarrow \quad \boxed{\alpha_H \lesssim 10^{-10}}$$

Beyond Horndeski is highly constrained as DE models



Resonant GW decay $\gamma \rightarrow \pi\pi$

- Take into account **large occupation** number of gravitons Creminelli, Tambalo, Vernizzi and VY 19
- **EoM** of $\pi(t, \vec{x})$ in the background of GW $\gamma_{ij} = M_{\text{Pl}}^2 h_0^+ \sin(\omega(t - z)) \epsilon_{ij}^+$

Resonant GW decay $\gamma \rightarrow \pi\pi$

- Take into account **large occupation** number of gravitons Creminelli, Tambalo, Vernizzi and VY 19
- **EoM** of $\pi(t, \vec{x})$ in the background of GW $\gamma_{ij} = M_{\text{Pl}} h_0^+ \sin(\omega(t - z)) \epsilon_{ij}^+$

$$\ddot{\pi} - c_s^2 \nabla^2 \pi - c_s^2 \beta \sin[\omega(t - z)] (\partial_x^2 - \partial_y^2) \pi = 0$$

Due to the coupling $\dot{\gamma}_{ij} \partial_i \pi \partial_j \pi$

$$\beta \equiv \frac{2\alpha_H \omega^2 M_{\text{Pl}} h_0^+}{\Lambda_3^3}$$

Resonant GW decay $\gamma \rightarrow \pi\pi$

- Take into account **large occupation** number of gravitons

Creminelli, Tambalo, Vernizzi and VY 19

- EoM** of $\pi(t, \vec{x})$ in the background of GW $\gamma_{ij} = M_{\text{Pl}} h_0^+ \sin(\omega(t - z)) \epsilon_{ij}^+$

$$\ddot{\pi} - c_s^2 \nabla^2 \pi - c_s^2 \beta \sin[\omega(t - z)] (\partial_x^2 - \partial_y^2) \pi = 0$$

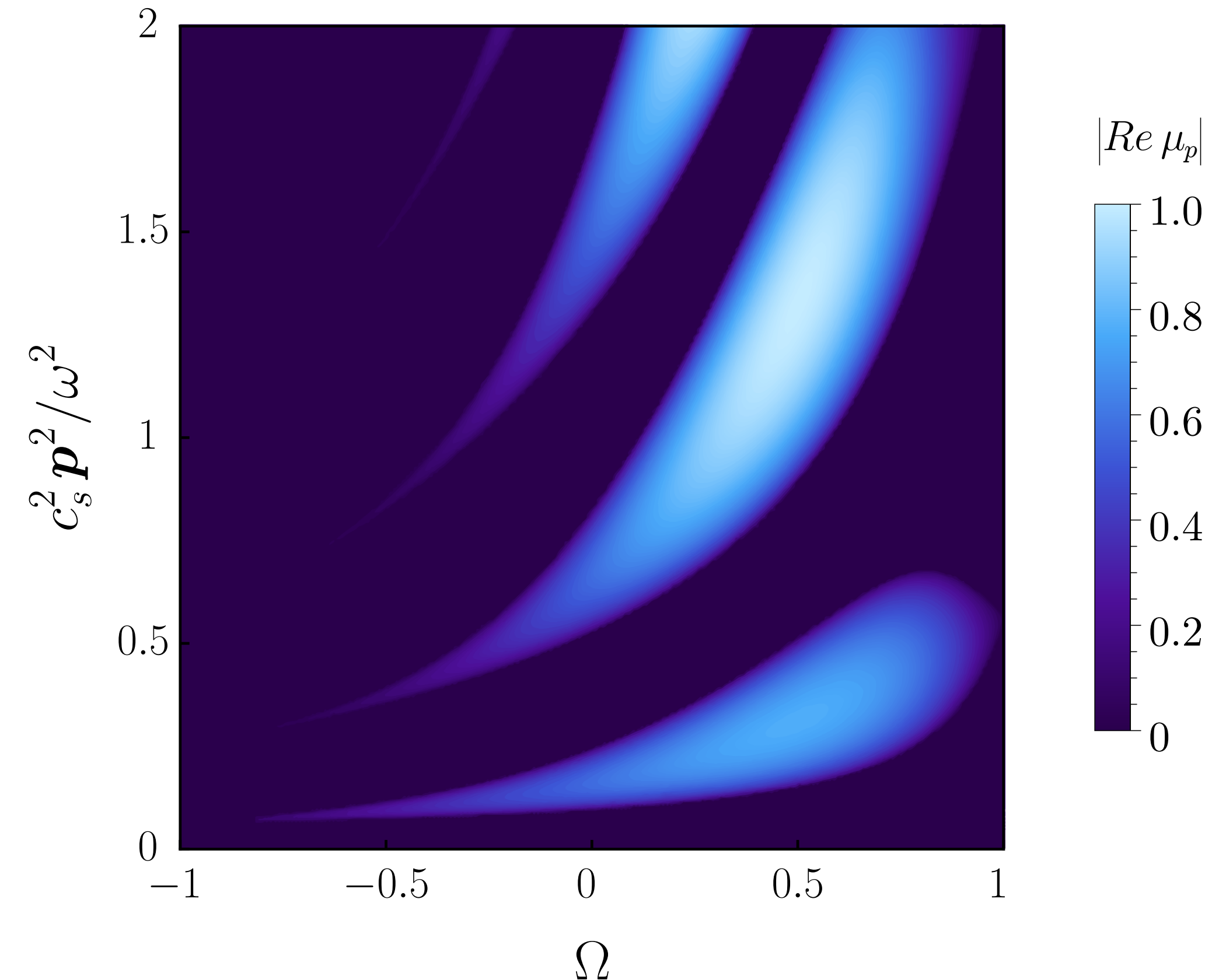
Due to the coupling $\dot{\gamma}_{ij} \partial_i \pi \partial_j \pi$

$$\beta \equiv \frac{2\alpha_H \omega^2 M_{\text{Pl}} h_0^+}{\Lambda_3^3}$$

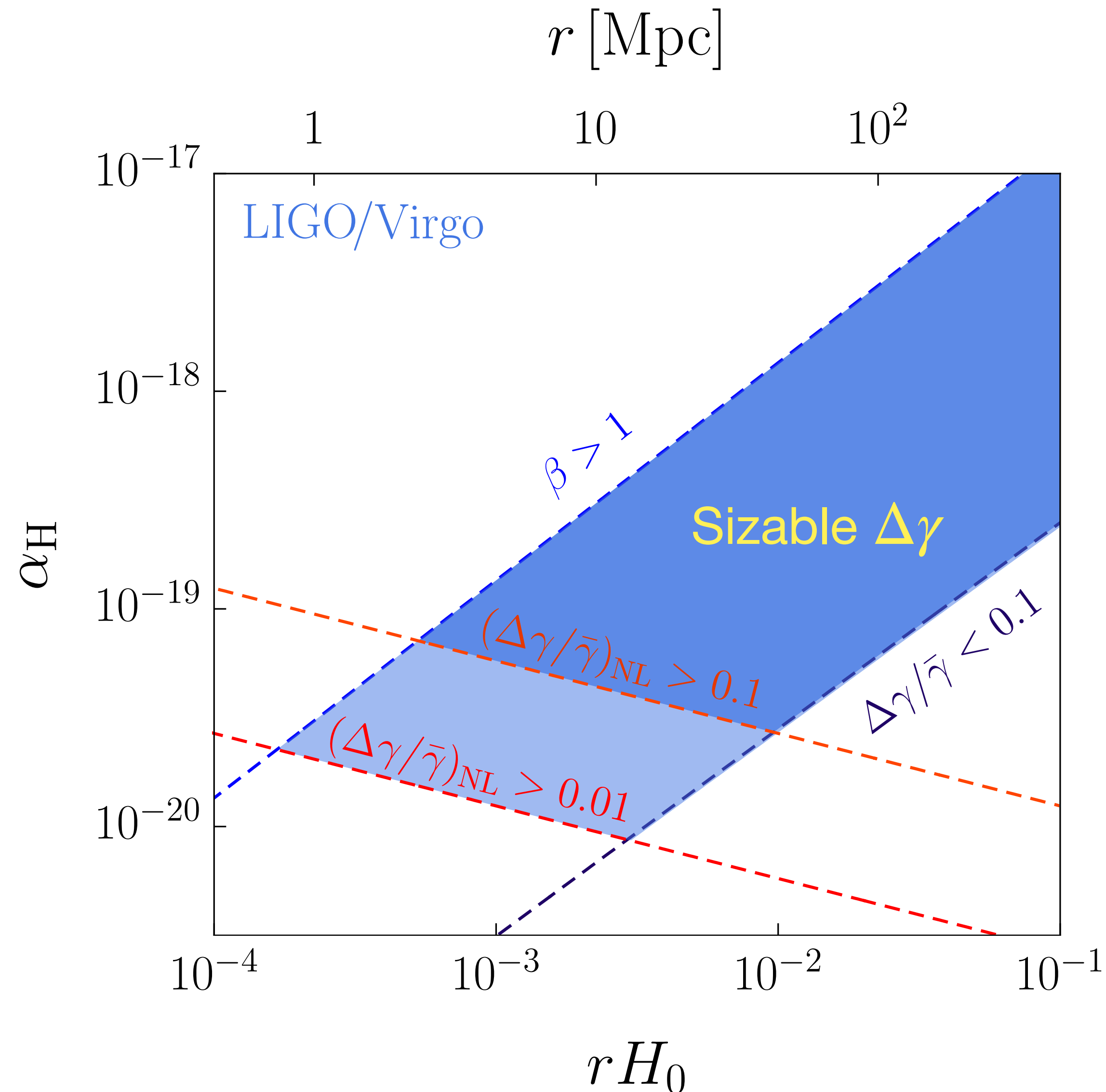
- $\beta < 1$: **Resonant** instability

$$\frac{d^2 \pi_{\vec{p}}}{d\tau^2} + [A - 2q \cos(2\tau)] \pi_{\vec{p}} = 0 \quad \Rightarrow \quad \pi_{\vec{p}} \sim e^{\mu\tau} \quad \mu \sim \beta$$

Floquet exponent



Resonant GW decay $\gamma \rightarrow \pi\pi$



- Modification to GWs: $\gamma_{ij} \equiv \bar{\gamma}_{ij} + \Delta\gamma_{ij}$

$$\Delta\gamma_{ij} \sim \frac{1}{\Lambda_{\star}^3} \partial_t^2 \left(\partial_i \pi \partial_j \pi \right)$$

- Neglect **nonlinearities** of π 's and
- Absence of sizable $\Delta\gamma_{ij}$ **rules out**

$$10^{-20} \lesssim |\alpha_H| \lesssim 10^{-17}$$

Perturbative bound: $|\alpha_H| \lesssim 10^{-10}$

DE Instabilities by GWs

- Large GW amplitude: $\beta > 1$

Creminelli, Tambalo, Vernizzi and VY 20

$$\ddot{\pi} - c_s^2 \nabla^2 \pi - c_s^2 \beta \sin[\omega(t - z)](\partial_x^2 - \partial_y^2)\pi = 0$$

DE Instabilities by GWs

- Large GW amplitude: $\beta > 1$

Creminelli, Tambalo, Vernizzi and VY 20

$$\ddot{\pi} - c_s^2 \nabla^2 \pi - \underline{c_s^2 \beta \sin[\omega(t - z)](\partial_x^2 - \partial_y^2) \pi} = 0 \quad \Rightarrow \quad \text{Gradient instability!}$$

DE Instabilities by GWs

- Large GW amplitude: $\beta > 1$

Creminelli, Tambalo, Vernizzi and VY 20

$$\ddot{\pi} - c_s^2 \nabla^2 \pi - \underline{c_s^2 \beta \sin[\omega(t - z)](\partial_x^2 - \partial_y^2) \pi} = 0 \quad \Rightarrow \quad \text{Gradient instability!}$$

- Too naive: one needs to consider **nonlinearities** of π

$$\mathcal{L}_\pi = -\frac{1}{2} \bar{\eta}^{\mu\nu} \partial_\mu \pi \partial_\nu \pi - \frac{1}{\Lambda_3^3} \square \pi (\partial \pi)^2 + \frac{\alpha_B}{\Lambda_2^2} \dot{\gamma}_{ij} \partial_i \pi \partial_j \pi + \frac{\alpha_B}{M_{\text{Pl}}} \pi \dot{\gamma}_{ij}^2$$

$$\Lambda_2 = 10^{-3} \text{ eV}$$

$$\Lambda_3 = 10^{-13} \text{ eV}$$

DE Instabilities by GWs

- Large GW amplitude: $\beta > 1$

Creminelli, Tambalo, Vernizzi and VY 20

$$\ddot{\pi} - c_s^2 \nabla^2 \pi - \underline{c_s^2 \beta \sin[\omega(t - z)](\partial_x^2 - \partial_y^2) \pi} = 0 \quad \Rightarrow \quad \text{Gradient instability!}$$

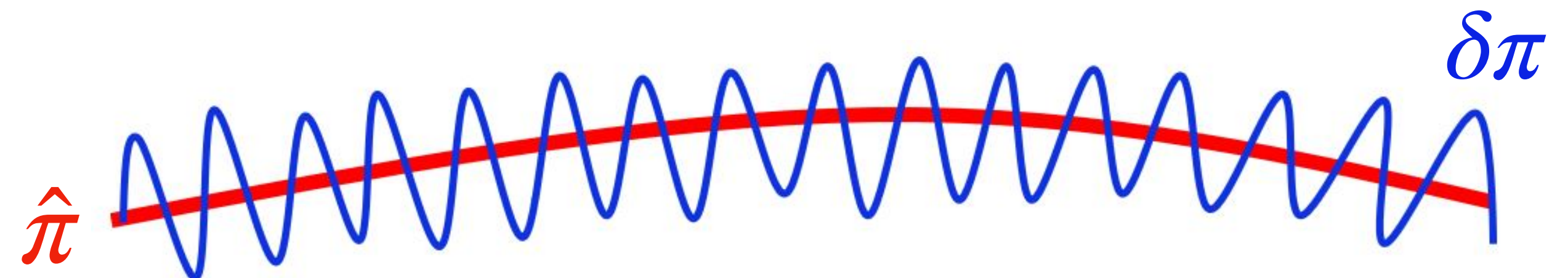
- Too naive: one needs to consider **nonlinearities** of π

$$\mathcal{L}_\pi = -\frac{1}{2} \bar{\eta}^{\mu\nu} \partial_\mu \pi \partial_\nu \pi - \frac{1}{\Lambda_3^3} \square \pi (\partial \pi)^2 + \frac{\alpha_B}{\Lambda_2^2} \dot{\gamma}_{ij} \partial_i \pi \partial_j \pi + \frac{\alpha_B}{M_{\text{Pl}}} \pi \dot{\gamma}_{ij}^2$$

$$\Lambda_2 = 10^{-3} \text{ eV}$$

$$\Lambda_3 = 10^{-13} \text{ eV}$$

- Expand around **nonlinear** solution: $\pi = \hat{\pi} + \delta\pi \quad \Rightarrow \quad \mathcal{L}_{\delta\pi} = Z^{\mu\nu}(x) \partial_\mu \delta\pi \partial_\nu \delta\pi$



DE Instabilities by GWs

- Large GW amplitude: $\beta > 1$

Creminelli, Tambalo, Vernizzi and VY 20

$$\ddot{\pi} - c_s^2 \nabla^2 \pi - c_s^2 \beta \sin[\omega(t - z)](\partial_x^2 - \partial_y^2)\pi = 0 \quad \Rightarrow \quad \text{Gradient instability!}$$

- Too naive: one needs to consider **nonlinearities** of π

$$\mathcal{L}_\pi = -\frac{1}{2}\bar{\eta}^{\mu\nu}\partial_\mu\pi\partial_\nu\pi - \frac{1}{\Lambda_3^3}\square\pi(\partial\pi)^2 + \frac{\alpha_B}{\Lambda_2^2}\dot{\gamma}_{ij}\partial_i\pi\partial_j\pi + \frac{\alpha_B}{M_{\text{Pl}}}\pi\dot{\gamma}_{ij}^2$$

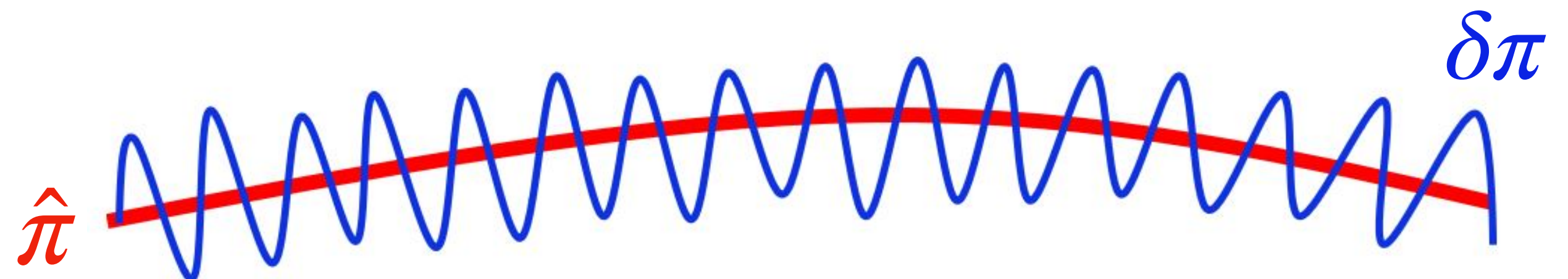
$$\Lambda_2 = 10^{-3} \text{ eV}$$

$$\Lambda_3 = 10^{-13} \text{ eV}$$

- Expand around **nonlinear** solution: $\pi = \hat{\pi} + \delta\pi \quad \Rightarrow \quad \mathcal{L}_{\delta\pi} = Z^{\mu\nu}(x) \partial_\mu\delta\pi\partial_\nu\delta\pi$

$$Z^{00} > 0: \text{No ghost}$$

$$Z^{0i}Z^{0j} - Z^{ij}Z^{00} > 0: \text{No gradient}$$

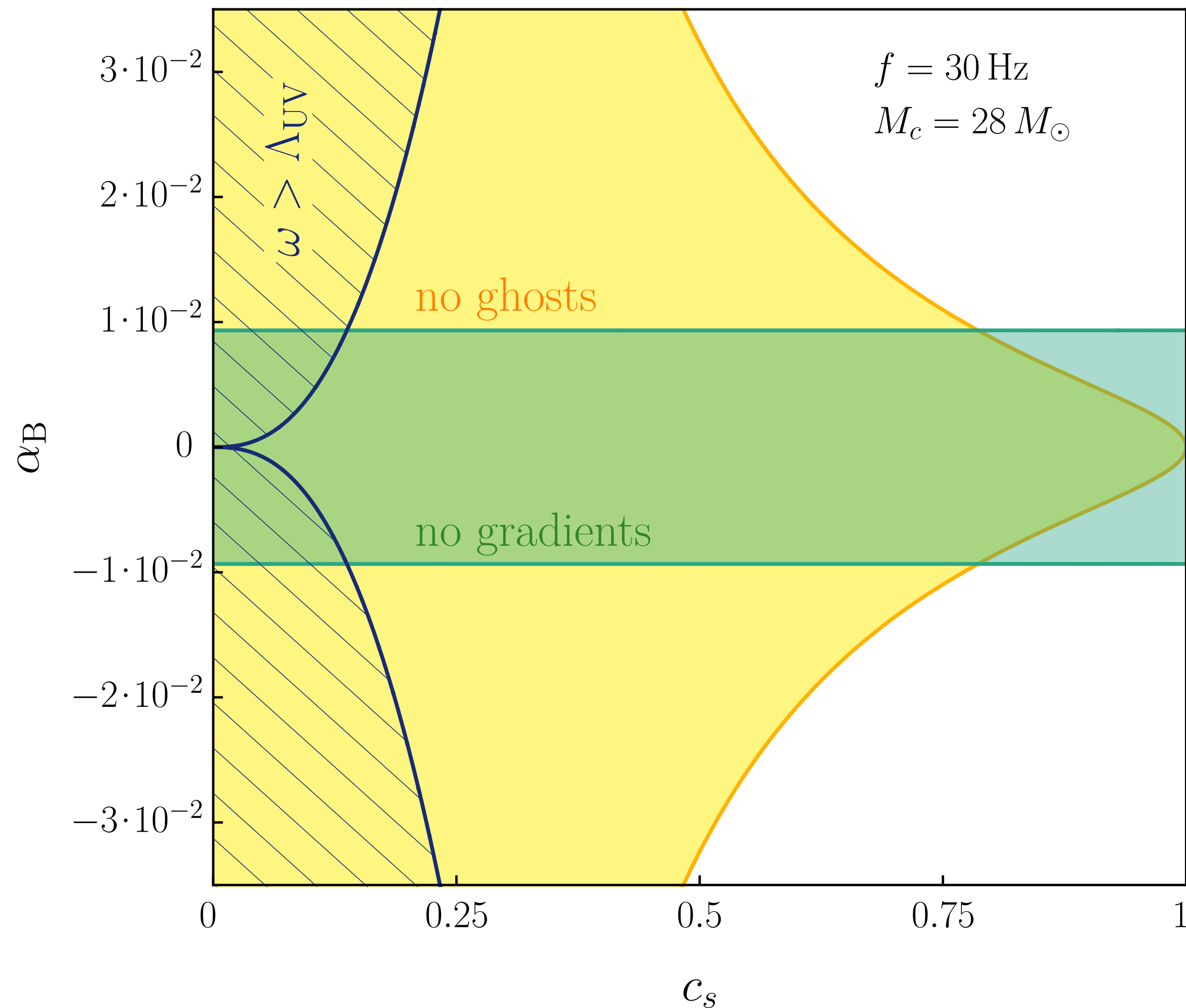


DE Instabilities by GWs

Creminelli, Tambalo, Vernizzi and VY 20

- Bound on α_B : cubic Horndeski

- Absence of ghost/gradient instabilities

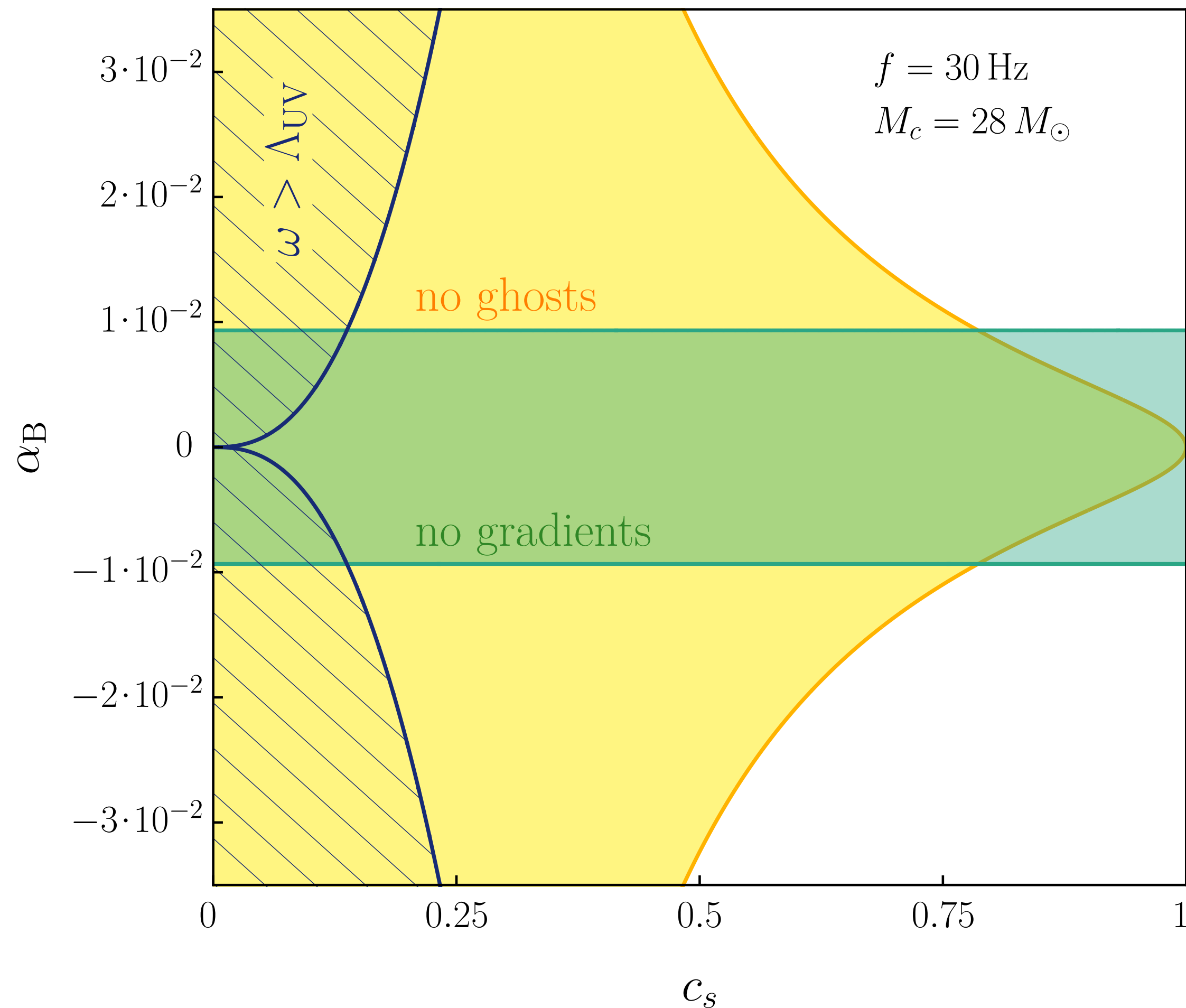


$$|\alpha_B| \lesssim 10^{-2}$$

DE Instabilities by GWs

Creminelli, Tambalo, Vernizzi and VY 20

- Bound on α_B : cubic Horndeski



- Absence of **ghost/gradient** instabilities

$$|\alpha_B| \lesssim 10^{-2}$$

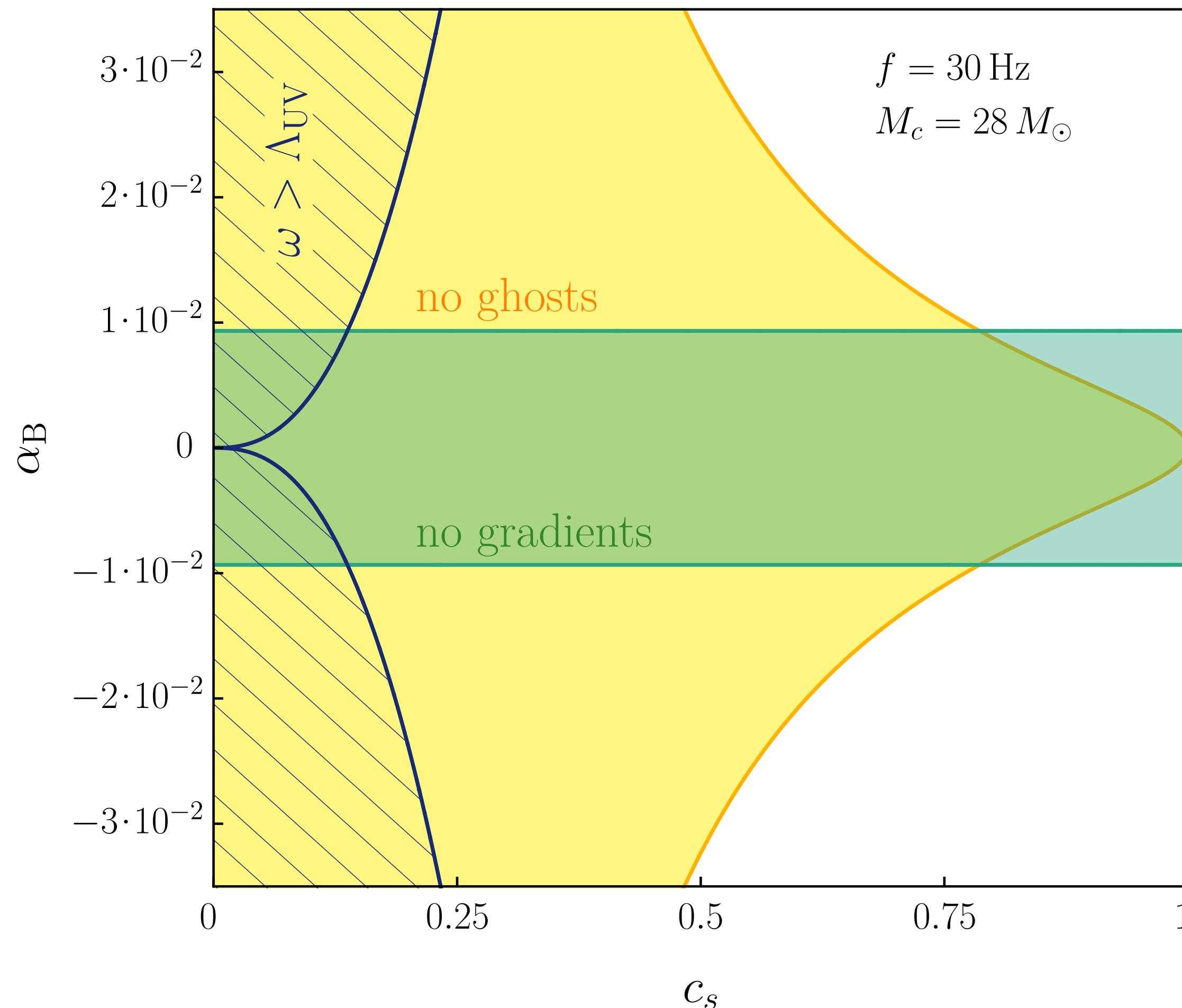
- Surviving **scalar-tensor** theories as DE models

$$S_{c_T^2=1}^{\text{EFT}} = \int d^4x \sqrt{-g} \left[\frac{M_*^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} + \frac{m_2^4(t)}{2} (\delta g^{00})^2 \right]$$

DE Instabilities by GWs

Creminelli, Tambalo, Vernizzi and VY 20

- Bound on α_B : cubic Horndeski



- Absence of ghost/gradient instabilities

$$|\alpha_B| \lesssim 10^{-2}$$

- Surviving scalar-tensor theories as DE models

$$S_{c_T^2=1}^{\text{EFT}} = \int d^4x \sqrt{-g} \left[\frac{M_*^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} + \frac{m_2^4(t)}{2} (\delta g^{00})^2 \right]$$

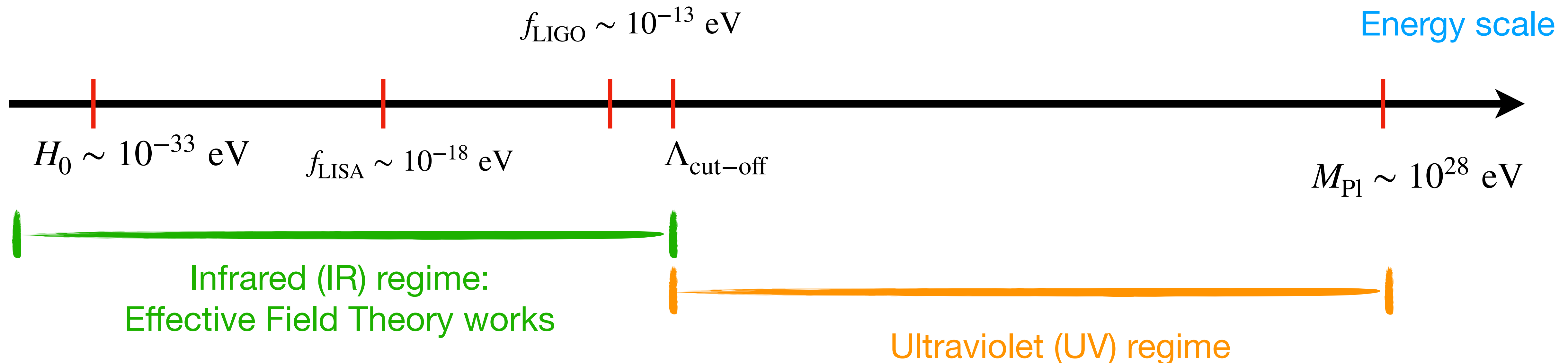


$$\mathcal{L} = G_2(\phi, X) + C(\phi, X) R + \frac{6C_{,X}(\phi, X)^2}{C(\phi, X)} \phi^{;\mu} \phi_{;\mu\nu} \phi_{;\lambda} \phi^{;\nu\lambda}$$

K-essence + non-minimally coupled with gravity

Modified Gravity as an EFT

- On large scale gravity is modified



- On small scales (solar system) gravity behaves the same as in GR

The presence of the fifth force (on small scales) can be typically suppressed by screening mechanism (but not necessary)

$$r_V \sim 100 \left(\frac{M}{M_{\odot}} \right)^{1/3} \text{ pc}$$

Backup

Conformal transf. and $f(R)$

- Under $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ $\sqrt{-\tilde{g}} = \Omega^4 \sqrt{-g}$ $\tilde{R} = \Omega^{-2} \left[R - 6g^{\mu\nu} \frac{\nabla_\mu \nabla_\nu \Omega}{\Omega} \right]$ Wald's GR book, app.D

- $f(R)$ gravity $S = \int d^4x \sqrt{-g} f(R) + S_m[g_{\mu\nu}; \psi_m]$

Introduce the **auxiliary** field $\Phi \Rightarrow S = \int d^4x \sqrt{-g} \left[f(\Phi) + \frac{df}{d\Phi} (R - \Phi) \right] + S_m[g_{\mu\nu}; \psi_m]$

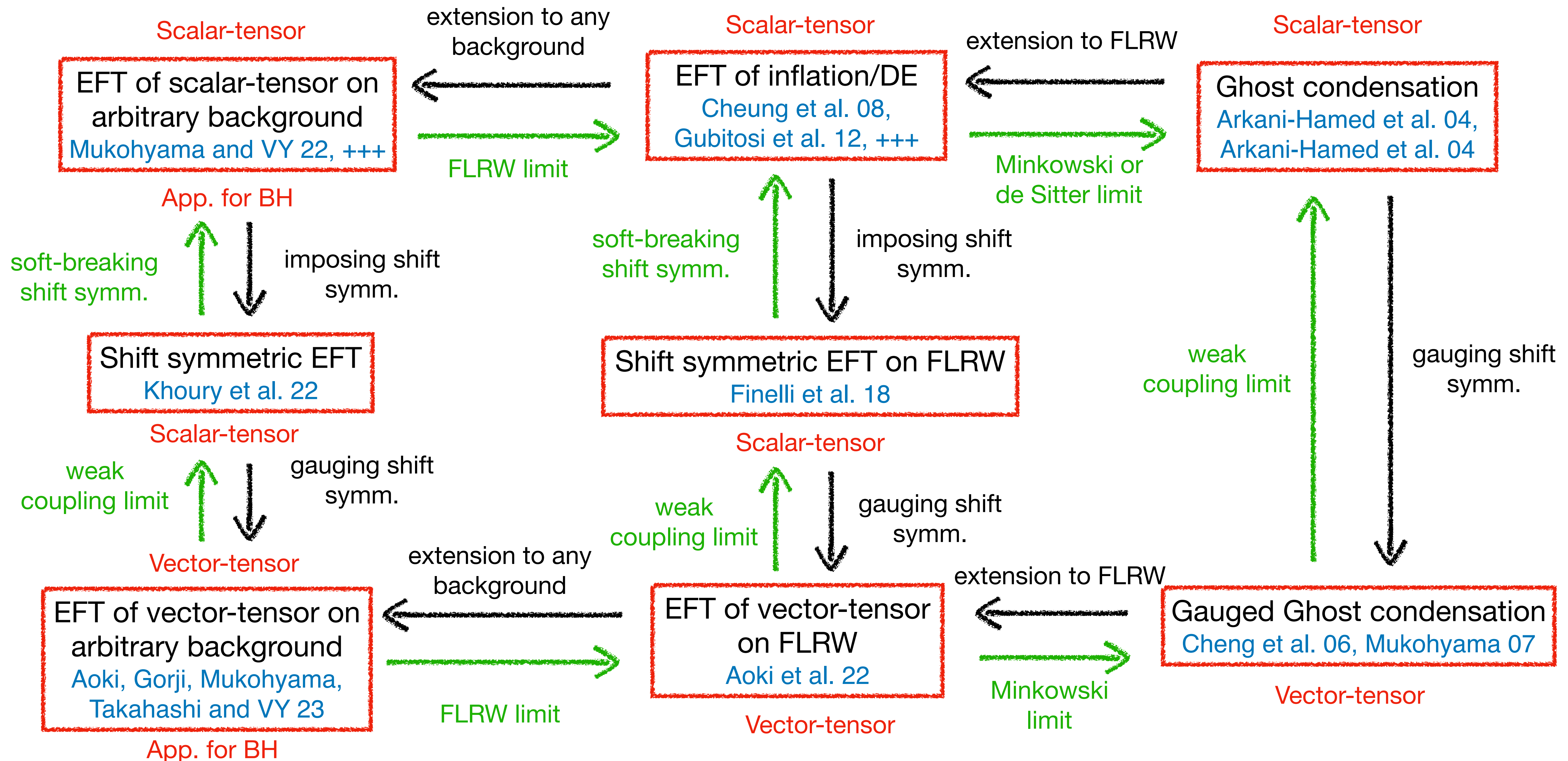
Φ is non-dynamical $\Rightarrow$ EoM of Φ gives $\Phi = R$ with $f'' \neq 0$

This can be recast into the form $S = \int d^4x \sqrt{-g} \left[F(\Phi) R - V(\Phi) \right] + S_m[g_{\mu\nu}; \psi_m]$

$$V(\Phi) = -f(\Phi) + \Phi f'(\Phi)$$

Use conformal transf. To go to Einstein frame

The Web of EFTs



Example of using the constraint $c_{\text{GW}}^2 = 1$

- Expand the Horndeski action on $\Phi(t)$: $S[h_{ij}] = \frac{M^2}{8} \int d^4x a^3(t) \left[\dot{h}_{ij}^2 - \frac{c_{\text{GW}}^2}{a^2(t)} (\partial_k h_{ij})^2 \right]$

$$1 - c_{\text{GW}}^2 = \frac{2m_4^2}{M^2}$$

$$m_4^2 = - [2XG_{4X} + XG_{5\Phi} + (H\dot{\Phi} - \ddot{\Phi})XG_{5X}] + X^2F_4 - 3H\dot{\Phi}X^2F_5$$

$$M^2 \equiv M_{\text{Pl}}^2 + 2m_4^2$$

$$X = (\partial_\mu \Phi)^2$$

- Impose $m_4^2 = 0$

$$\dot{h}_{ij}^2 - \frac{1}{a^2(t)} (\partial_k h_{ij})^2$$

$$L_2 = G_2(\Phi, X) \quad L_3 = G_3(\Phi, X) \square \Phi$$

$$\dot{h}_{ij}^2$$

$$2G_{4X} - XF_4 + G_{5\Phi} = 0$$

$$L_4 = G_4(\Phi, X)R - 2G_{4X}(\Phi, X)[(\square\Phi)^2 - \Phi_{\mu\nu}\Phi^{\mu\nu}] - F_4(X, \Phi)\epsilon^{\mu\nu\rho}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\Phi_{\mu}\Phi_{\mu'}\Phi_{\nu\nu'}\Phi_{\rho\rho'}$$

$$L_5 = G_5(\Phi, X)G_{\mu\nu}\Phi^{\mu\nu} + \frac{1}{3}G_{5X}(\Phi, X)[(\square\Phi)^3 - 3(\square\Phi)\Phi_{\mu\nu}\Phi^{\mu\nu} + 2\Phi_{\mu\nu}\Phi^{\sigma\mu}\Phi^{\nu}_{\sigma}]$$

$$F_5(\Phi, X)\epsilon^{\mu\nu\rho\sigma}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\Phi_{\mu}\Phi_{\mu'}\Phi_{\nu\nu'}\Phi_{\rho\rho'}\Phi_{\sigma\sigma'}$$

Creminelli and Vernizzi 17, Ezquiaga and Zumalacárregui 17, Baker et al. 17, Sakstein and Jain 17