

Large scale structure and computational cosmology

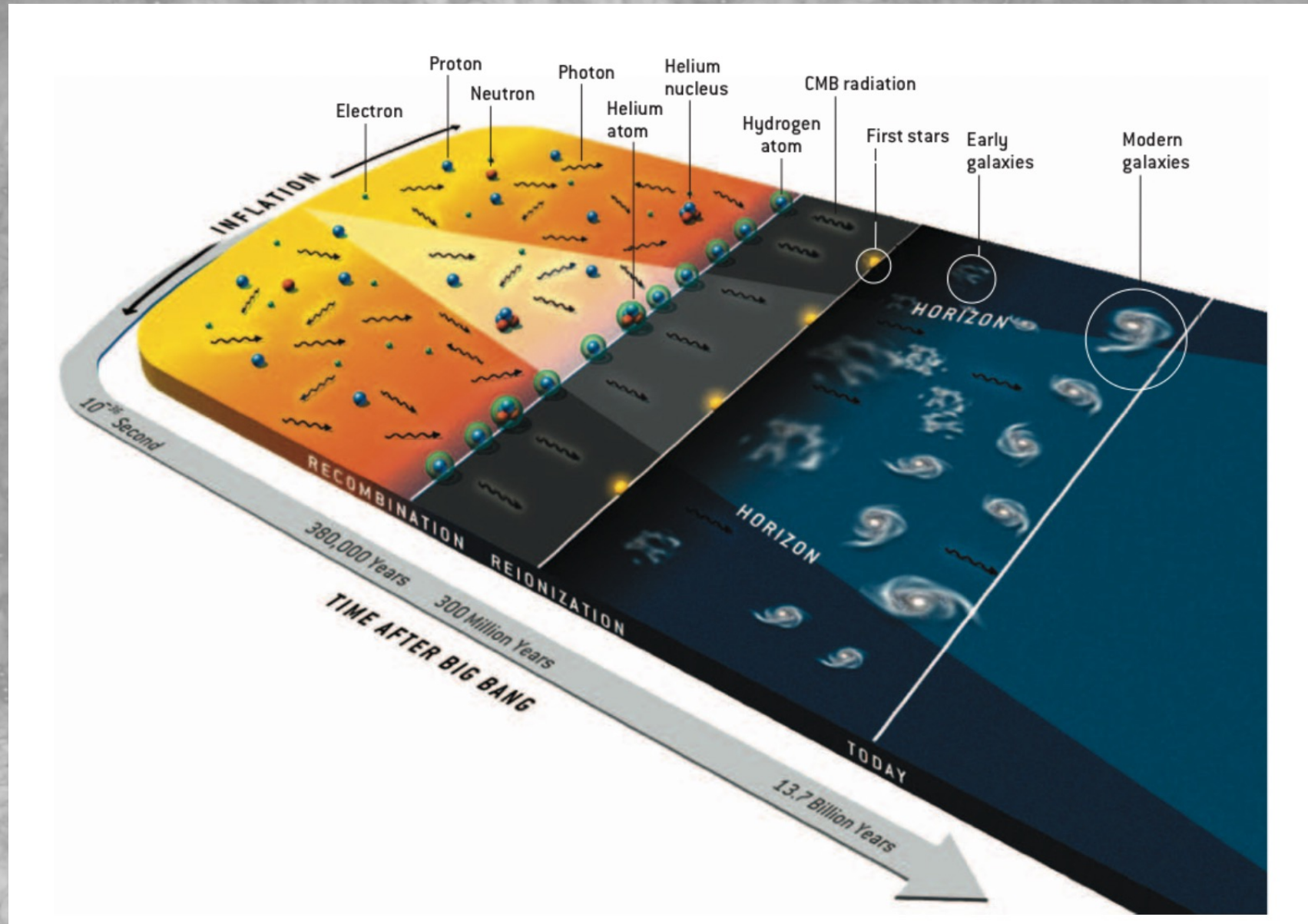
Dr Noam I Libeskind

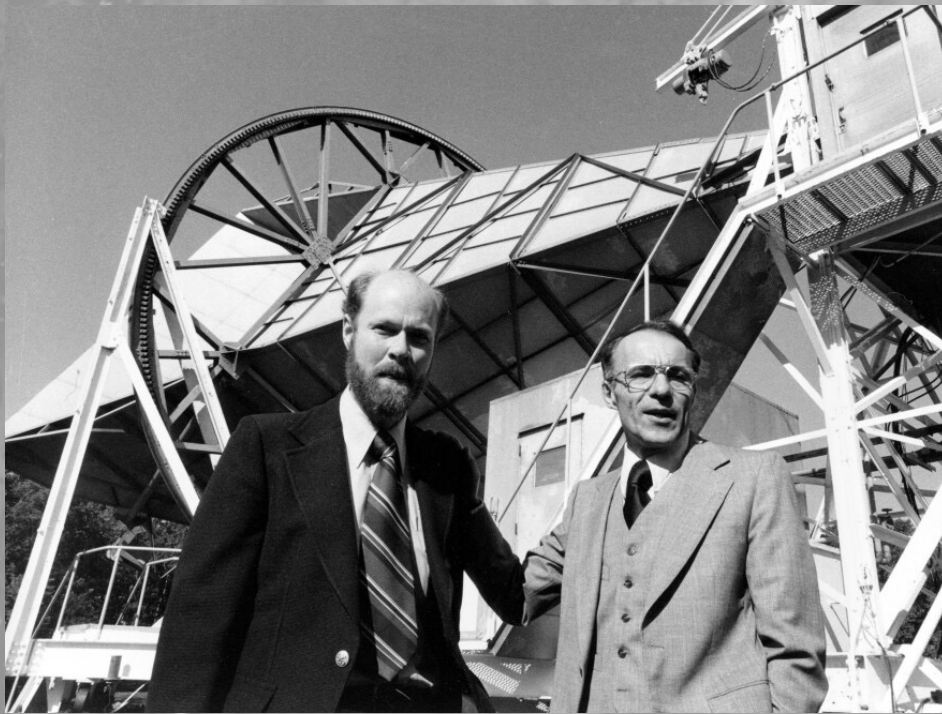
Cosmography and Large-scale structure



Leibniz-Institut für
Astrophysik Potsdam

The (hot) big bang model

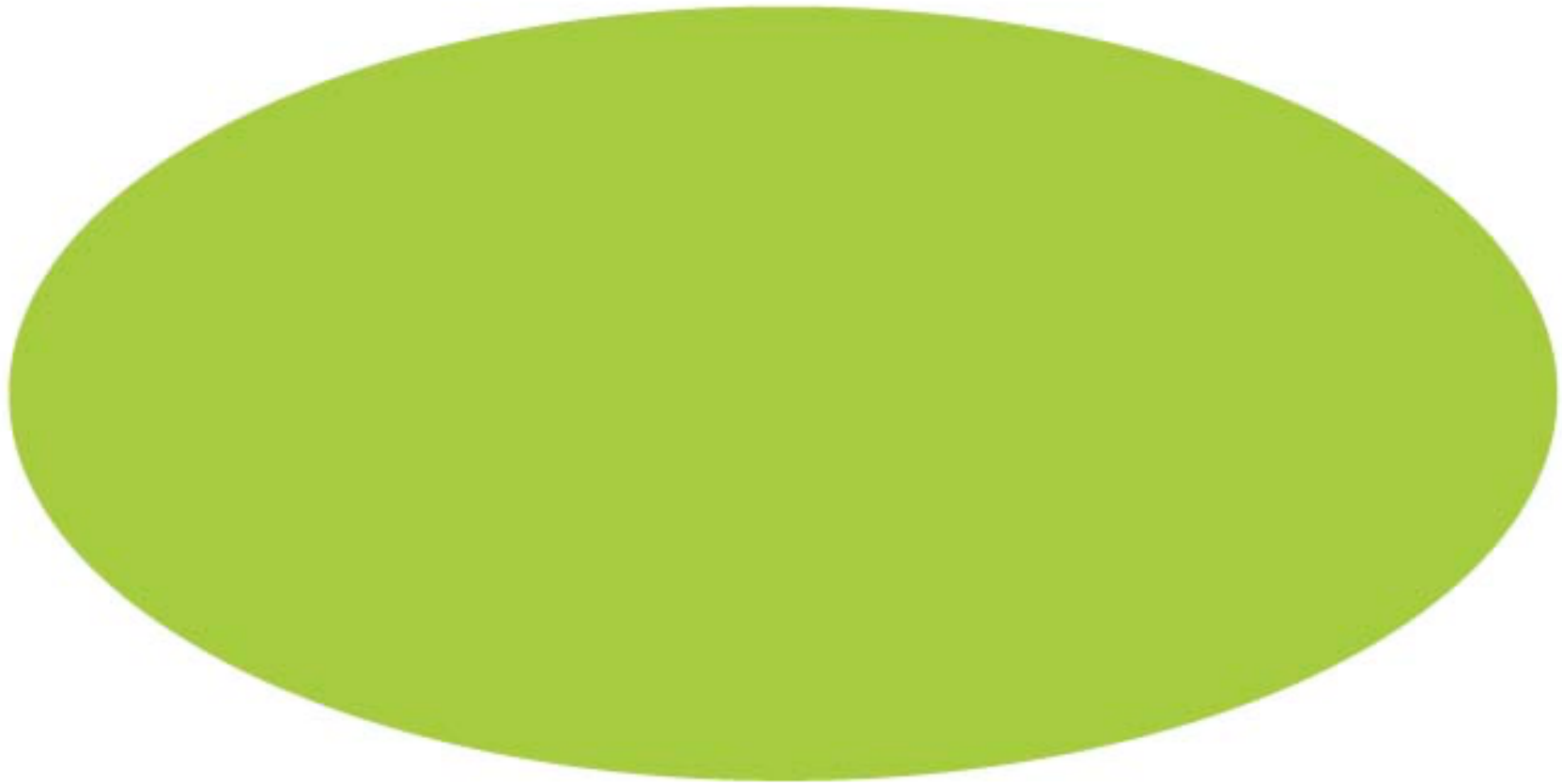




Penzias & Wilson's Horn Antenna

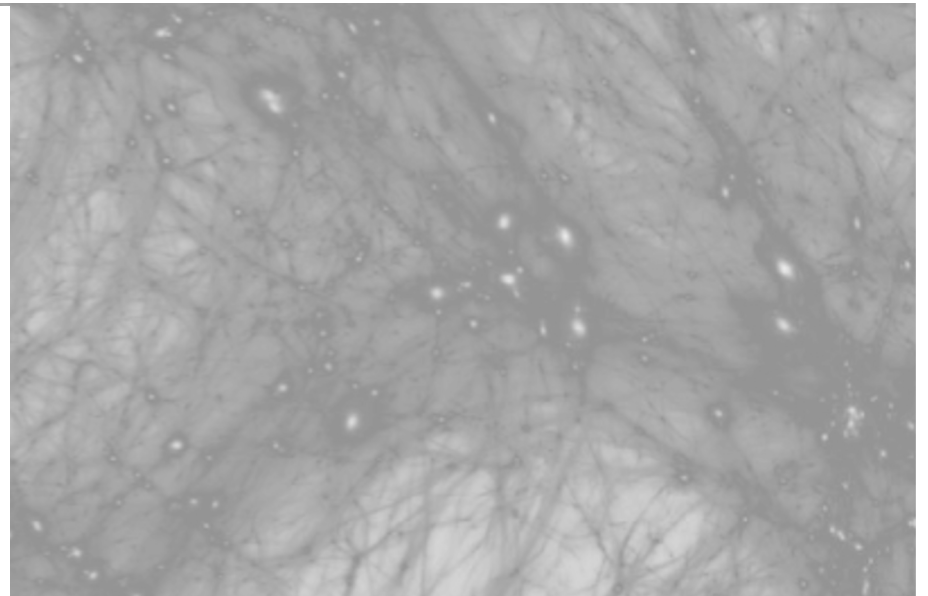


ISOTROPY OF THE COSMIC MICROWAVE BACKGROUND

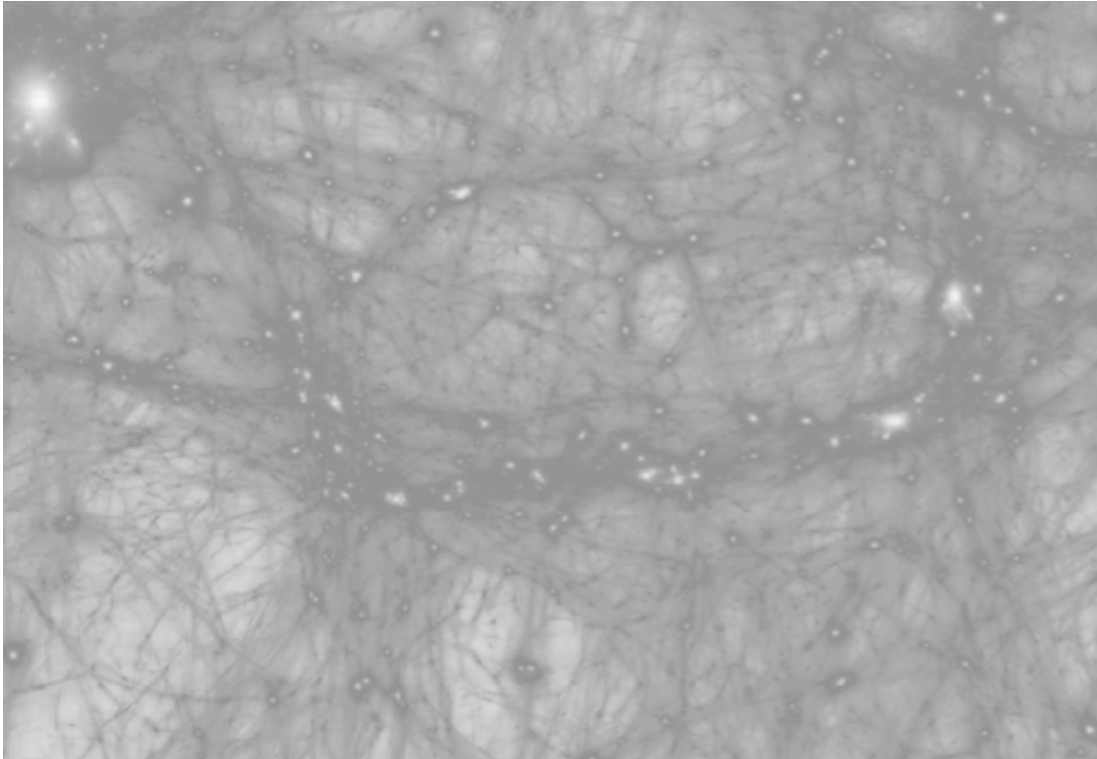


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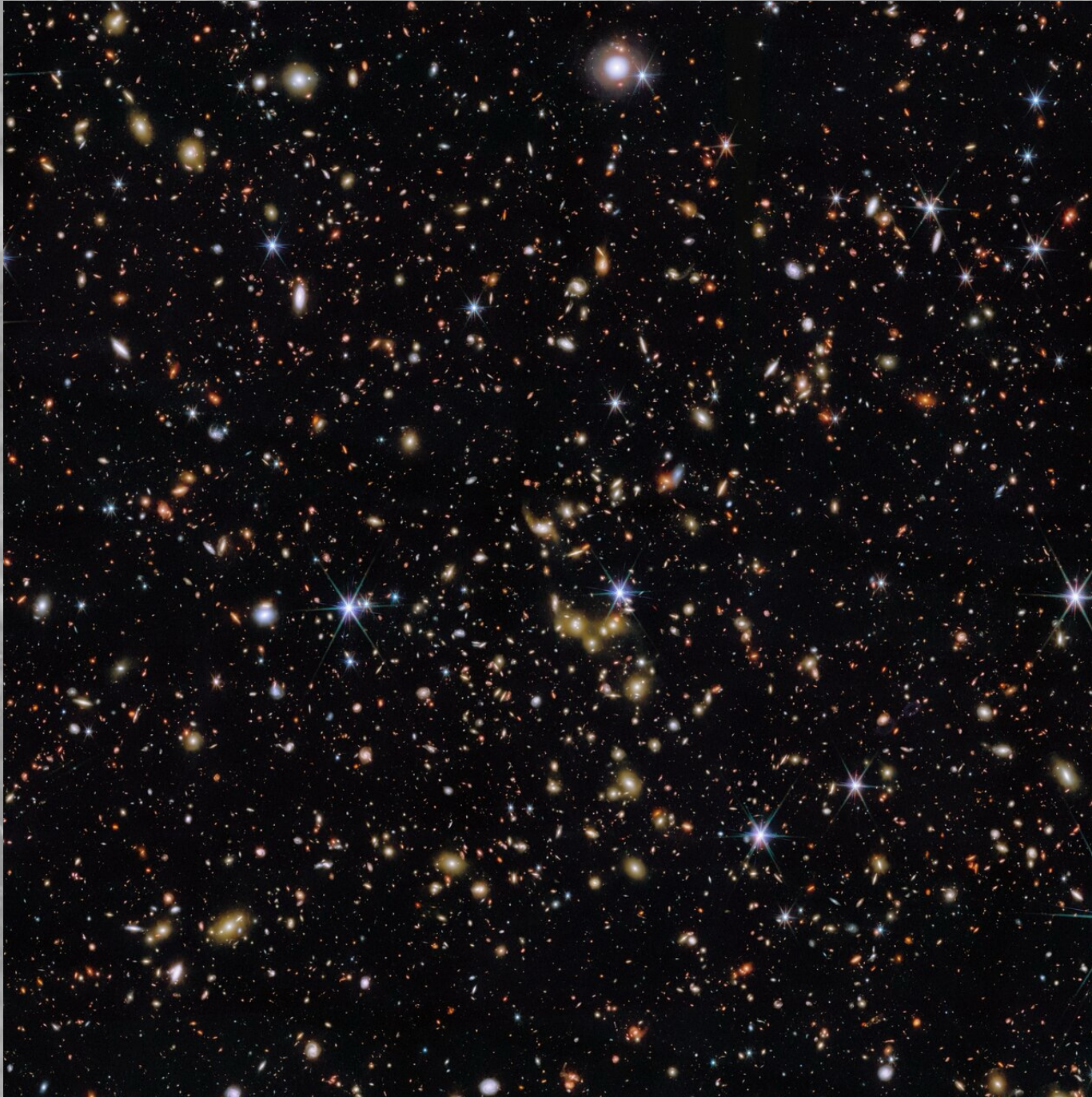
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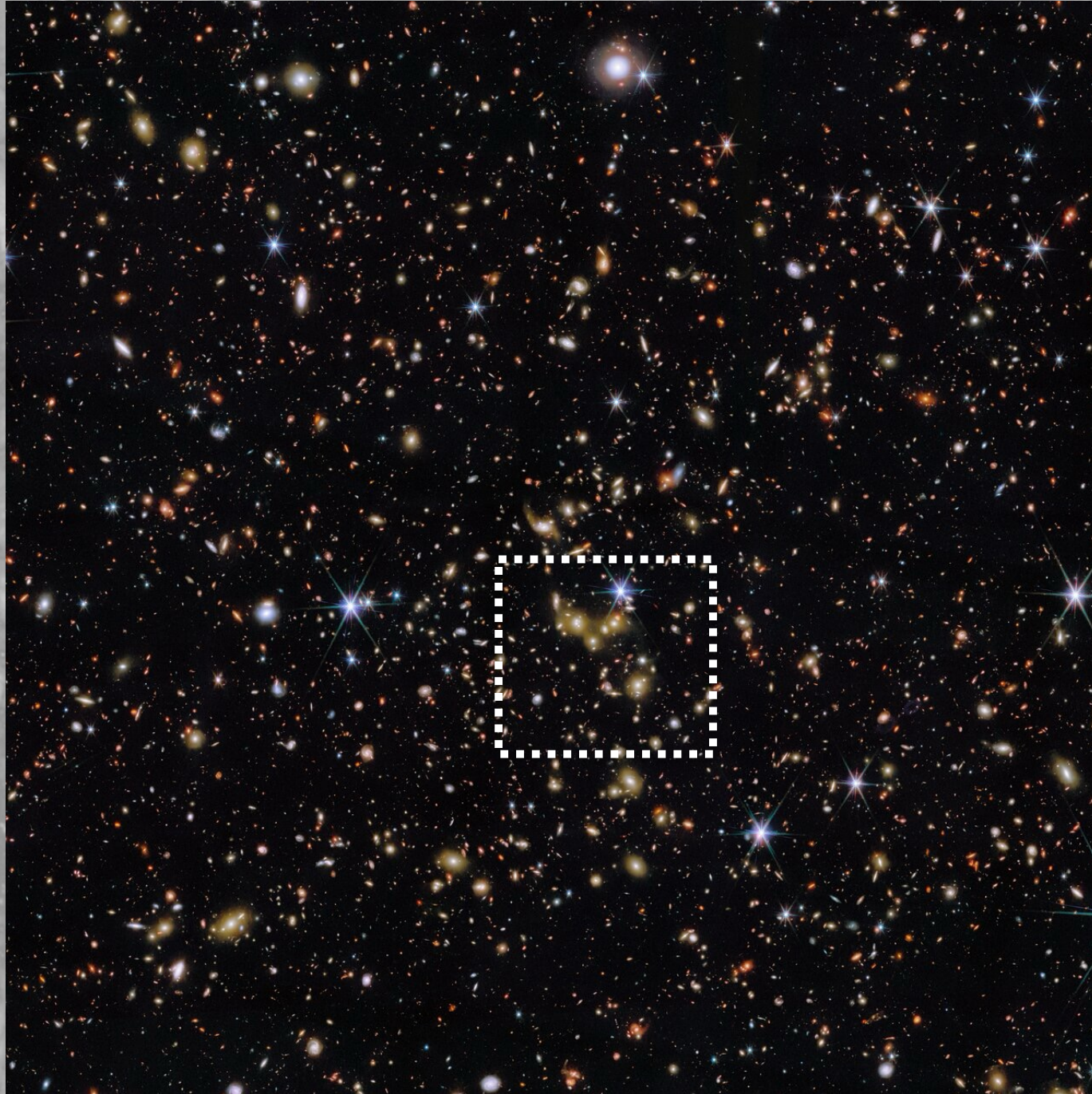
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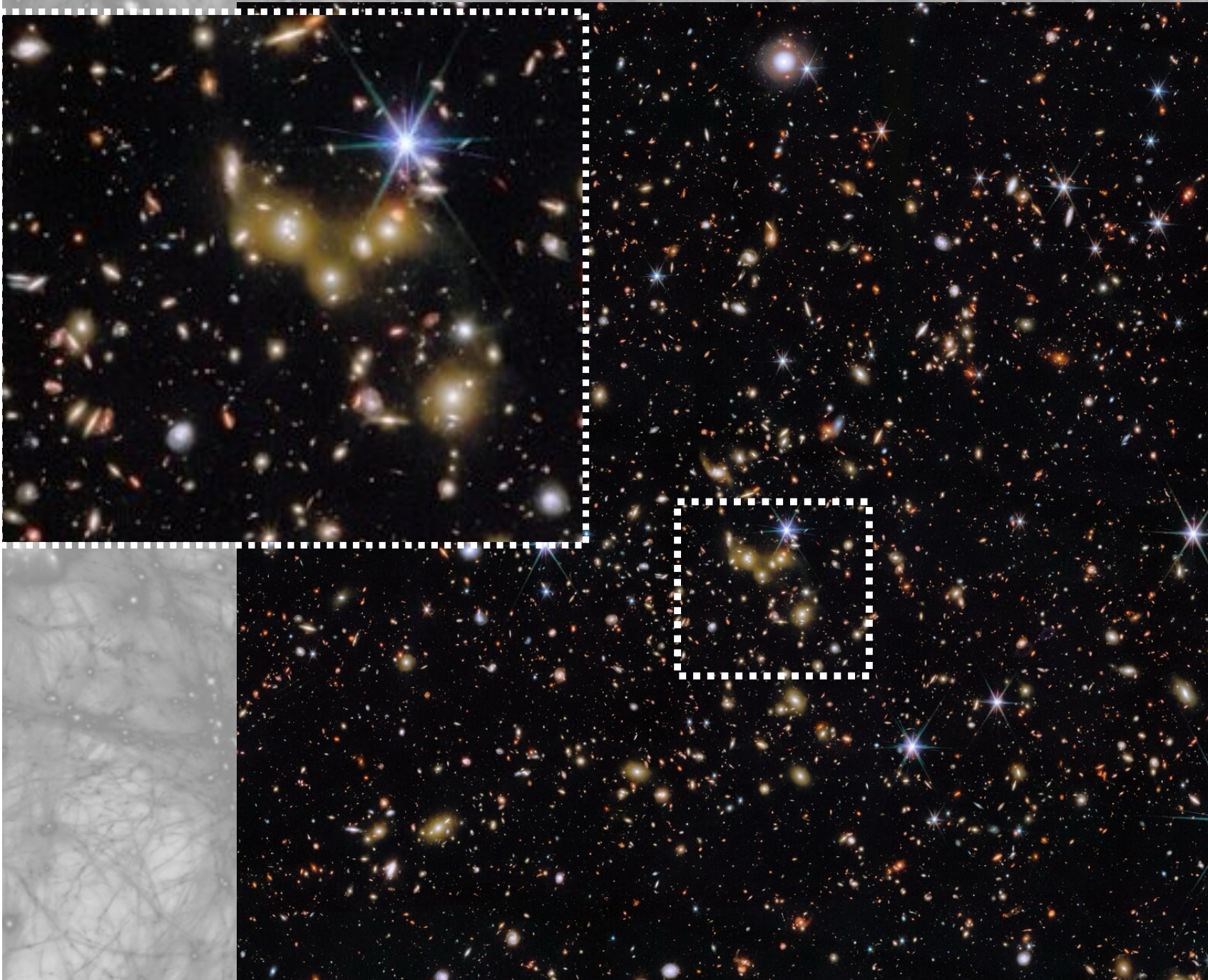
The Universe today is not simple – its quite complex



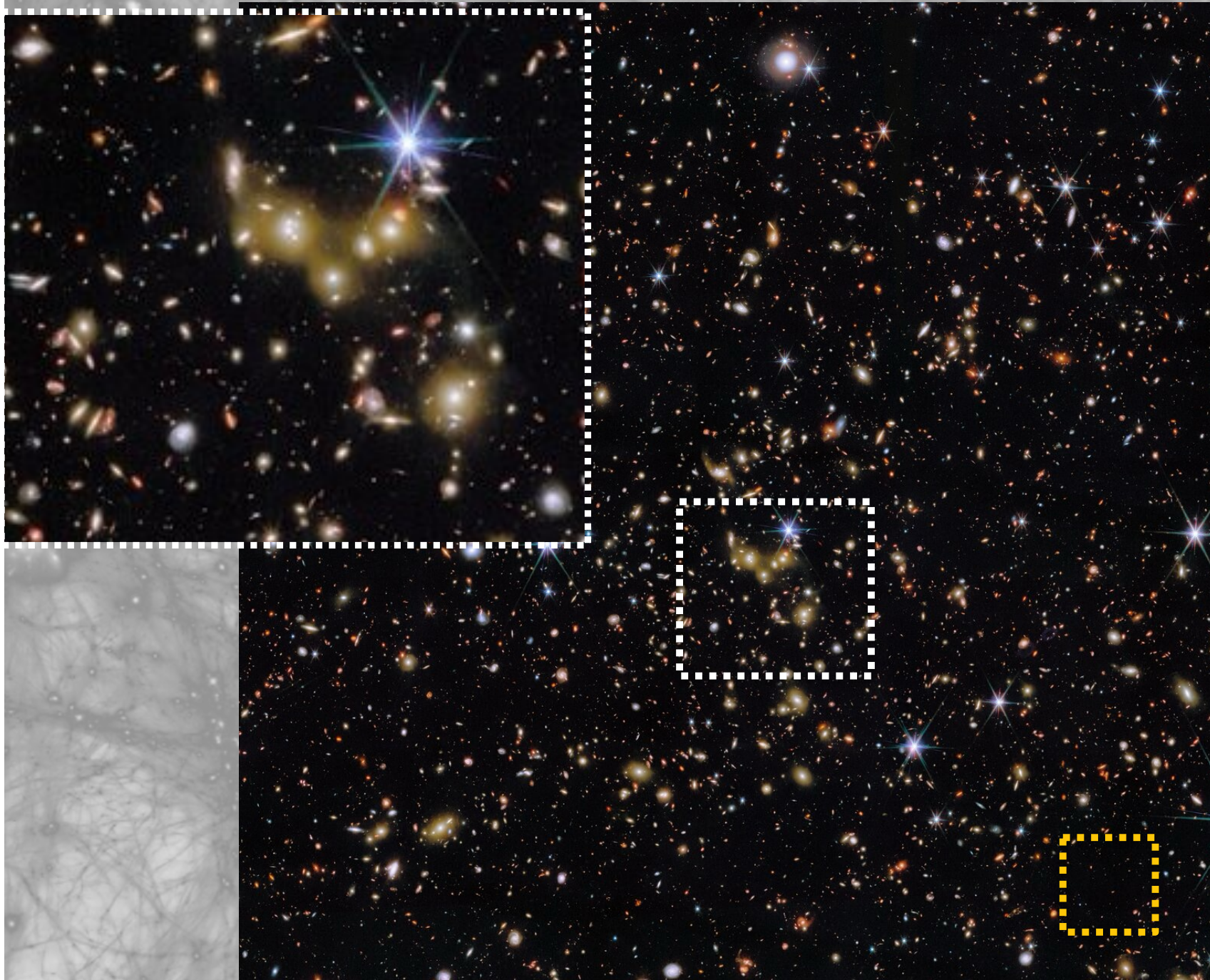
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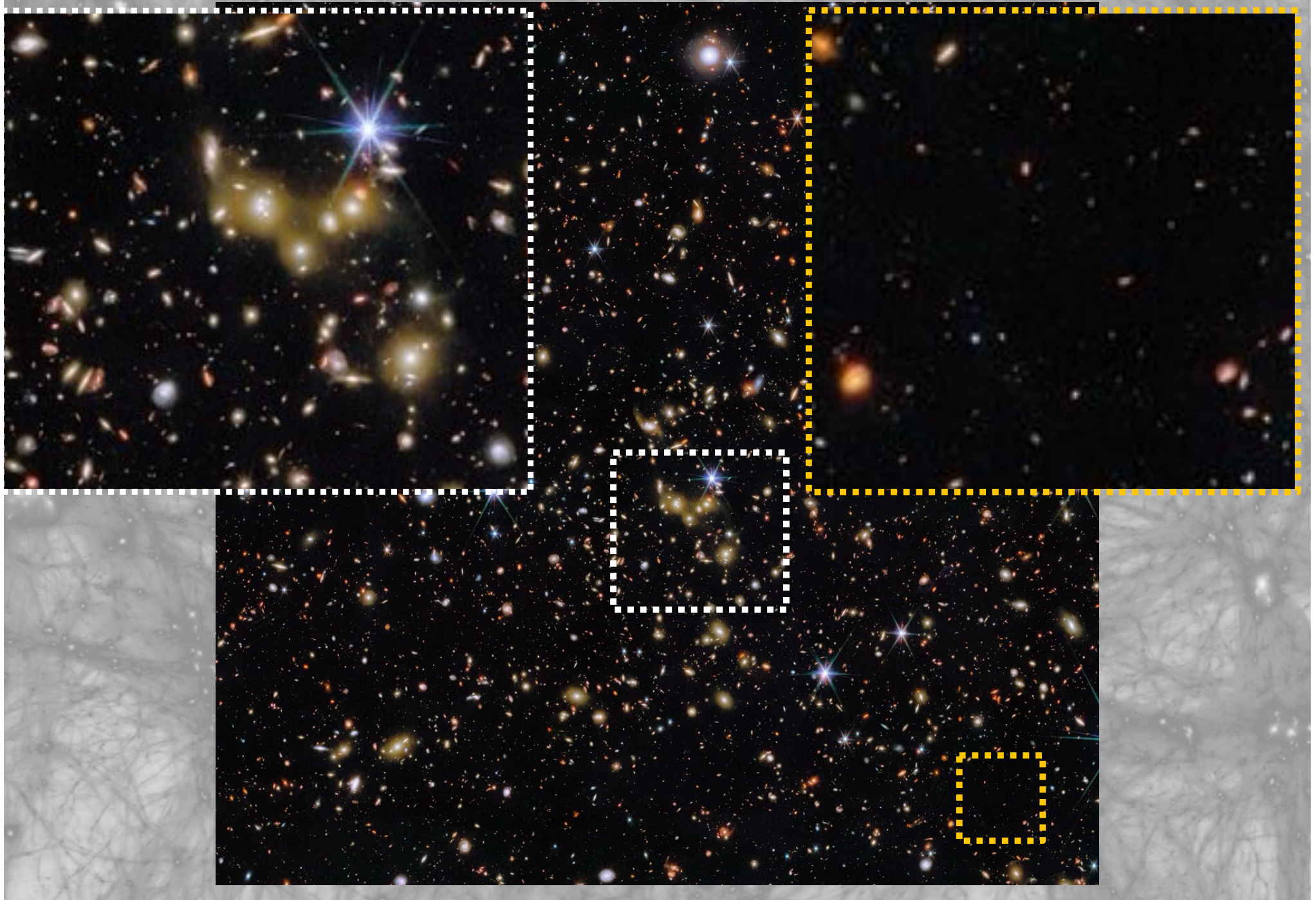
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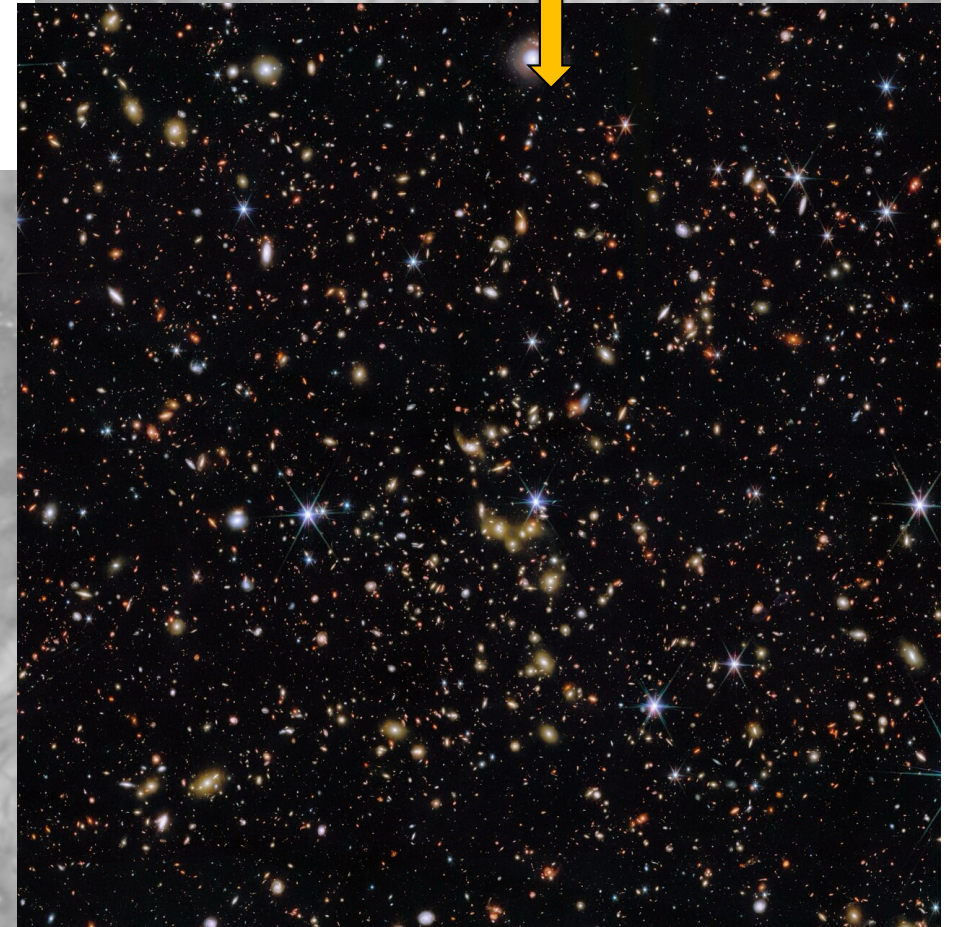
ISOTROPY OF THE COSMIC MICROWAVE BACKGROUND



MAP990004

Where did the richness structure of the
cosmic web come from?

How did small scale
inhomogeneities
arise from a
homogenous start



Mon. Not. R. astr. Soc. (1972) 160, Short Communication.

A HYPOTHESIS, UNIFYING THE STRUCTURE AND THE ENTROPY OF THE UNIVERSE

Ya. B. Zeldovich

(Received 1972 September 4)

SUMMARY

A hypothesis about the averaged initial state and its perturbations is put forward, describing the entropy of the hot Universe (due to damping of short waves) and its structure (clusters of galaxies due to long wave perturbations).

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A hypothesis is put forward, assuming that initially, near the cosmological singularity, the Universe was filled with cold baryons. The averaged evolution was described by the uniform isotropic expansion, according to Friedmann solution and the equation of state of cold baryons.

Superimposed on this averaged picture are initial fluctuations of baryon density and corresponding fluctuations of the metric.

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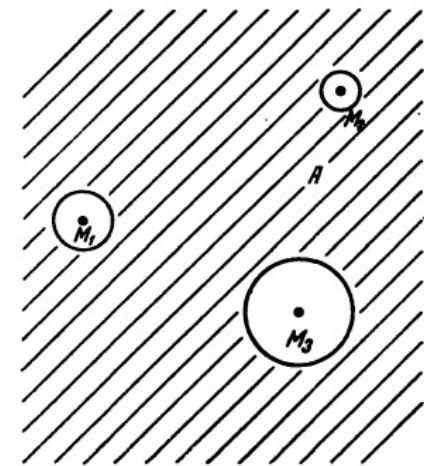


Fig. 1. Vacuole model with delayed cores M_1 , M_2 , M_3 . A) expanding matter of metagalaxy.

DETECTION OF ANISOTROPY IN THE
COSMIC BLACKBODY RADIATION

G. F. Smoot, M. V. Gorenstein, and R. A. Muller

July 6, 1977

Prepared for the U. S. Energy Research and
Development Administration under Contract W-7405-ENG-48

For Reference

Not to be taken from this room



LBL-6468

C.1

~~check figure for
systematic deviation~~

Draft of 6/13/77

MASTER

ApJ?

DETECTION OF ANISOTROPY IN THE COSMIC BLACKBODY RADIATION*

G.F. Smoot, M.V. Gorenstein and R.A. Muller

University of California
Lawrence Berkeley Laboratory and Space Sciences Laboratory
Berkeley, California 94720

$$T(\hat{r}) = T_0 + T_1 \cos(\hat{r}, \hat{r}_{\max})$$

ABSTRACT

We have detected anisotropy in the cosmic blackbody radiation with a 33 GHz (0.9 cm) twin-antenna Dicke radiometer flown aboard a U-2 aircraft to an altitude of 20 km. In data spanning approximately two-thirds of the northern hemisphere, we observe an anisotropy which is well-fit by a first-order spherical harmonic with an amplitude of $(3.2 \pm 0.6) \times 10^{-3} \text{K}$, and an axis of symmetry in the direction $(10.8 \pm 0.5 \text{ hr R.A.}, 5^\circ \pm 10^\circ \text{ dec})$. When expected backgrounds are subtracted, the amplitude is $(3.5 \pm 0.6) \times 10^{-3} \text{K}$. This observation is readily interpreted as due to motion of the earth relative to the radiation with a velocity of $390 \pm 60 \text{ km/sec}$.

direction of maximum temp

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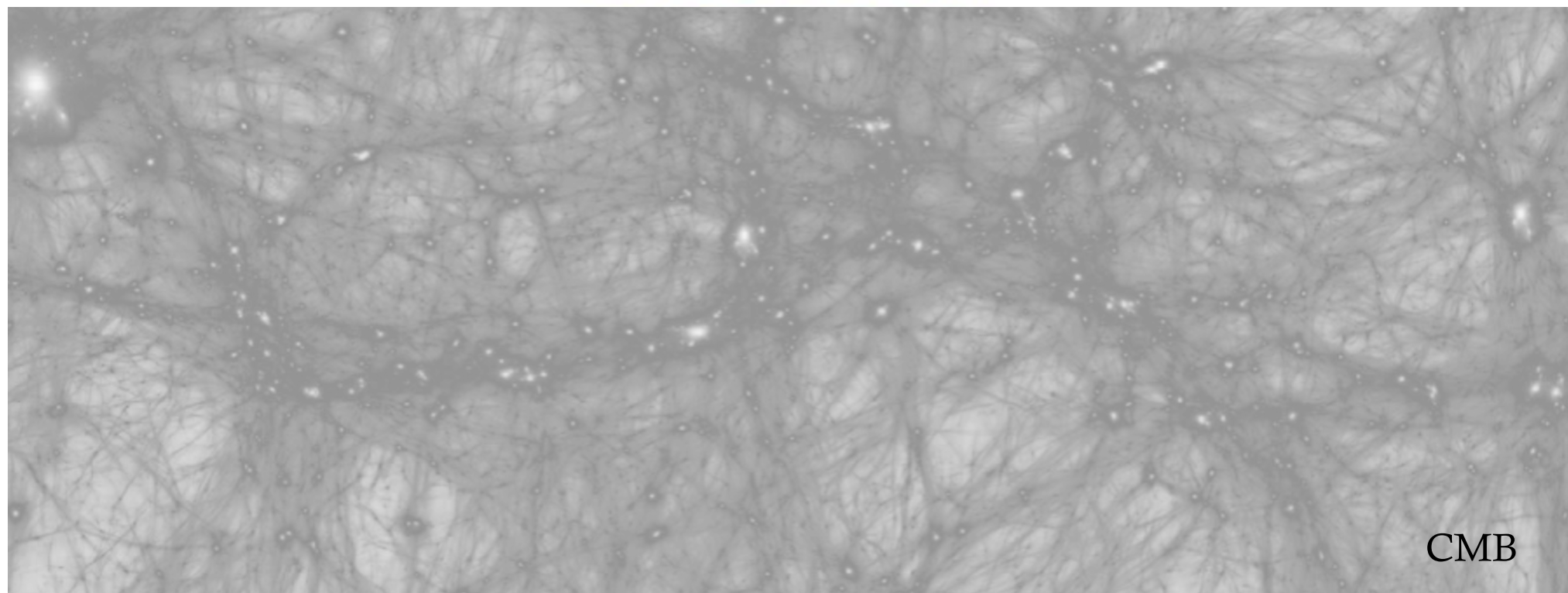
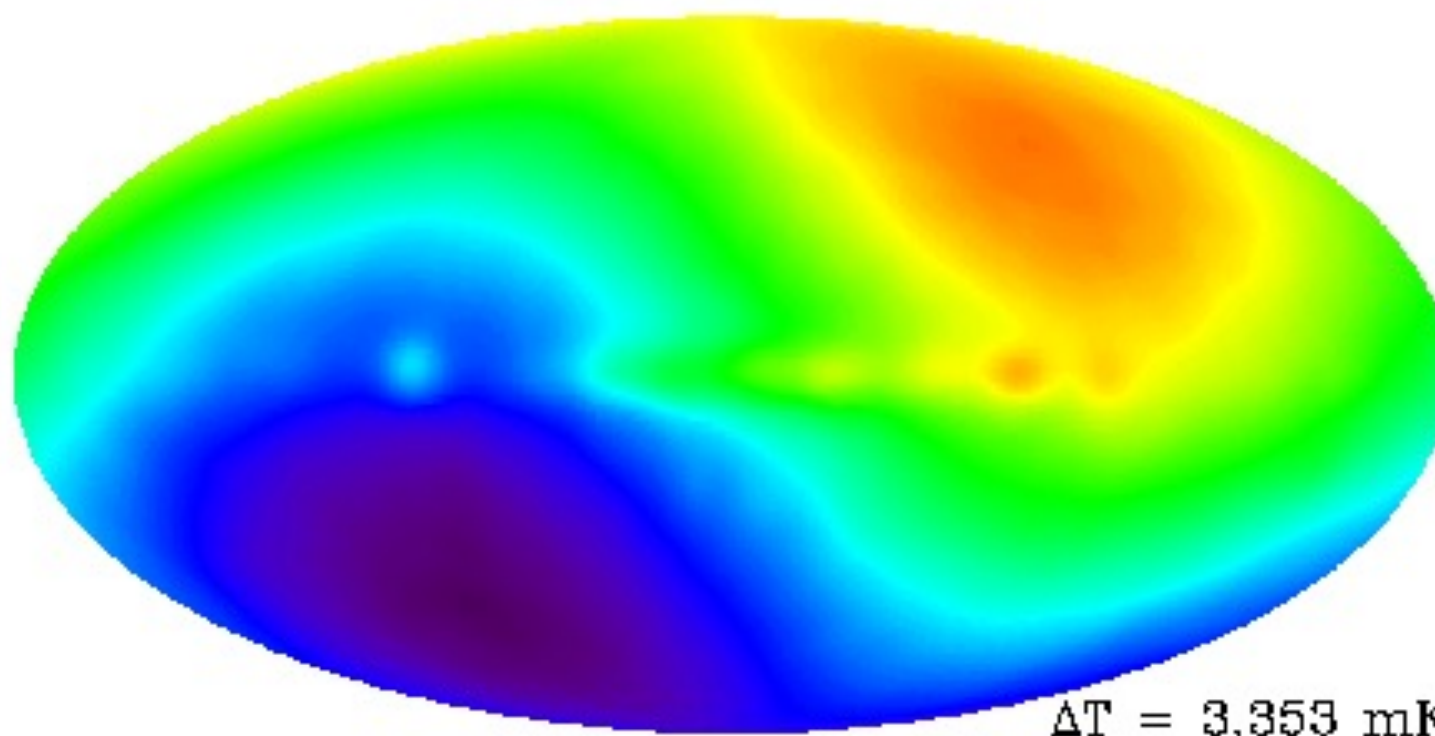
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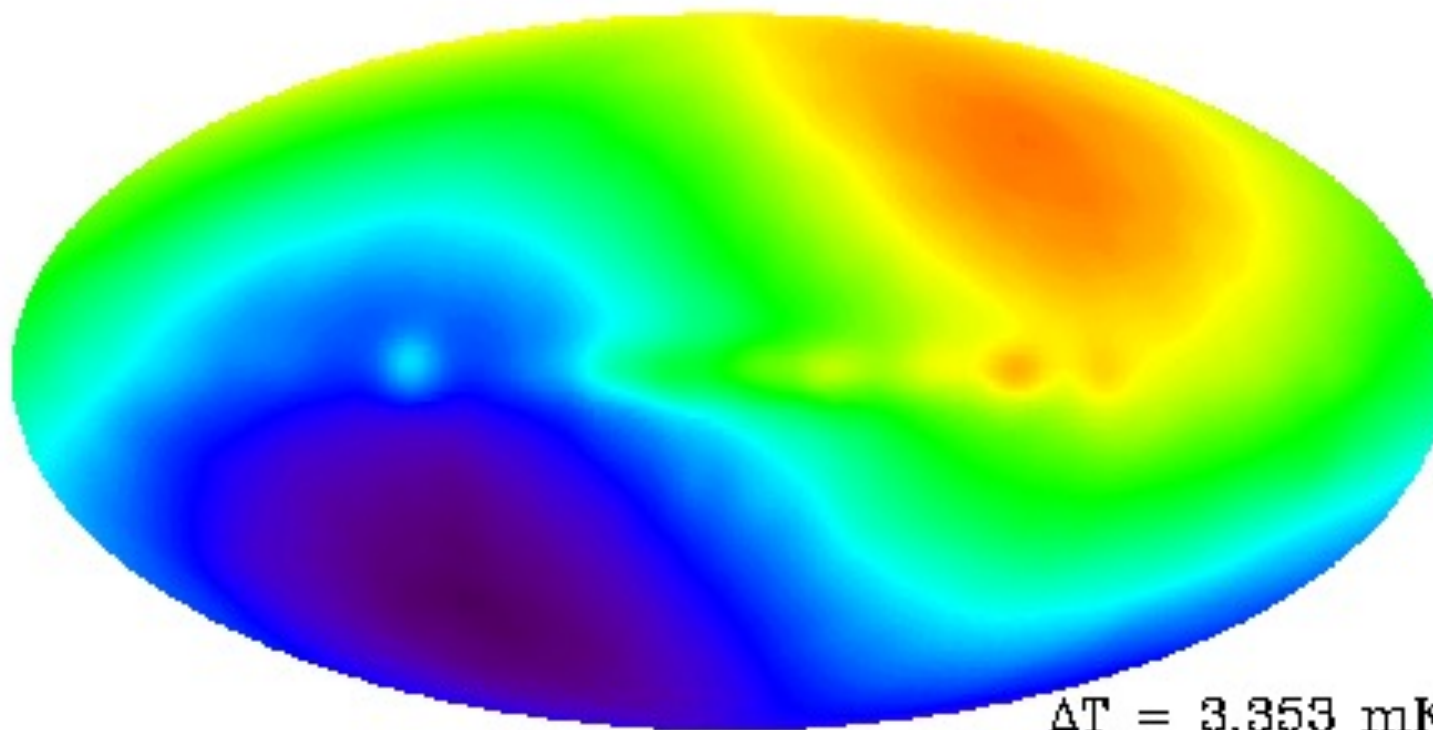
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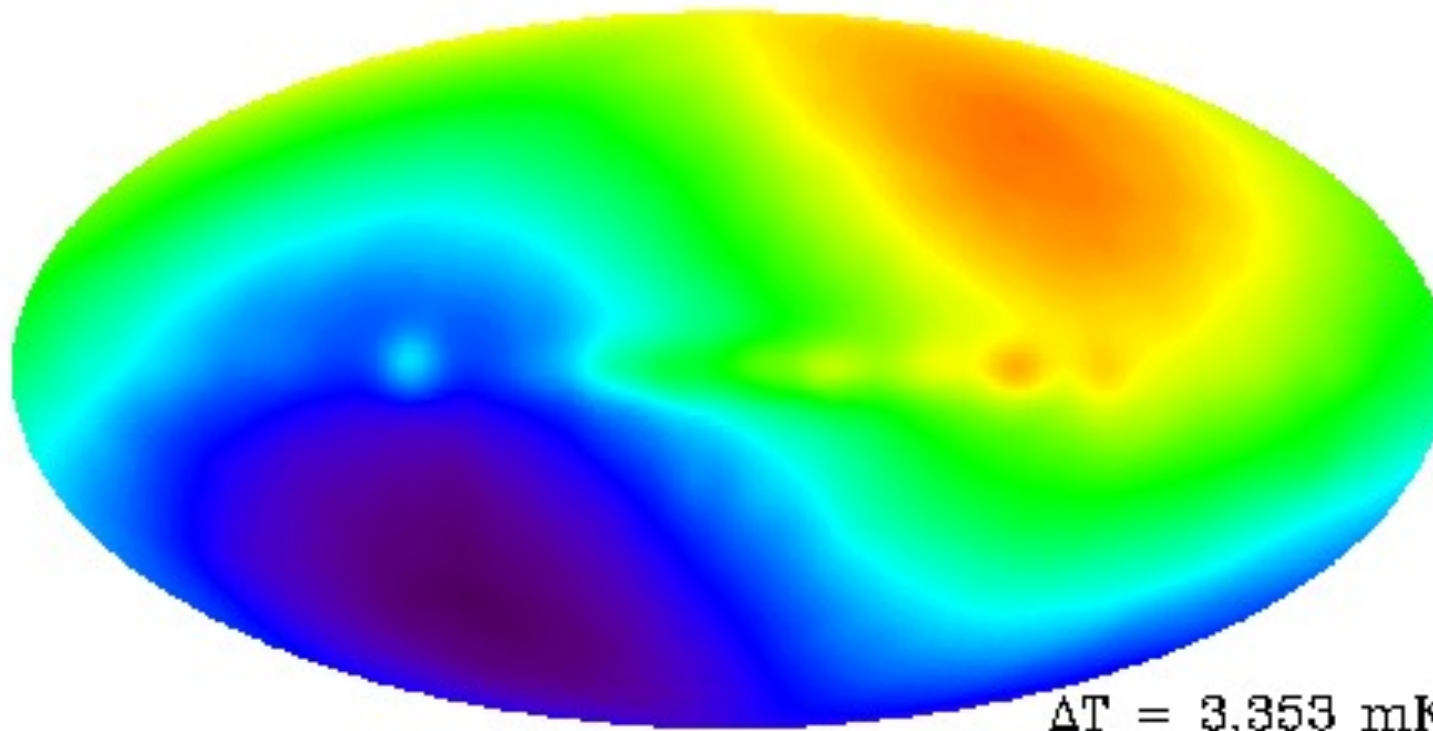


$$\Delta T = 3.353 \text{ mK}$$

the cosmic blackbody

radiation is isotropic to one part in 3000.

CMB



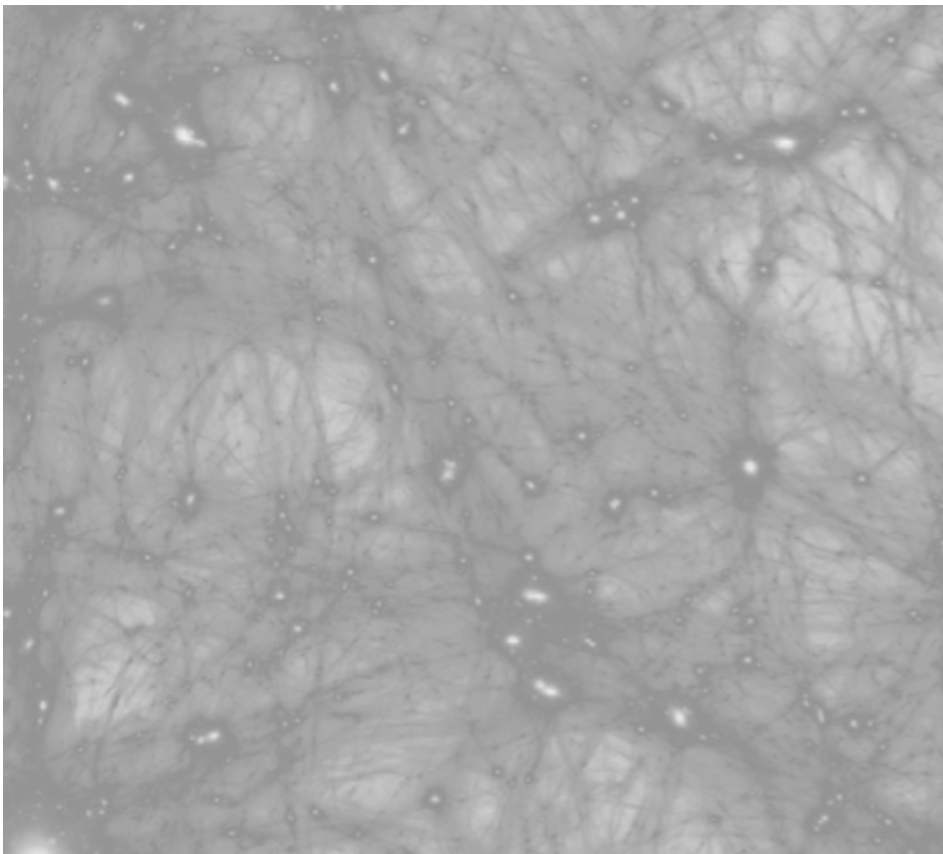
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If we subtract from our measured velocity the component due to the rotation of the Milky-Way galaxy⁹ $\approx 300 \text{ km/sec}$, we calculate the net motion of the Milky-Way with respect to the canonical reference frame of cosmology to be $\approx 600 \text{ km/sec}$ in the direction (R.A. = 10.4 hr, dec. = -18°). These various velocities are summarized in Table I. The large peculiar velocity of the Milky Way galaxy is unexpected, and presents a challenge to cosmological theory.

CMB



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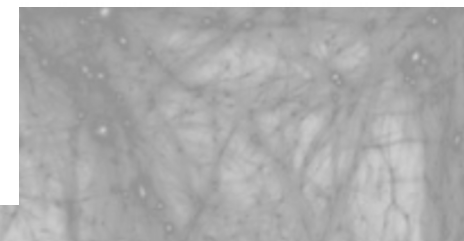
Nature Vol. 270 3 November 1977

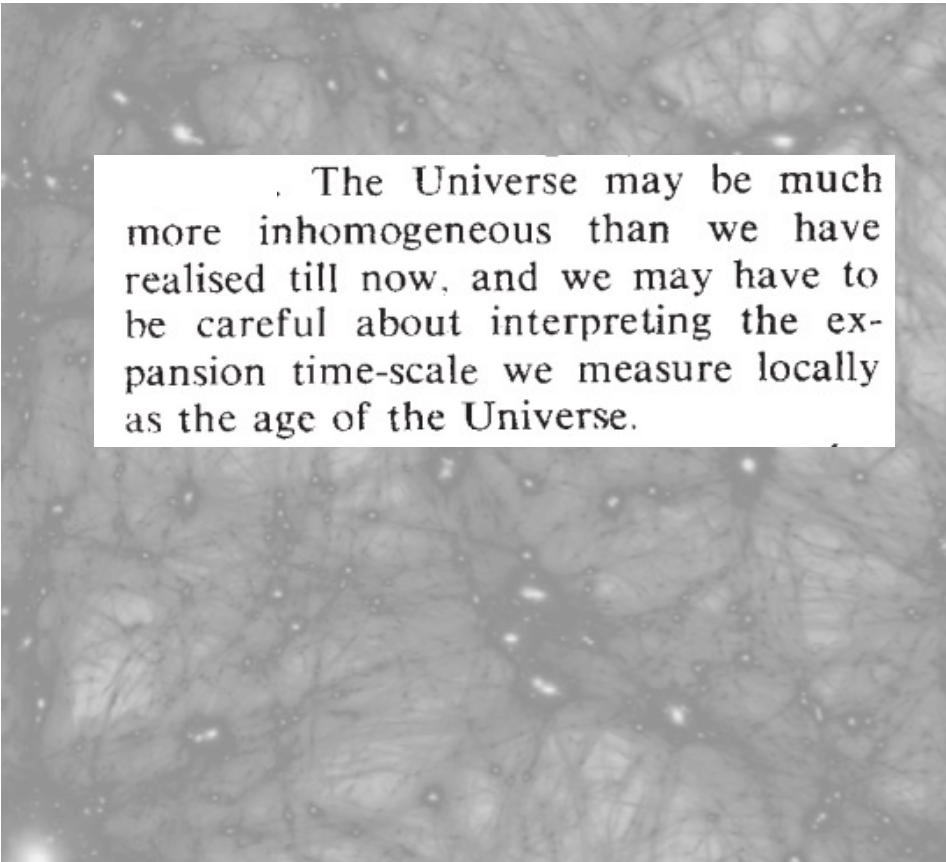
news and views

Aether drift detected at last

from Michael Rowan-Robinson

LBL-6468
C.1





The Universe may be much more inhomogeneous than we have realised till now, and we may have to be careful about interpreting the expansion time-scale we measure locally as the age of the Universe.

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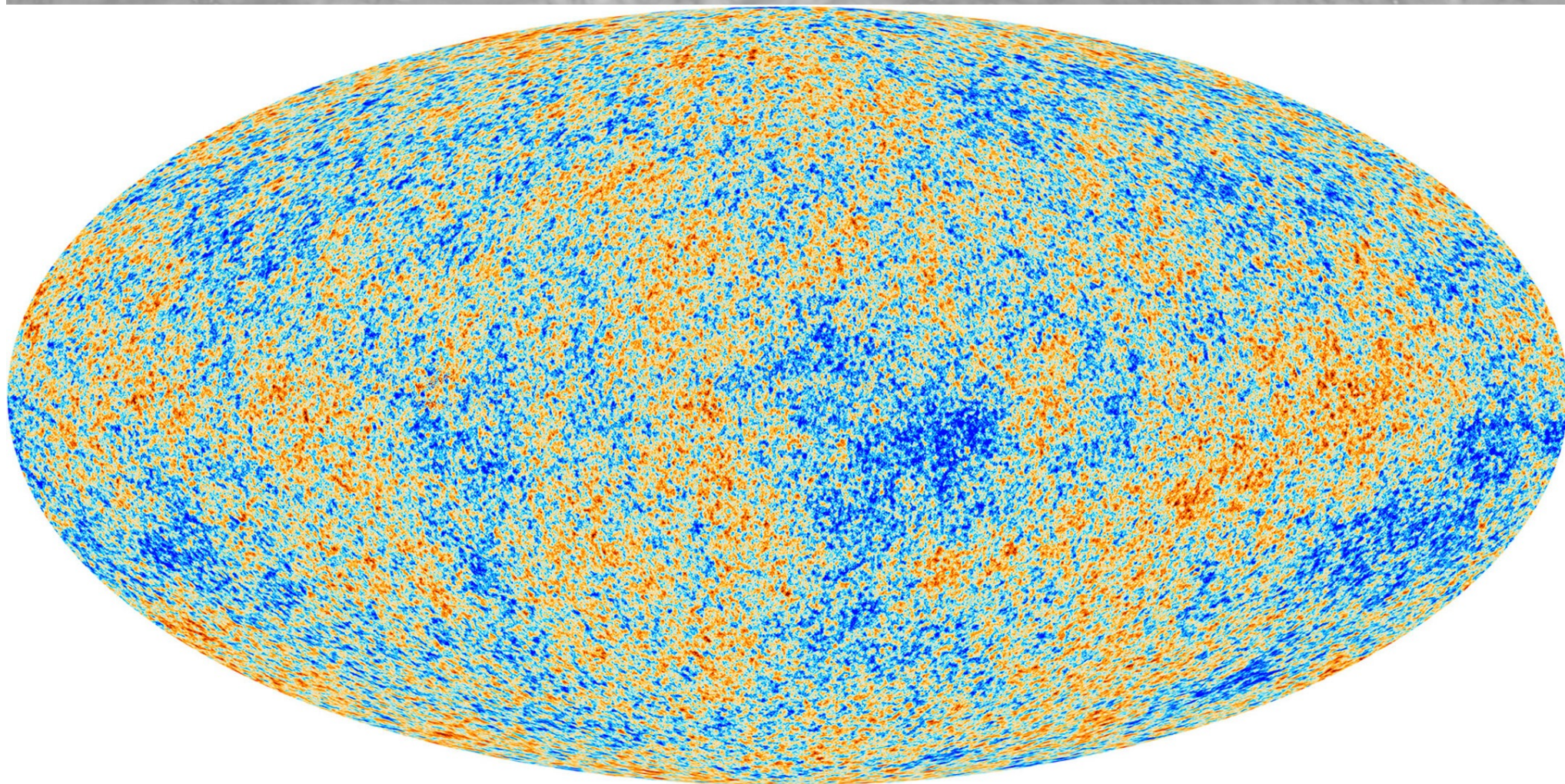
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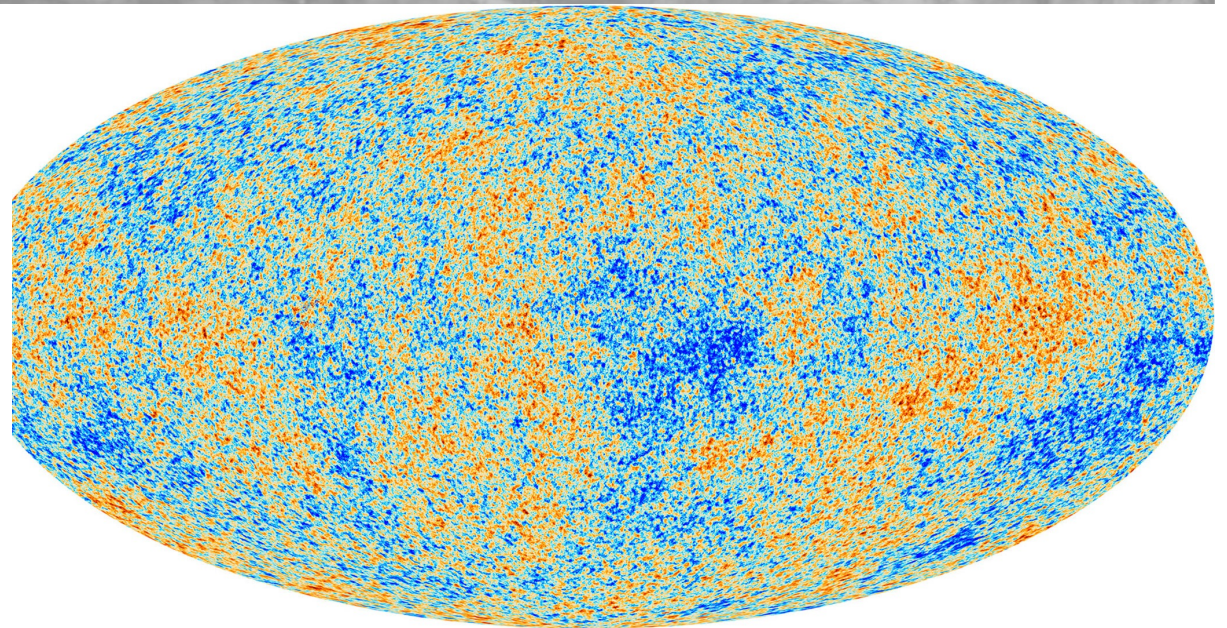
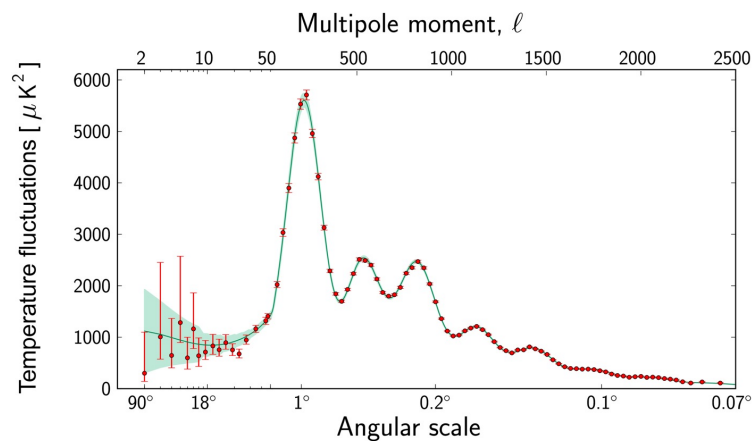
LBL-6468
C.1



Planck 2019

The Cosmic Microwave Background

- Very simple: Universe was born 13.7 bn years ago, possibly in a singularity
- It was hot
- Matter and radiation were smoothly distributed
- It was very smooth but not perfectly smooth – some inhomogeneities: the initial conditions of structure formation



How did a simple initial state evolve into something so complicated

A problem of classical physics

- governing equations (non-relativistic fluid with pressure)

- Poisson's equation

$$\Delta\Psi = 4\pi G\rho$$

- continuity equation

$$\frac{\partial\rho}{\partial t} + \nabla \cdot (\rho\vec{v}) = 0$$

- conservation of momentum

$$\frac{\partial\vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\nabla\Psi - \frac{\nabla p}{\rho}$$

- equation of state

$$p = c_s^2 \rho$$

(c_s : sound speed)

How did a simple initial state evolve into something so complicated

A problem of classical physics

▪ governing equations (non-relativistic fluid with pressure)

• Poisson's equation

• continuity equation

• conservation

• equation of state

Complex partial
differential equations
that requires a numerical
solution

$$\Psi = \frac{1}{\rho}$$

$$= c_s^2 \rho$$

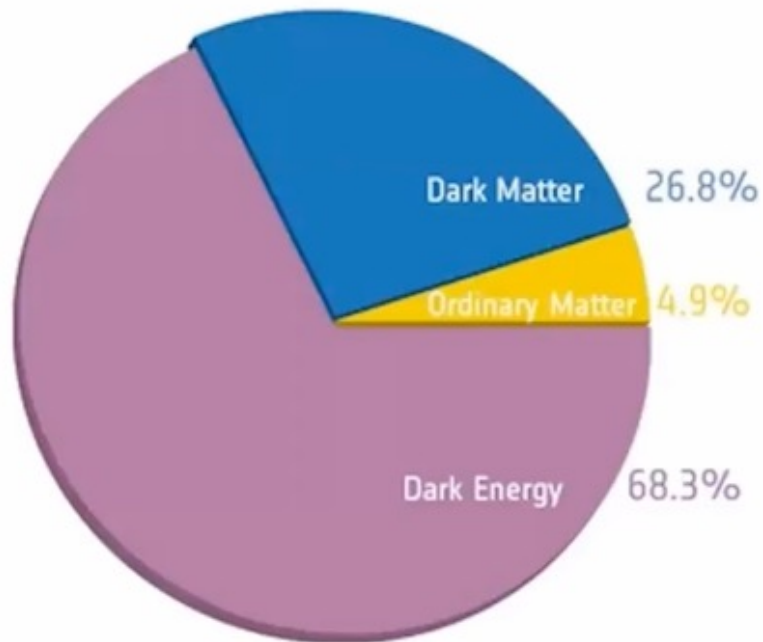
(c_s : sound speed)

Vlasov poisson eq

But how can one even approach the idea of simulating the entire universe?

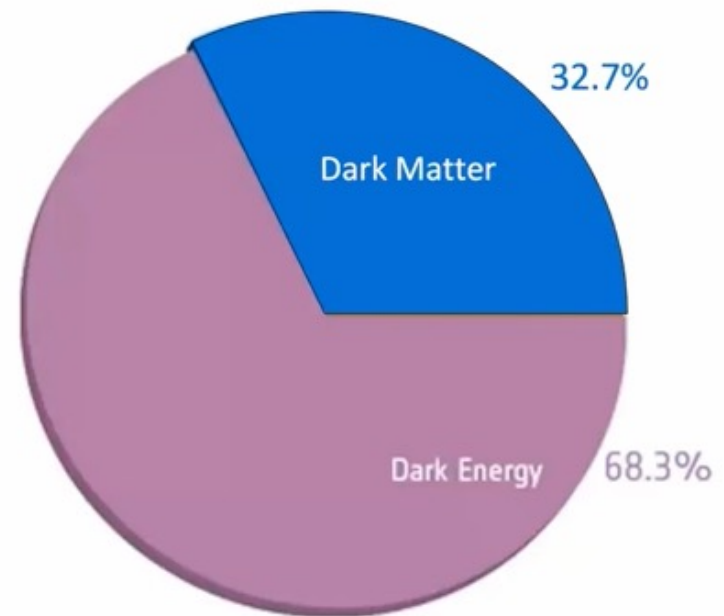
First: make some simplifying assumptions

**Composition of the Universe today
(Planck Λ CDM model)**



Isn't dark matter complicated?
No its just gravitating !

Fiducial Dark Matter-Only Universe



Isn't dark energy complicated?
No it just changes $a(t)$!

The first paper suggesting simulations of gravity



Erik Holmberg

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND
ASTRONOMICAL PHYSICS

VOLUME 94

NOVEMBER 1941

NUMBER 3

ON THE CLUSTERING TENDENCIES AMONG THE NEBULAE

II. A STUDY OF ENCOUNTERS BETWEEN LABORATORY MODELS OF STELLAR SYSTEMS BY A NEW INTEGRATION PROCEDURE

ERIK HOLMBERG

ABSTRACT

In a previous paper¹ the writer discussed the possibility of explaining the observed clustering effects among extragalactic nebulae as a result of captures. The present investigation deals with the important problem of whether the loss of energy resulting from the tidal disturbances at a close encounter between two nebulae is large enough to effect a capture. The tidal deformations of two models of stellar systems, passing each other at a small distance, are studied by reconstructing, piece by piece, the orbits described by the individual mass elements. The difficulty of integrating the total gravitational force acting upon a certain element at a certain point of time is solved by replacing gravitation by light. The mass elements are represented by light-bulbs, the candle power being proportional to mass, and the total light is measured by a photocell (Fig. 1). The nebulae are assumed to have a flattened shape, and each is represented by 37 light-bulbs. It is found that the tidal deformations cause an increase in the attraction between the two objects, the increase reaching its maximum value when the nebulae are separating, i.e., after the passage. The resulting loss of energy (Fig. 6) is comparatively large and may, in favorable cases, effect a capture. The spiral arms developing during the encounter (Figs. 4) represent an interesting by-product of the investigation. The direction of the arms depends on the direction of rotation of the nebulae with respect to the direction of their space motions.

I. THE EXPERIMENTAL ARRANGEMENTS

The present paper is a study of the tidal disturbances appearing in stellar systems which pass one another at small distances. These tidal disturbances are of some importance since they are accompanied by a loss of energy which may result in a capture between the two objects. In a previous paper¹ the writer discussed the clustering tendencies among extragalactic nebulae. A theory was put forth that the observed clustering effects are the result of captures between individual nebulae. The capture theory seems to be able to account not only for double and multiple nebulae but also for the large extragalactic clusters. The present investigation tries to give an answer to the important question of whether the loss of energy accompanying a close encounter between two nebulae is large enough to effect a capture.

A study of tidal disturbances is greatly facilitated if it can be restricted to only two dimensions, i.e., to nebulae of a flattened shape, the principal planes of which coincide with the plane of their hyperbolic orbits. In order to reconstruct the orbit described by

¹ *Mt. W. Contr.*, No. 633; *Ap. J.*, 92, 200, 1940.

The first paper suggesting simulations of gravity



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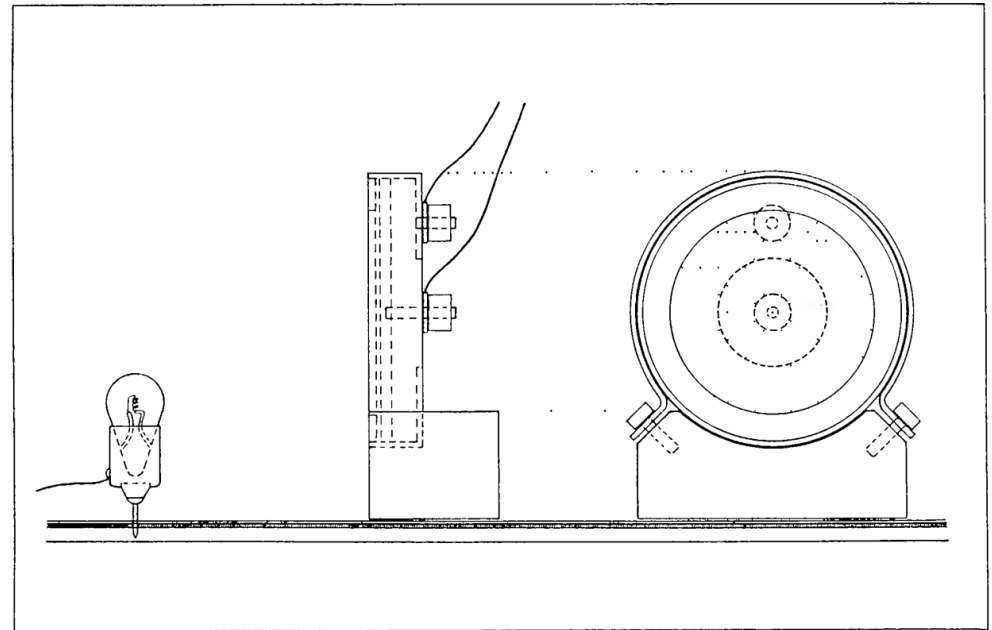
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“Gravity solver”



Erik Holmberg



G. 1.—Cross-section of light-bulb and photocell (half-size)

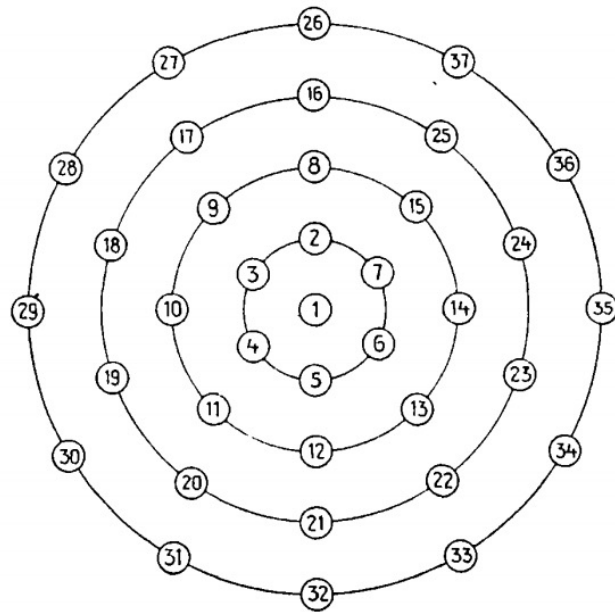


FIG. 3

“Initial conditions”

Light “dilution” is the same as gravity ie $\sim r^{-2}$

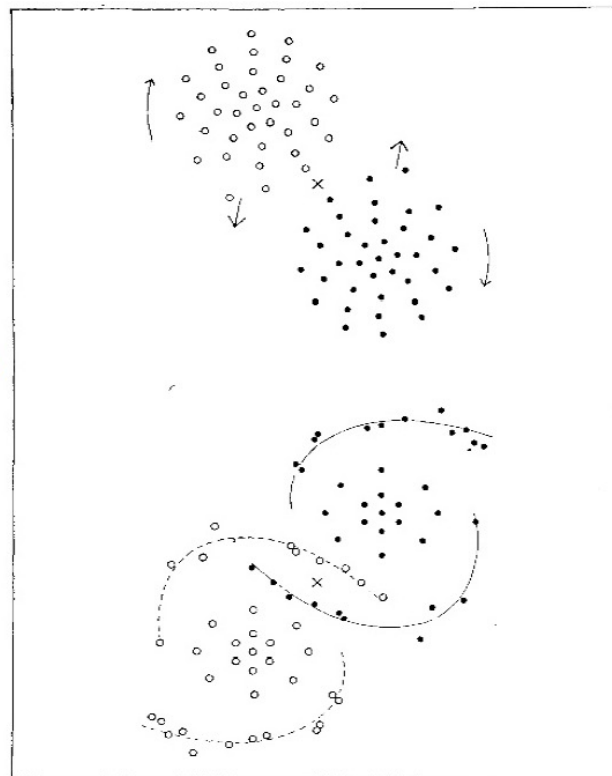
$$N = 2 \times 37$$



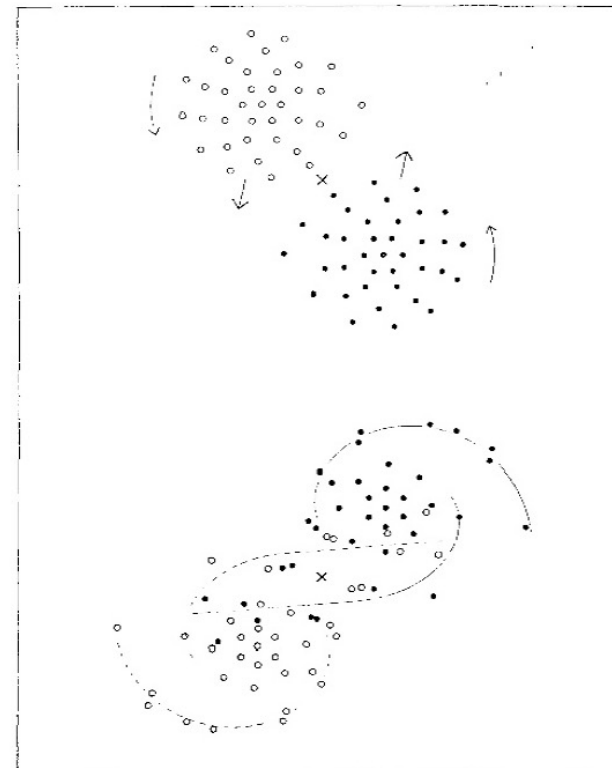
Erik Holmberg

- replacing gravity by light (same $1/r^2$ law)
- formation of tidal features

4m



3m



- gravity of N bodies

$$\boxed{m_i \ddot{\vec{r}}_i = \vec{F}(\vec{r}_i) \quad \forall i \in N}$$

- the “brute force approach” scales like N^2 :

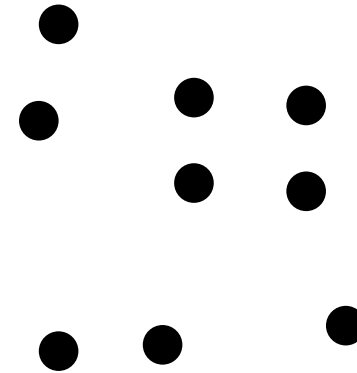
$$\vec{F}(\vec{r}_i) = - \sum_{i \neq j} \frac{G m_i m_j}{(r_i - r_j)^3} (\vec{r}_i - \vec{r}_j)$$

the summation over (N-1) particles has to be done for all N particles:

\Rightarrow number of floating point operations $\propto N(N-1) \propto N^2$

- gravity of N bodies

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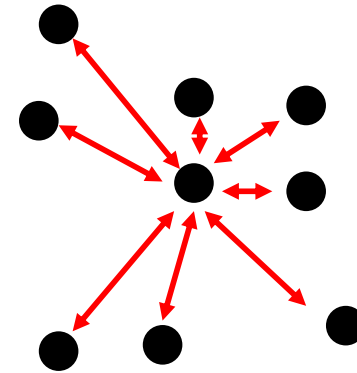
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N-1

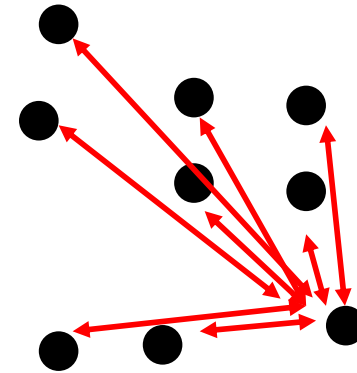
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$$(N-1) + (N-1) + \dots$$

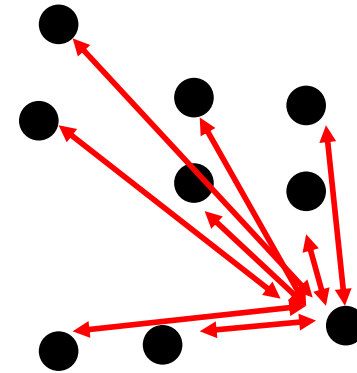
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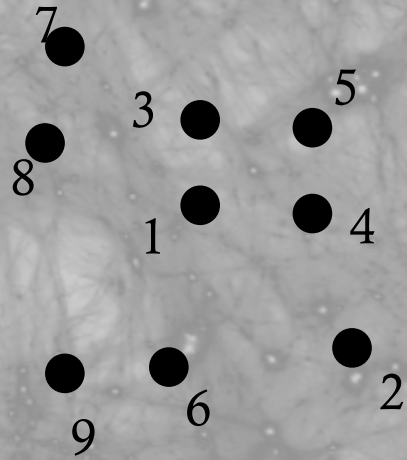
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$$\sim N^2$$

$$\vec{F}(\vec{r}_i) = - \sum_{i \neq j} \frac{G m_i m_j}{(r_i - r_j)^3} (\vec{r}_i - \vec{r}_j)$$

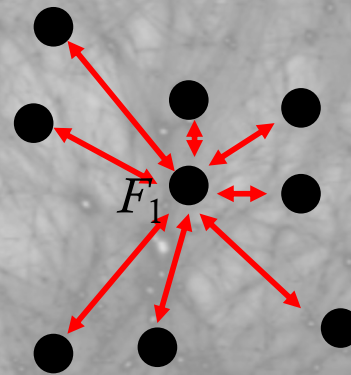
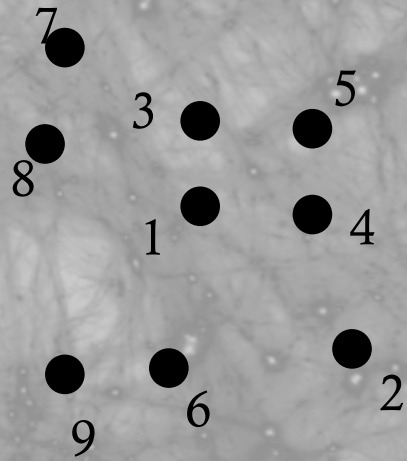
the summation over (N-1) particles has to be done for all N particles:

$$\Rightarrow \text{number of floating point operations} \propto N(N-1) \propto N^2$$



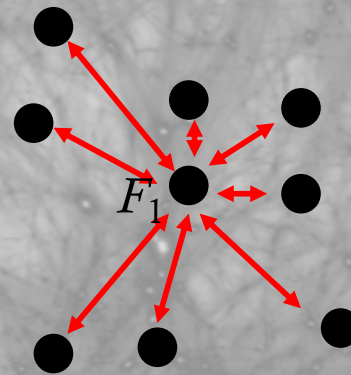
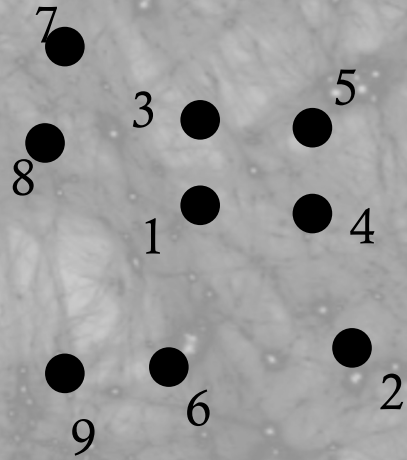
$$\vec{F}(\vec{r}_i) = - \sum_{i \neq j} \frac{G m_i m_j}{(r_i - r_j)^3} (\vec{r}_i - \vec{r}_j)$$

| Particle | r | Force | acceleration | displacement |
|----------|-----|-------|--------------|--------------|
| 1 | | | | |
| 2 | | | | |
| ... | | | | |
| N | | | | |



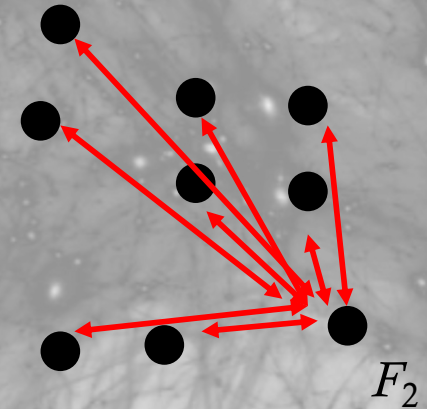
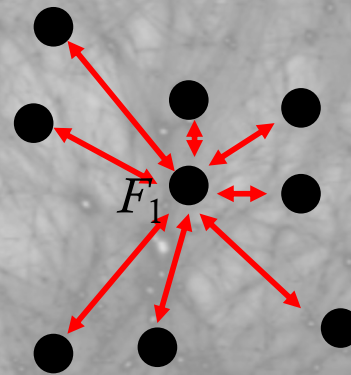
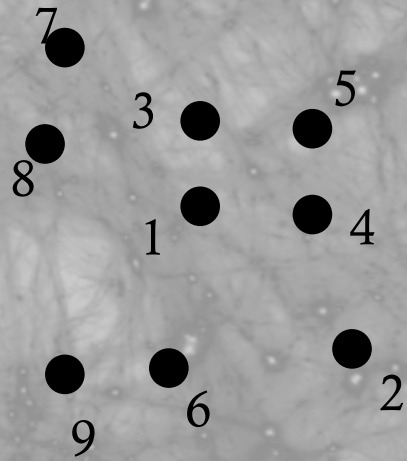
$$\vec{F}(\vec{r}_i) = - \sum_{i \neq j} \frac{G m_i m_j}{(r_i - r_j)^3} (\vec{r}_i - \vec{r}_j)$$

| Particle | r | Force | acceleration | displacement |
|----------|-----|-------|--------------|--------------|
| 1 | 0 | 0 | 0 | 0 |
| 2 | | | | |
| ... | | | | |
| N | | | | |



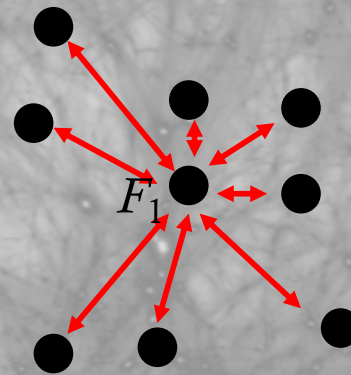
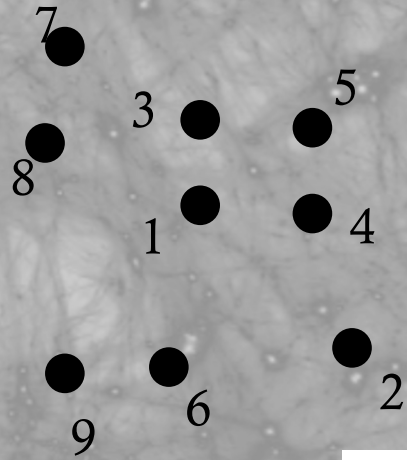
$$\vec{F}(\vec{r}_i) = - \sum_{i \neq j} \frac{G m_i m_j}{(r_i - r_j)^3} (\vec{r}_i - \vec{r}_j)$$

| Particle | r | Force | acceleration | displacement |
|----------|-------|---------------------|-------------------|-----------------------------------|
| 1 | 0 | 0 | 0 | 0 |
| 2 | r_2 | $F_2 = G m_2 / r_2$ | $a_2 = F_2 / m_2$ | $\Delta x_2 = a_2 (\Delta t_2)^2$ |
| ... | | | | |
| N | r_N | $F_N = G m_N / r_N$ | $a_N = F_N / m_N$ | $\Delta x_N = a_N (\Delta t_N)^2$ |



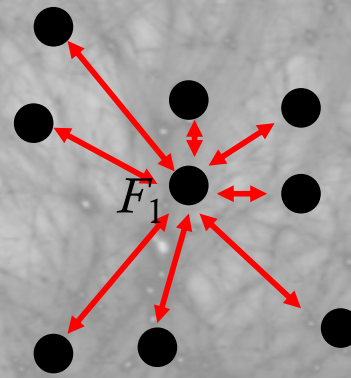
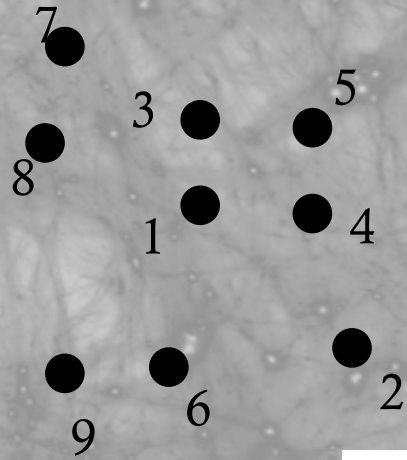
$$\vec{F}(\vec{r}_i) = - \sum_{i \neq j} \frac{G m_i m_j}{(r_i - r_j)^3} (\vec{r}_i - \vec{r}_j)$$

| Particle | r | Force | acceleration | displacement |
|----------|-------|---------------------|-------------------|-----------------------------------|
| 1 | r_1 | $F_1 = G m_1 / r_1$ | $a_1 = F_1 / m_1$ | $\Delta x_1 = a_1 (\Delta t_1)^2$ |
| 2 | 0 | 0 | 0 | 0 |
| ... | | | | |
| N | r_N | $F_N = G m_N / r_N$ | $a_N = F_N / m_N$ | $\Delta x_N = a_N (\Delta t_N)^2$ |



$$\vec{F}(\vec{r}_i) = - \sum_{i \neq j} \frac{G m_i m_j}{(r_i - r_j)^3} (\vec{r}_i - \vec{r}_j)$$

| Particle | r | Force | acceleratio n | Velocity | displacement |
|----------|-------|---------------------|-------------------|---------------------------------|---|
| 1 | 0 | 0 | 0 | 0 | 0 |
| 2 | r_2 | $F_2 = G m_2 / r_2$ | $a_2 = F_2 / m_2$ | $\Delta v_2 = a_2 (\Delta t_2)$ | $\Delta x_2 = \Delta v_2 \Delta t_2 + a_2 (\Delta t_2)$ |
| ... | | | | | |
| N | r_N | $F_N = G m_N / r_N$ | $a_N = F_N / m_N$ | $\Delta v_1 = a_N (\Delta t_N)$ | $\Delta x_N = \Delta v_N \Delta t_N + a_N (\Delta t_N)$ |

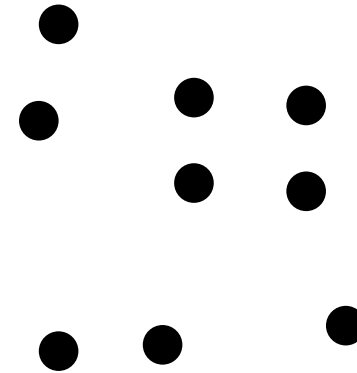


$$\vec{F}(\vec{r}_i) = - \sum_{i \neq j} \frac{G m_i m_j}{(r_i - r_j)^3} (\vec{r}_i - \vec{r}_j)$$

| Particle | r | Force | acceleratio n | Velocity | displacement |
|----------|-------|---------------------|-------------------|---------------------------------------|---|
| 1 | r_1 | $F_1 = G m_1 / r_1$ | $a_1 = F_1 / m_1$ | $\Delta v_1 = v_1 + a_2 (\Delta t_2)$ | $\Delta x_2 = \Delta v_2 \Delta t_2 + a_2 (\Delta t_2)$ |
| 2 | 0 | 0 | 0 | 0 | 0 |
| ... | | | | | |
| N | r_N | $F_N = G m_N / r_N$ | $a_N = F_N / m_N$ | $\Delta v_1 = v_N + a_N (\Delta t_N)$ | $\Delta x_N = \Delta v_N \Delta t_N + a_N (\Delta t_N)$ |

- gravity of N bodies

$$\boxed{m_i \ddot{\vec{r}}_i = \vec{F}(\vec{r}_i) \quad \forall i \in N}$$

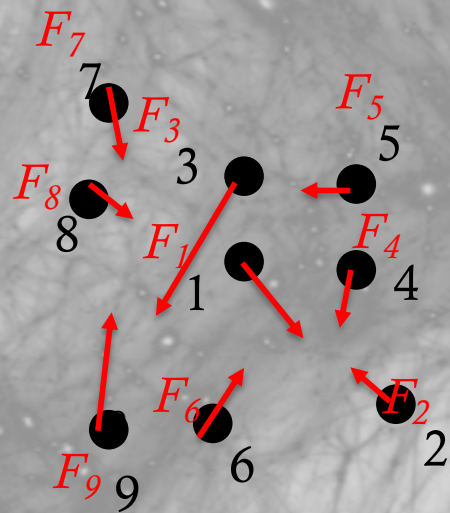
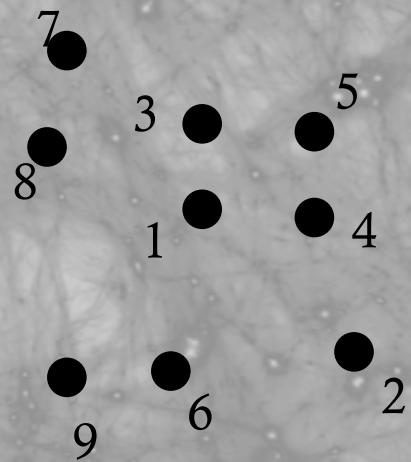


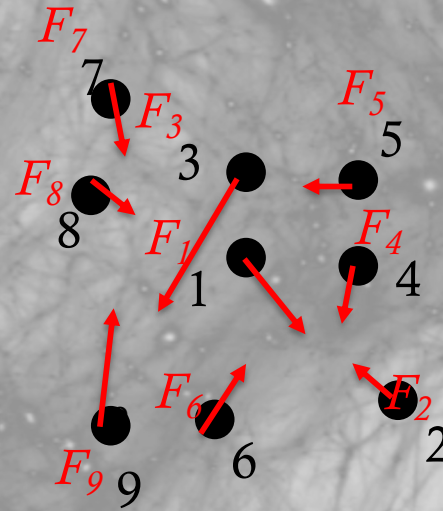
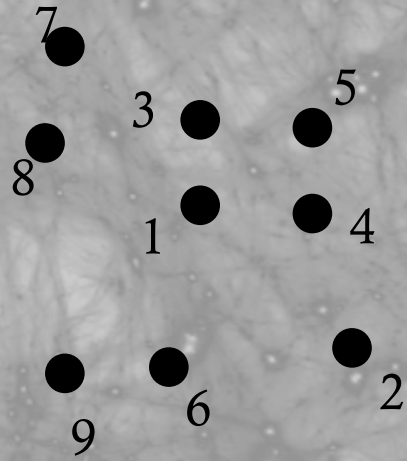
- the “brute force approach” scales like N^2 :

$$\vec{F}(\vec{r}_i) = - \sum_{i \neq j} \frac{G m_i m_j}{(r_i - r_j)^3} (\vec{r}_i - \vec{r}_j)$$

the summation over (N-1) particles has to be done for all N particles:

\Rightarrow number of floating point operations $\propto N(N-1) \propto N^2$





$$a_1 = F_1 / m_1$$

$$\Delta x_1 = a_1 (\Delta t)^2$$

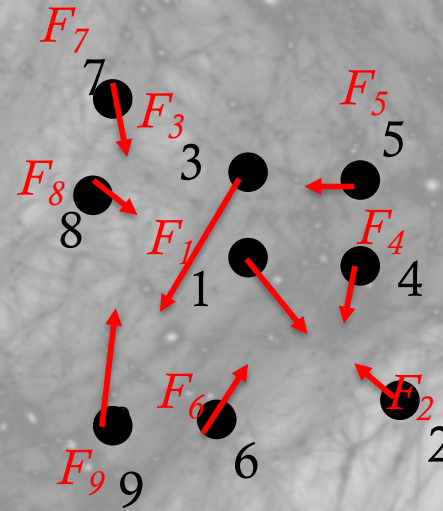
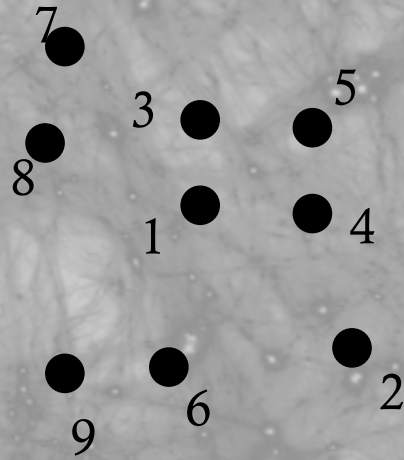
$$a_2 = F_2 / m_2$$

$$\Delta x_2 = a_2 (\Delta t)^2$$

...

$$a_N = F_N / m_N$$

$$\Delta x_N = a_N (\Delta t)^2$$



$$a_1 = F_1 / m_1$$

$$\Delta x_1 = a_1 (\Delta t)^2$$

$$a_1 = F_1 / m_1$$

$$\Delta x_1 = v_1 \Delta t + a_1 (\Delta t)^2$$

$$a_2 = F_2 / m_2$$

$$\Delta x_2 = a_2 (\Delta t)^2$$

$$a_2 = F_2 / m_2$$

$$\Delta x_2 = v_2 \Delta t + a_2 (\Delta t)^2$$

...

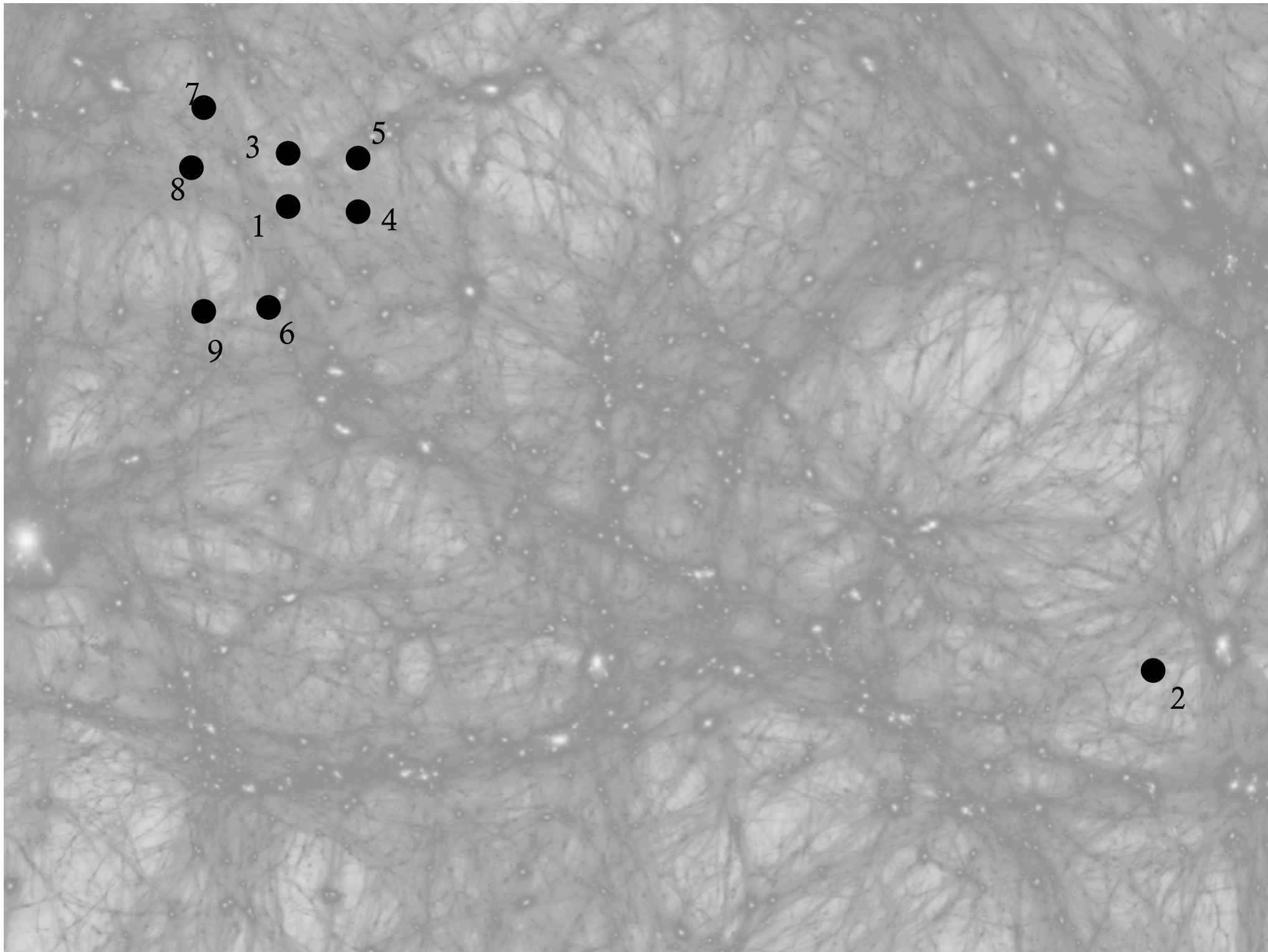
$$a_N = F_N / m_N$$

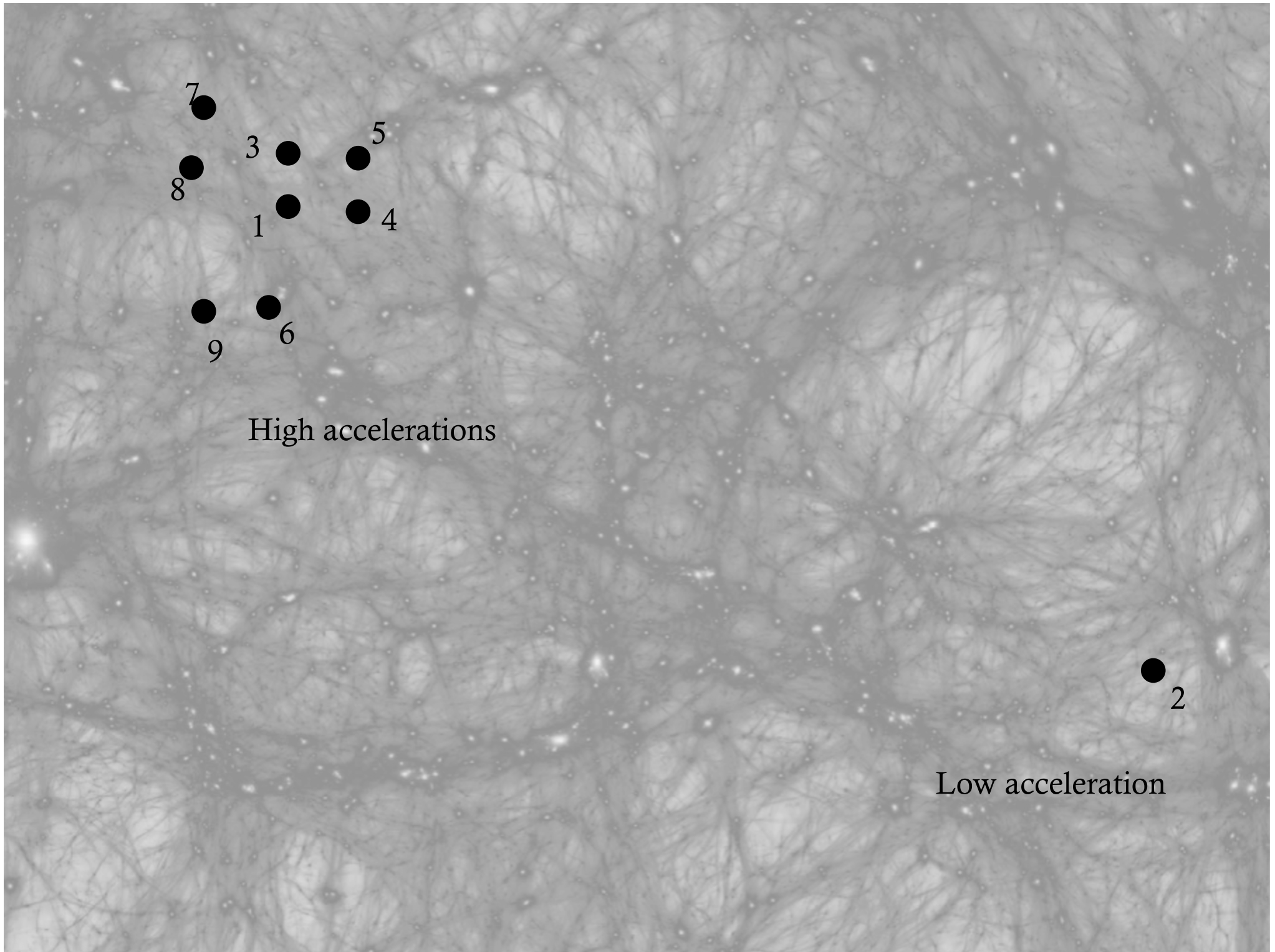
$$\Delta x_N = a_N (\Delta t)^2$$

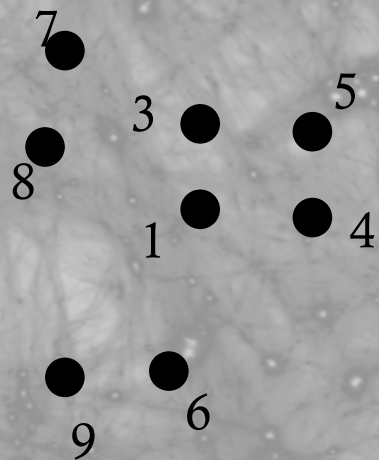
...

$$a_N = F_N / m_N$$

$$\Delta x_N = v_N \Delta t + a_N (\Delta t)^2$$







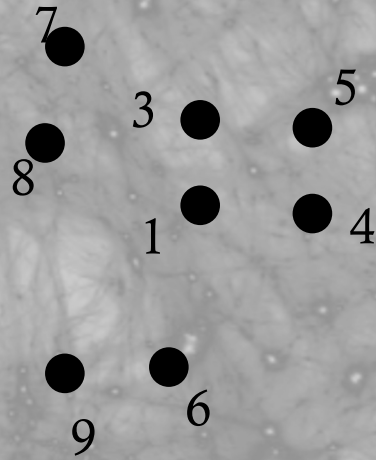
High accelerations

Time step should be small



Low acceleration

Time step can be longer



High accelerations

Time step should be small

$$\Delta t = \alpha \sqrt{\epsilon / |\mathbf{a}|}$$

Epsilon – softening
Alpha – “tolerance” parameter



Low acceleration

Time step can be longer

- ordinary differential equation

$$\frac{df}{dt} = G(f, t)$$

- ordinary differential equation

$$\frac{df}{dt} = G(f, t)$$

$$\Rightarrow \frac{\Delta f}{\Delta t} = \frac{f(t_{i+1}) - f(t_i)}{t_{i+1} - t_i} = \frac{f_{i+1} - f_i}{t_{i+1} - t_i} = G(f_i, t_i)$$

- ordinary differential equation

$$\frac{df}{dt} = G(f, t)$$

$$\Rightarrow \frac{\Delta f}{\Delta t} = \frac{f(t_{i+1}) - f(t_i)}{t_{i+1} - t_i} = \frac{f_{i+1} - f_i}{t_{i+1} - t_i} = G(f_i, t_i)$$

$$\Rightarrow f_{i+1} = f_i + \Delta t G(f_i, t_i)$$

▪ ordinary differential equation

- Euler scheme

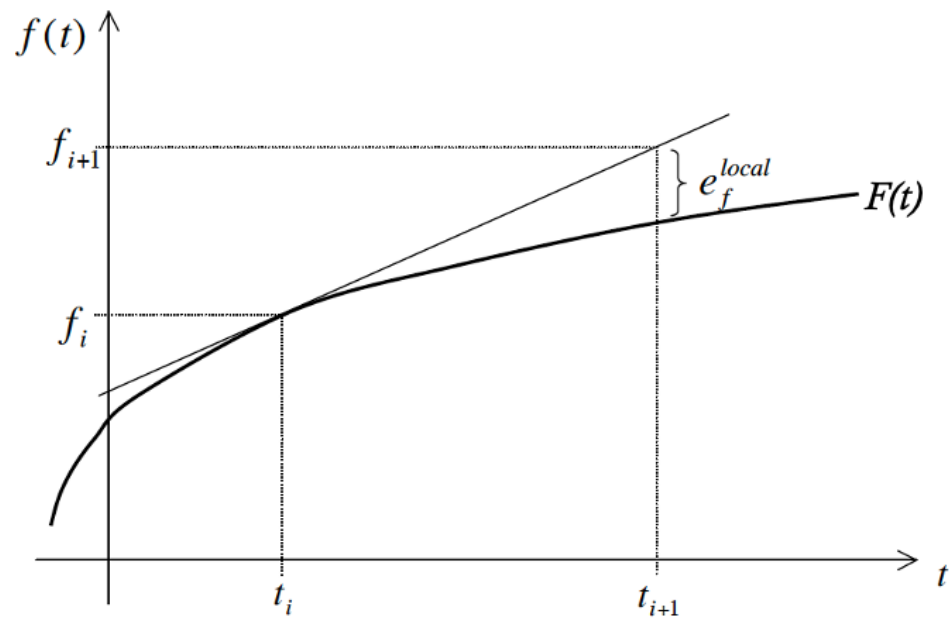
$$f_{i+1} = f_i + \Delta t G(f_i, t_i)$$

$$\frac{df}{dt} = G(f, t)$$

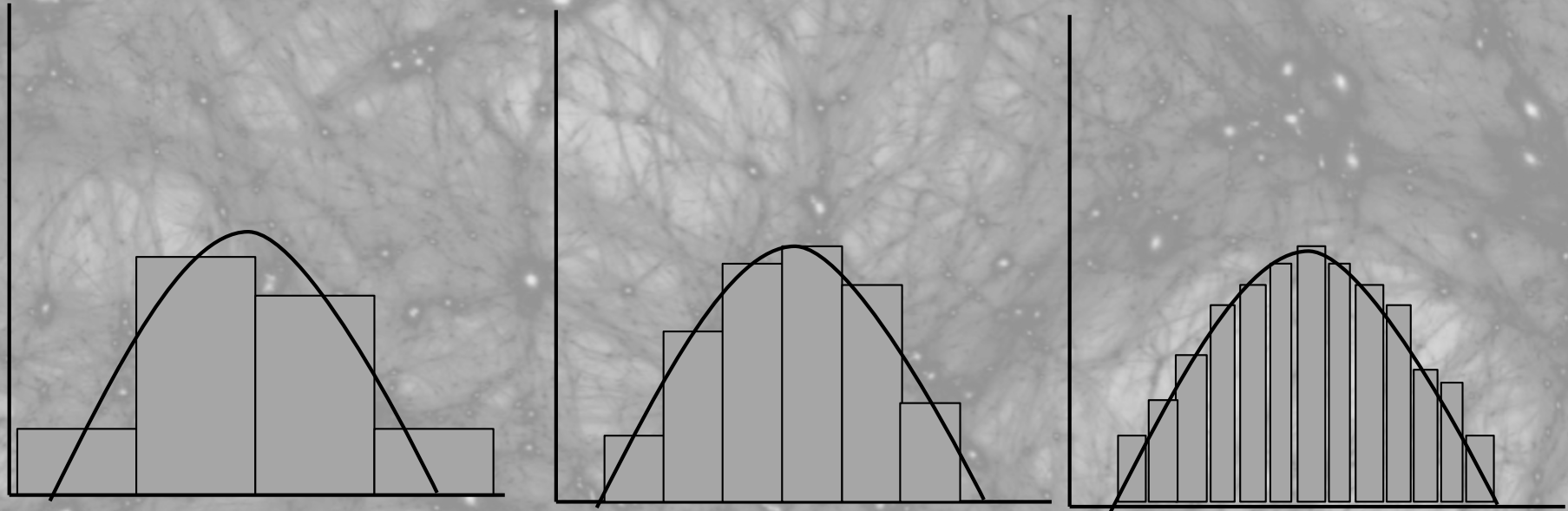
first term in Taylor expansion of $f(t)$ about t_i !

$$\Rightarrow f_{i+1} = f_i + \Delta t G(f_i, t_i)$$

■ ordinary differential equation

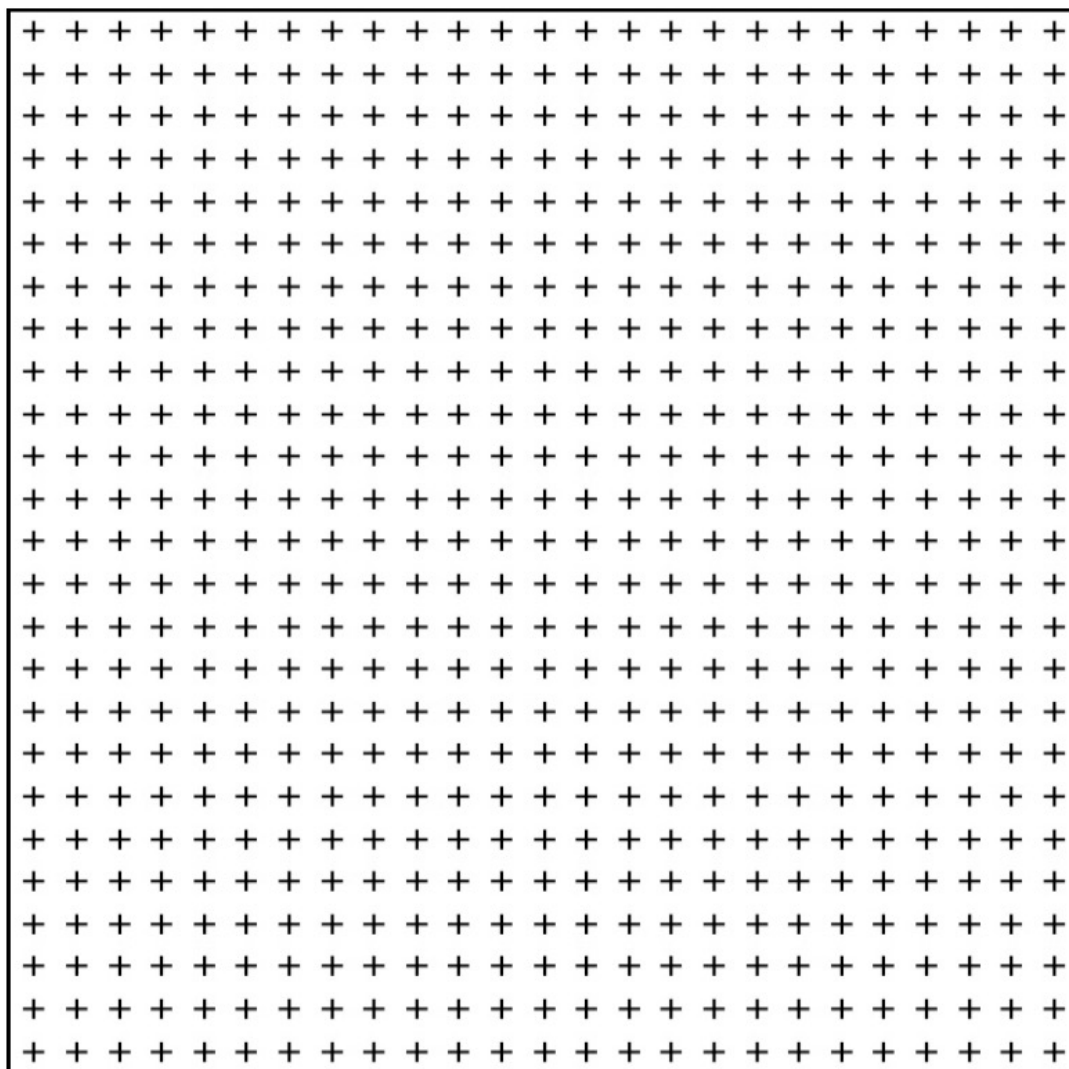


$$\Rightarrow f_{i+1} = f_i + \Delta t G(f_i, t_i)$$

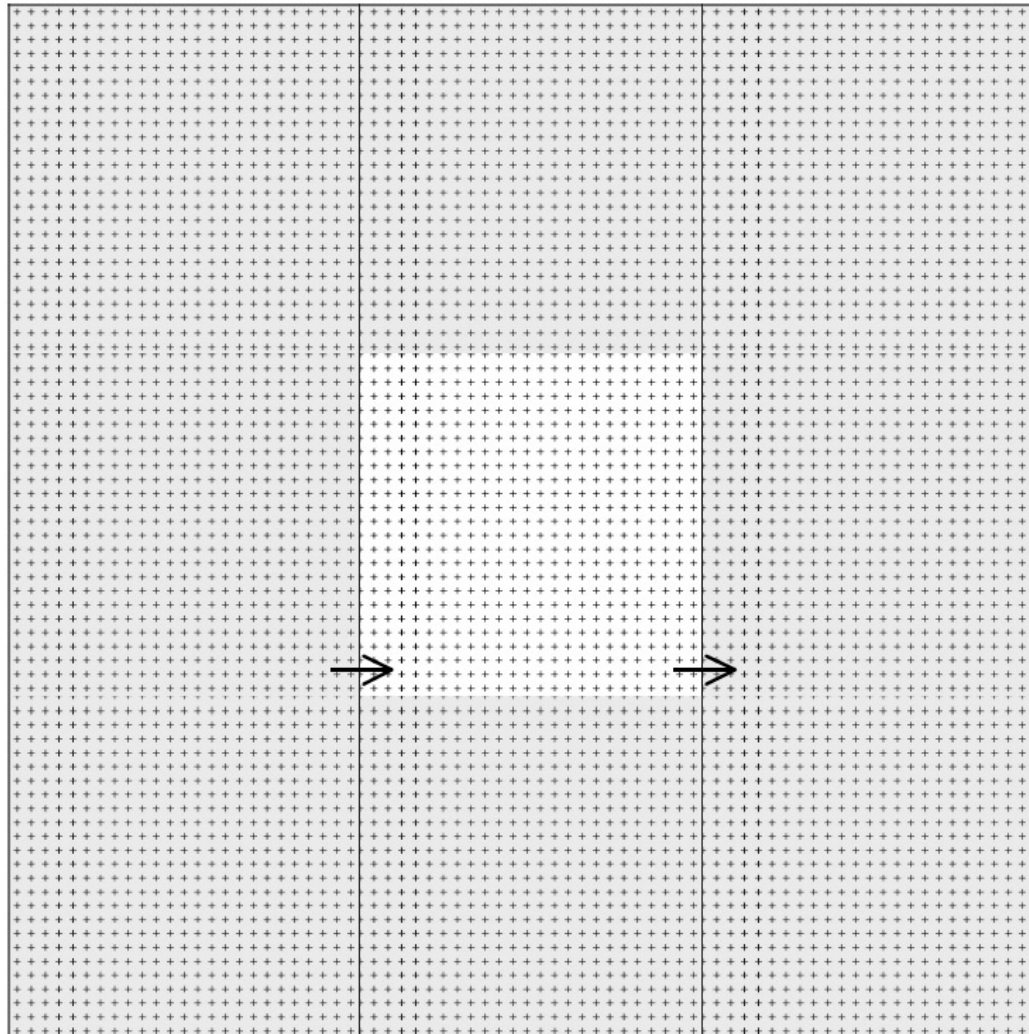


$$\int f(x)dx = \lim_{\Delta x \rightarrow 0} f(x)\Delta x$$

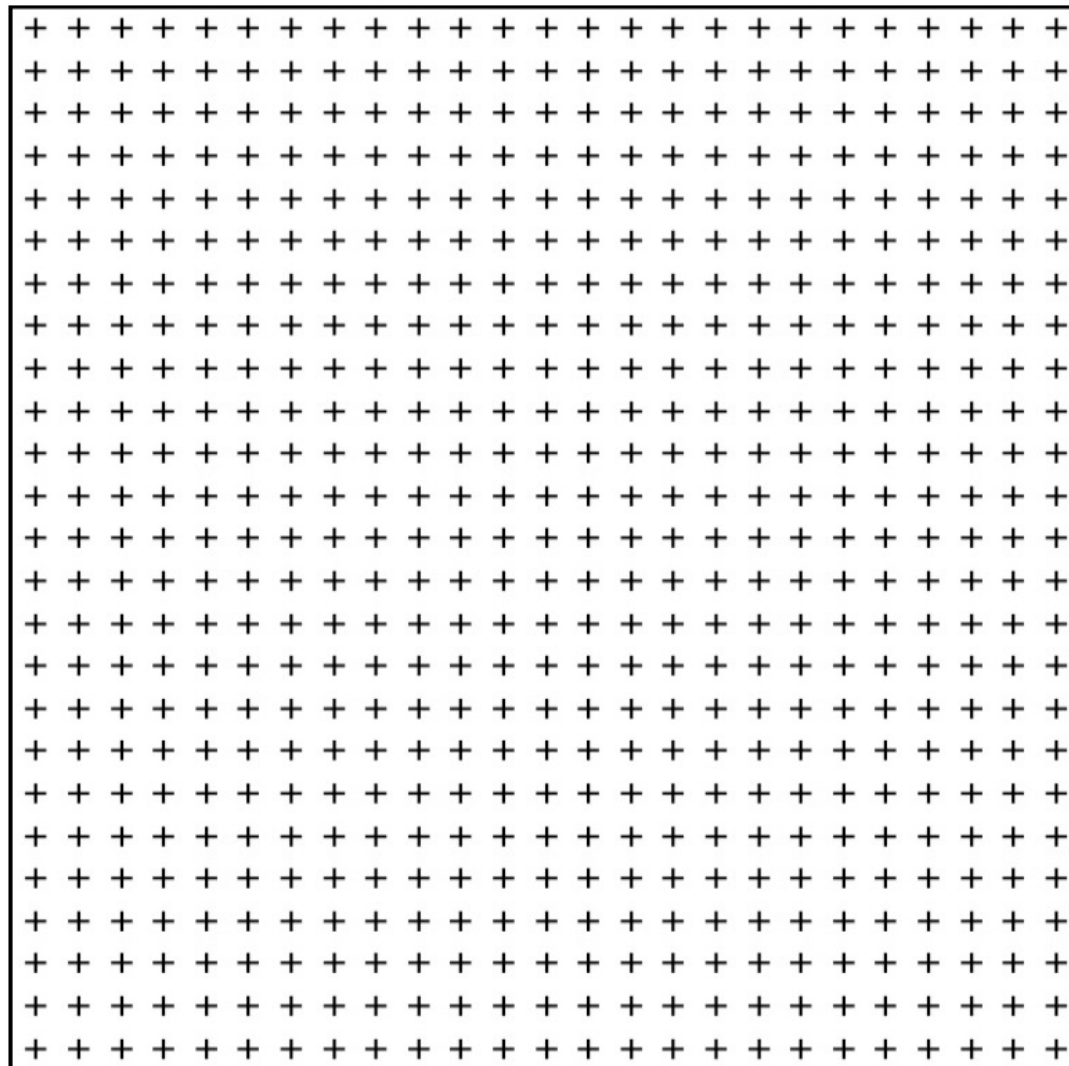
Error is thus a feature of numerical integration



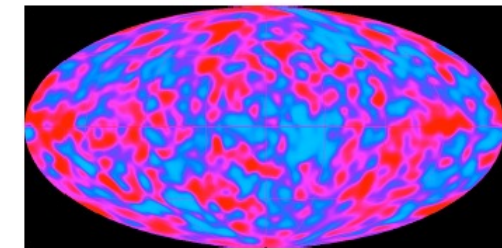
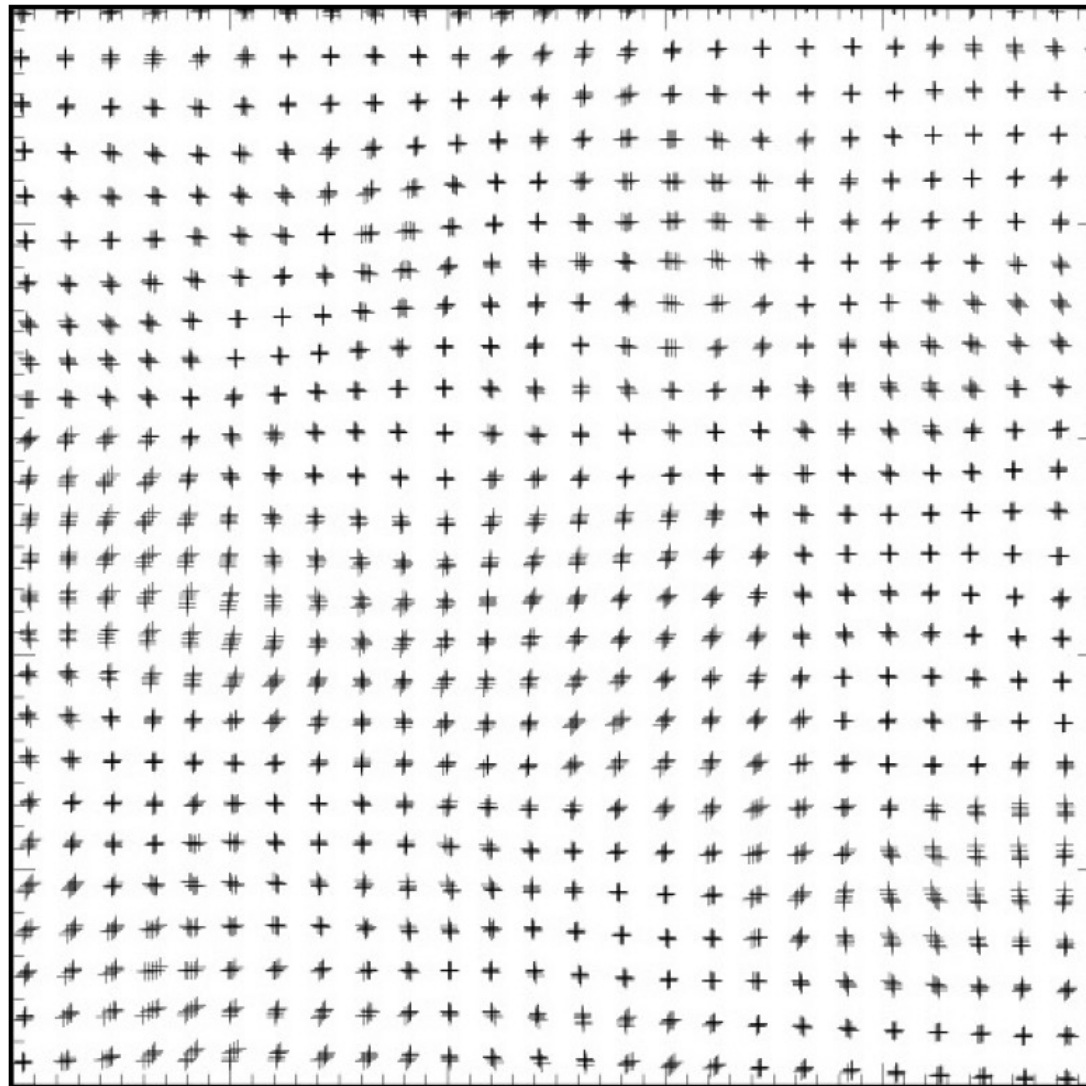
homogeneous
&
isotropic



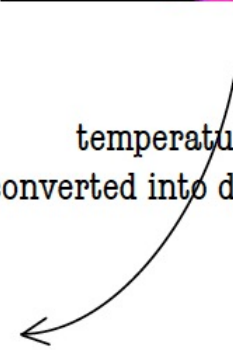
infinite
(periodic boundary conditions)

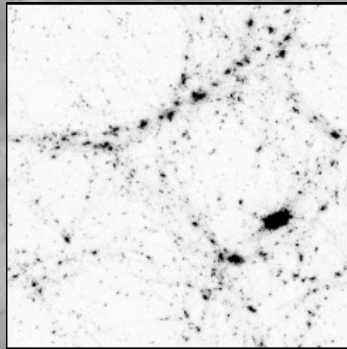


homogeneous
&
isotropic

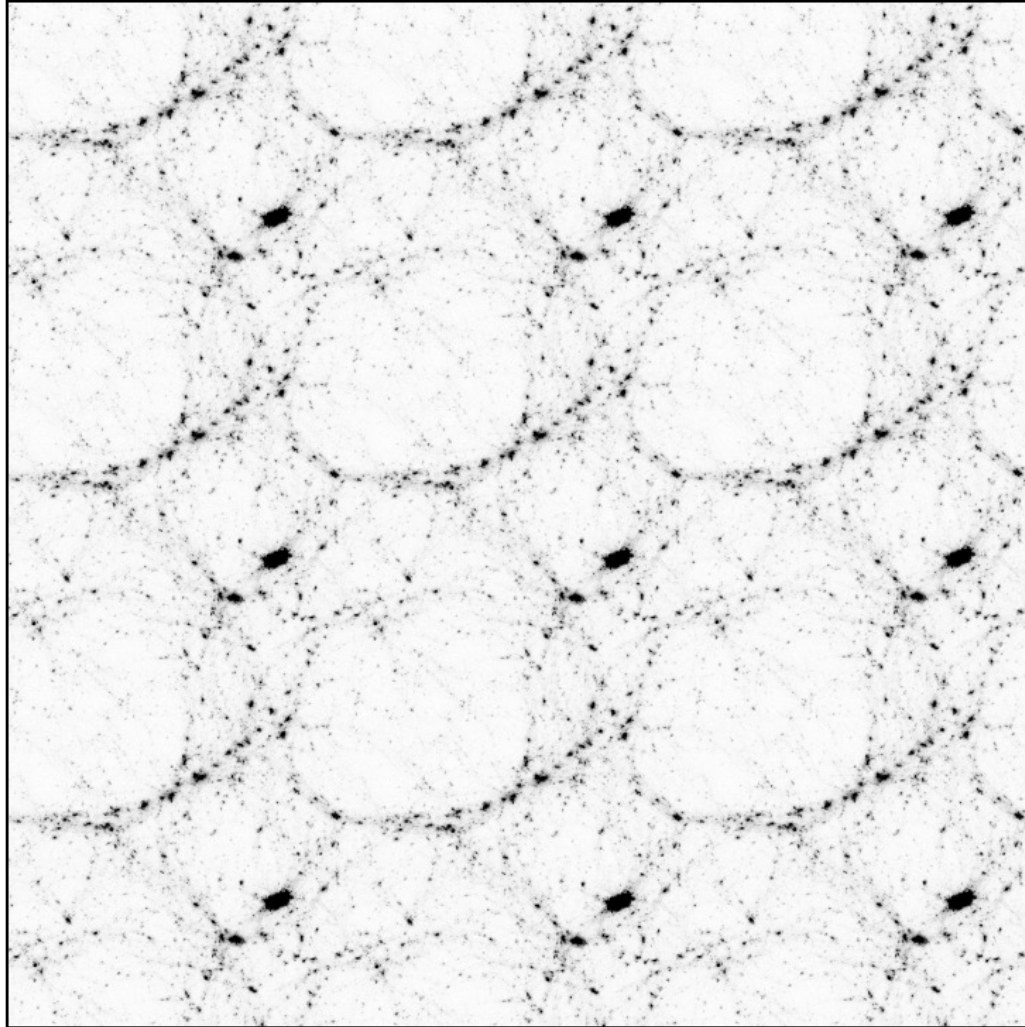


temperature fluctuations
converted into density perturbations





infinite
(periodic boundary conditions)



infinite
(periodic boundary conditions)

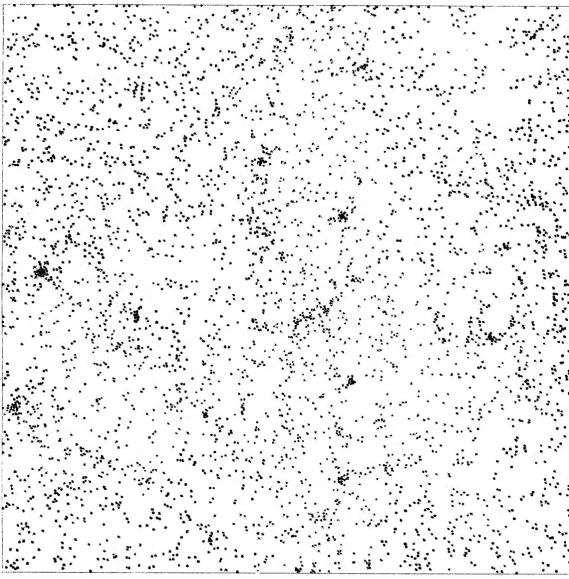
Three-dimensional numerical model of the formation of large-scale structure in the Universe

A. A. Klypin and S. F. Shandarin *The Keldysh Institute of Applied Mathematics, Academy of Sciences of USSR, Miusskaja Sq. 4, Moscow 125047, USSR*

Received 1982 November 15; in original form 1982 April 28

Summary. The first results of numerical fully three-dimensional simulations of formation and evolution of the large-scale structure of the Universe are presented. The simulations were carried out in the framework of the adiabatic scenario of galaxy formation.

The model contains $32^3 = 32\,768$ collisionless particles interacting only gravitationally. Equal mass particles are moving in a collective gravitational field which is smoothed at small scales. Evolution of perturbations is followed in an expanding cosmological model.



(a)

Figure 2. Three snapshots of the system. (a) Scale factor $a = 6.2 a_{\text{start}}$ and $(\delta\rho/\rho)_{r_0} = 1.6$; (b) $a = 13.6 a_{\text{start}}$; $(\delta\rho/\rho)_{r_0} \approx 3.3$; (c) $a = 18 a_{\text{start}}$; $(\delta\rho/\rho)_{r_0} \approx 4.8$. Only each eighth particle is plotted in the figures.

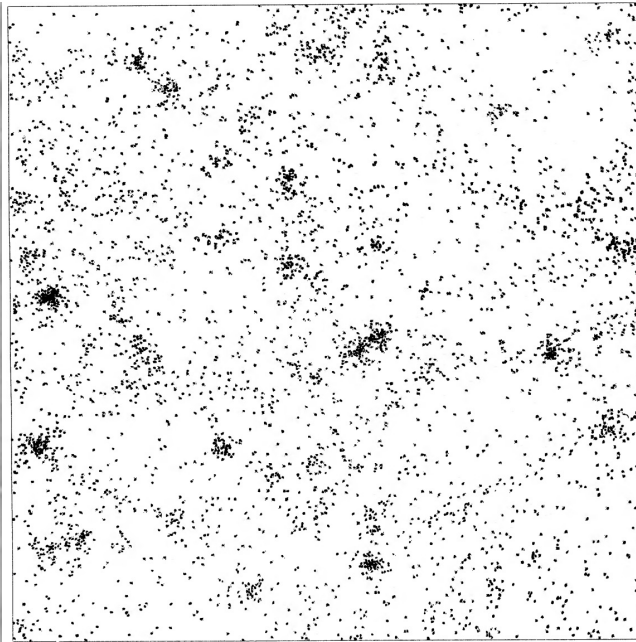


Figure 2(b)

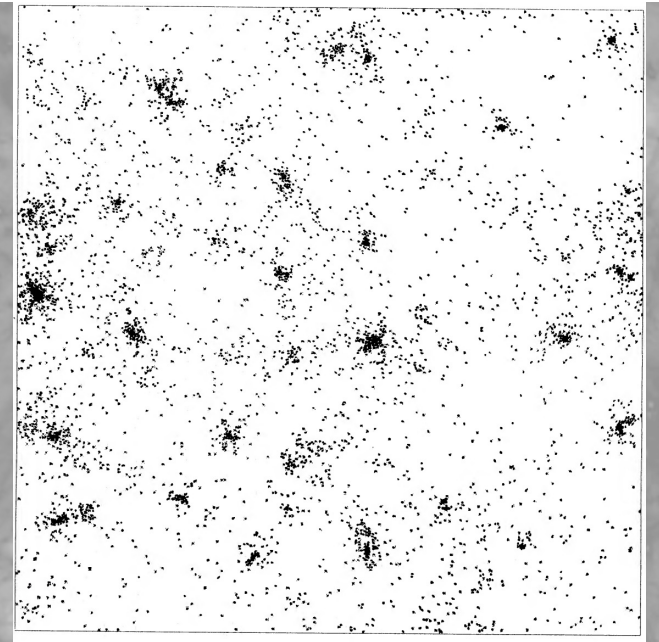


Figure 2 (c)

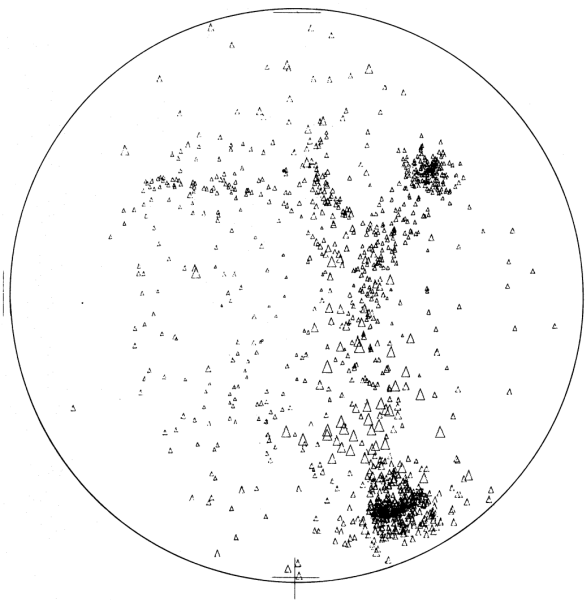


Figure 3 (b)

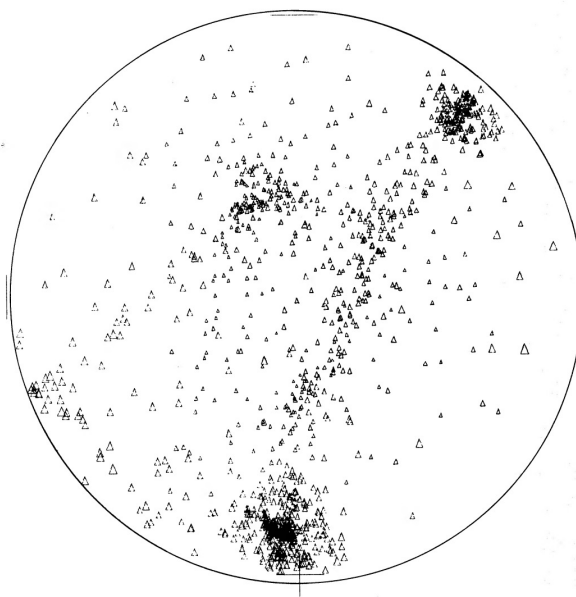
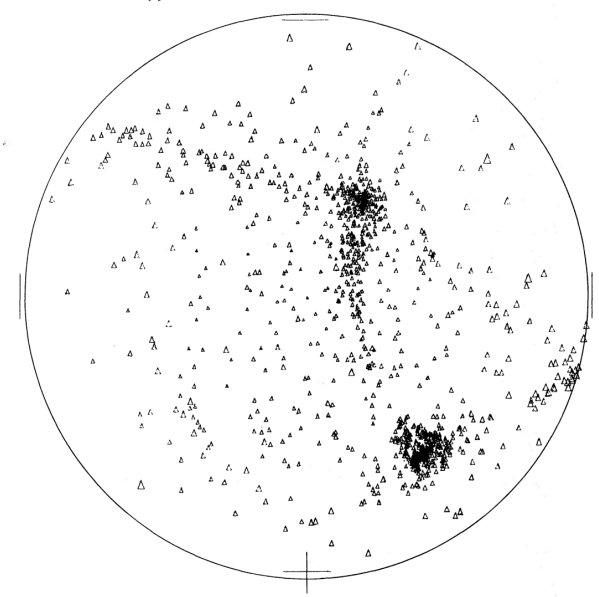


Figure 3 (c)



(a)

Figure 3. A small fragment of the particle distribution plotted in Fig. 2(b), when $a = 13.6 a_{\text{start}}$. As opposed to Fig. 2 here all particles in the sphere with radius $R = 6r_0 = 30 h^{-1} \text{ Mpc}$ are plotted. Every particle is depicted as a triangle whose size is inversely proportional to distance from an observer. The observer is situated at a distance $1.5 R$ from the centre of the sphere.

In Fig. 3 three different projections of the particle inside the sphere are shown. They were obtained with two successive rotations by 45° around an axis designated by +. One sees that within the sphere there are two rich clusters and two chains. A chain of particles connects the clusters, while another one begins in the bottom cluster, then goes up and left in Fig. 3(c) and leaves the sphere (at the upper left of Fig. 3a) without touching the upper cluster. A very complicated spatial distribution of the particles makes it too difficult to realize the relative location of the chains.

A more effective but much more complicated way is to draw a surface of a constant density level. In Fig. 4 a part of a surface defined as $\tilde{\rho} = 2.5 \tilde{\bar{\rho}}$ ($\tilde{\rho}$ is the mean density, $\tilde{\bar{\rho}} = 1$) is shown. It is depicted inside the same sphere. Two dots show the cluster centres. The chains in the figure touch each other near the upper cluster. This is the result of a coarse-grained grid, which was used to define the surface.

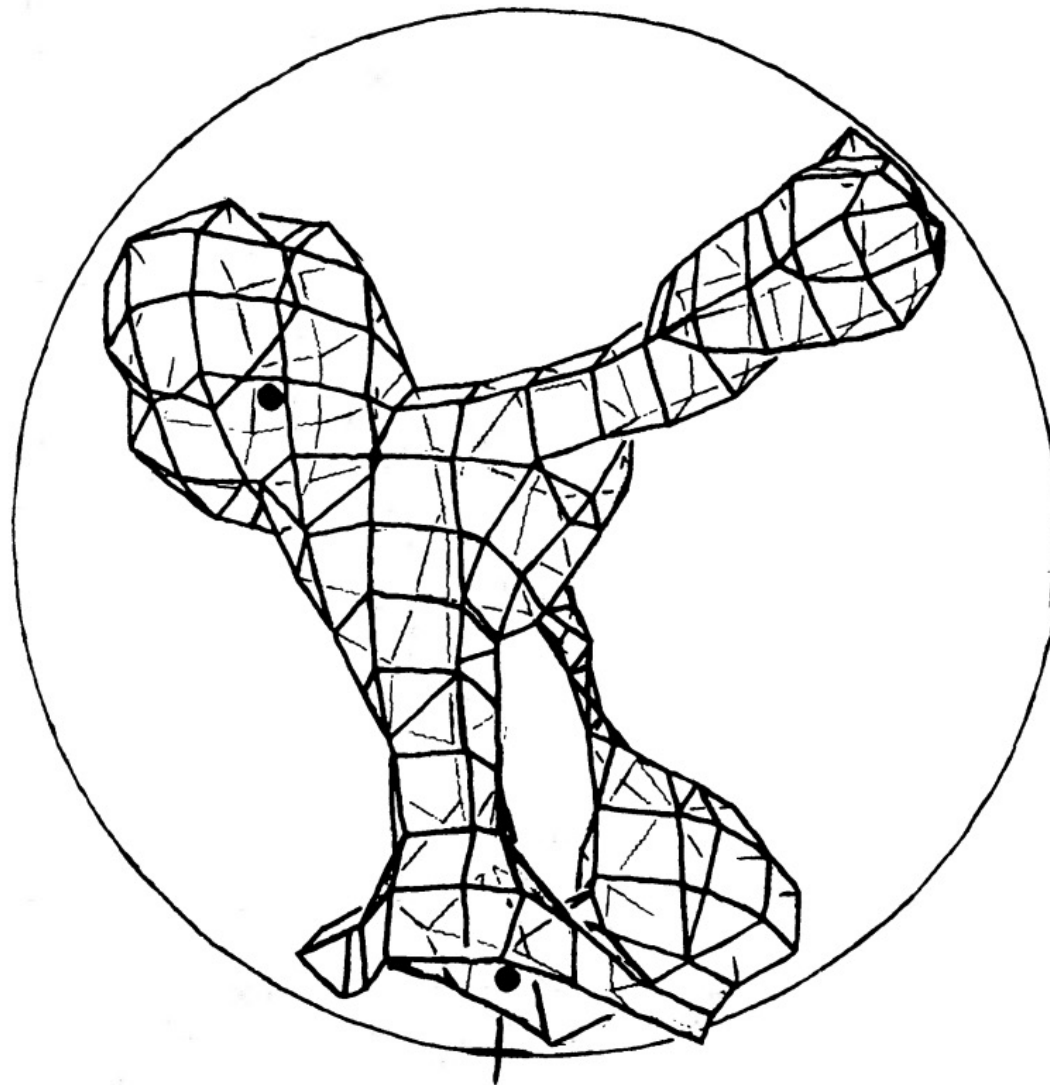
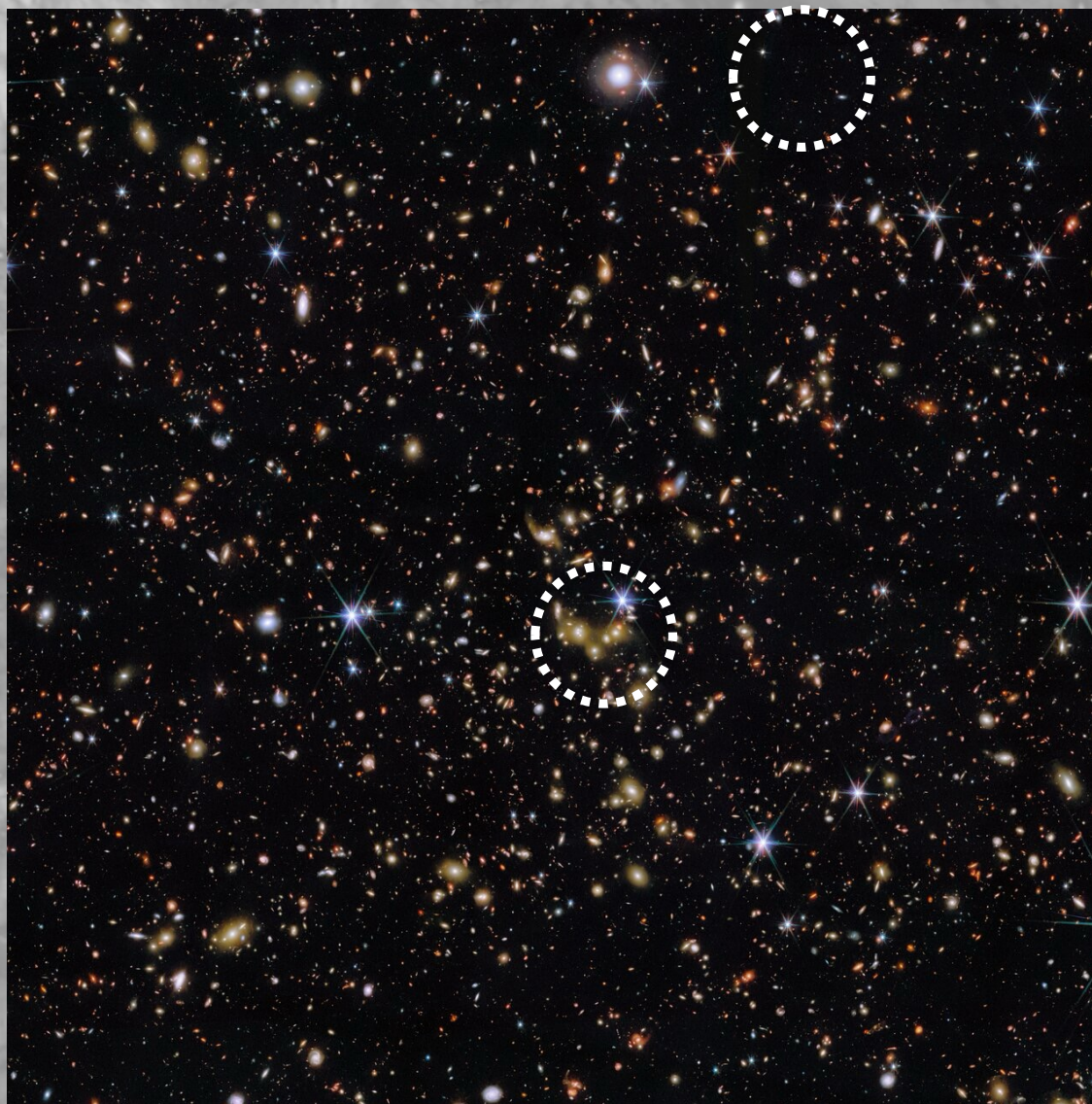


Figure 4. A surface of constant density level is plotted for the same region as that in Fig. 3.

The cosmic Chicken

Simulations: how can we use them?

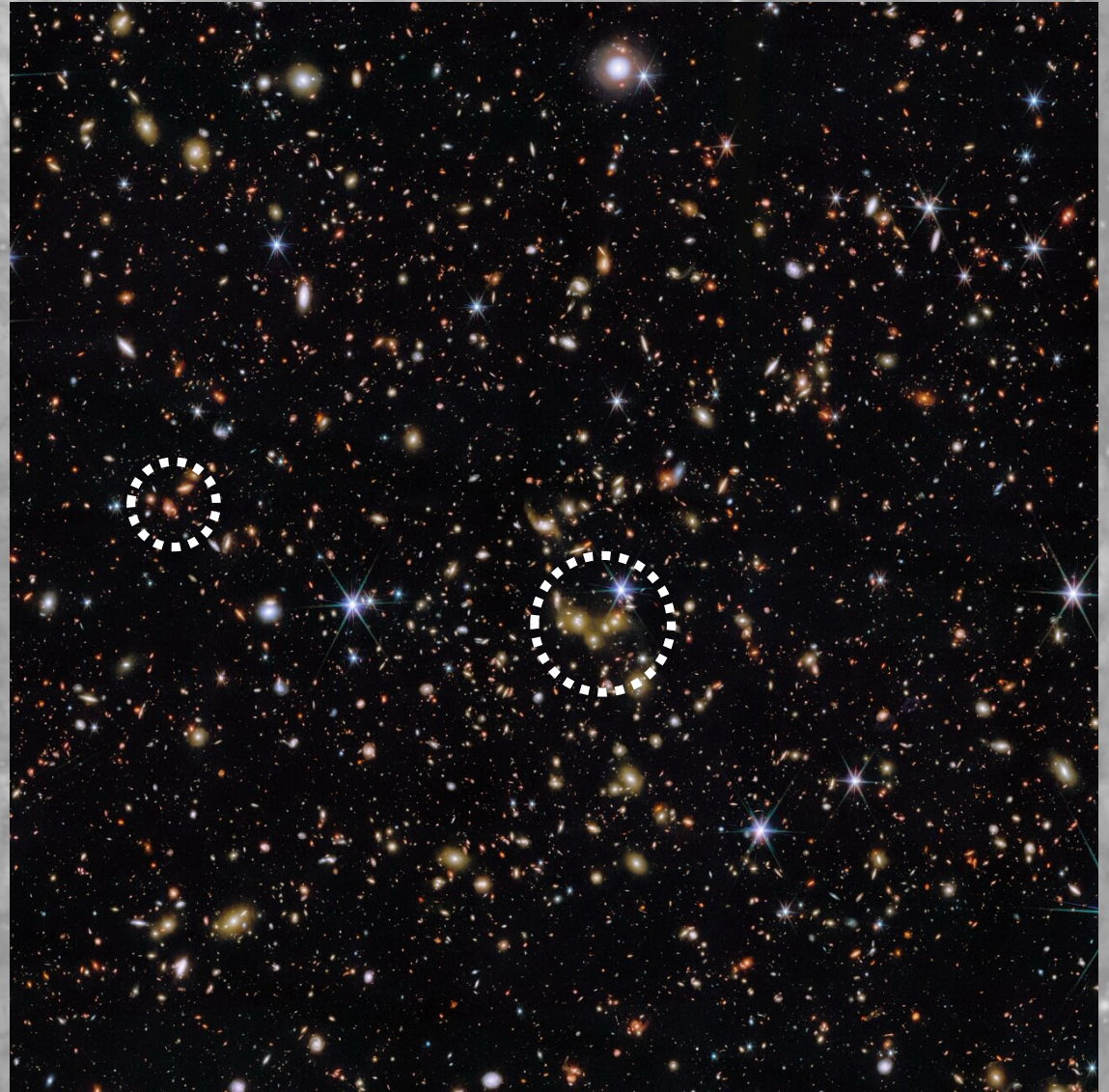
What is the spatial distribution of points? Are they clustered?



Simulations: how can we use them?

What is the spatial distribution of points? Are they clustered?

If they are clustered, are all the clusters the same size?

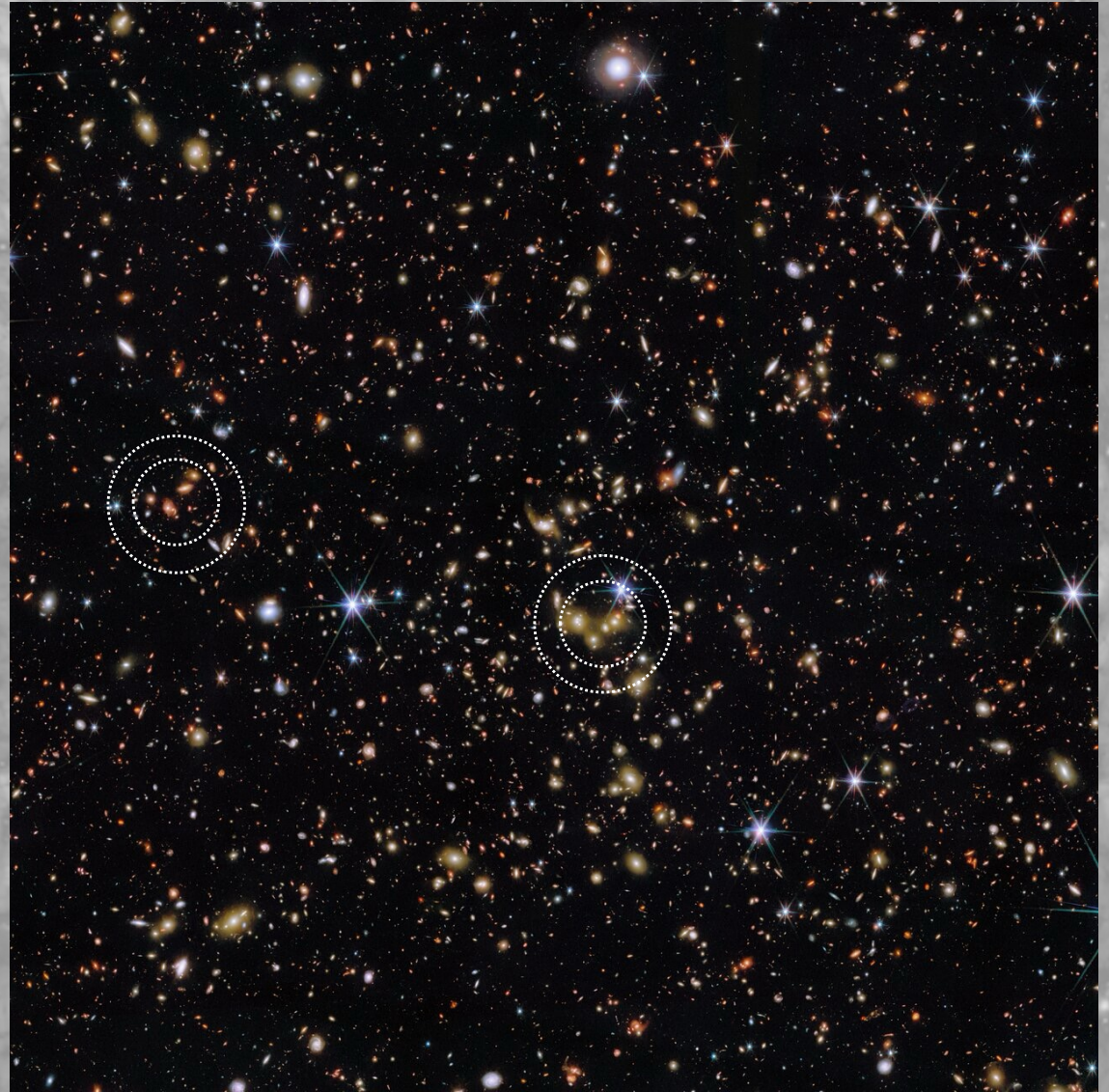


Simulations: how can we use them?

What is the spatial distribution of points? Are they clustered?

If they are clustered, are all the clusters the same size?

What is the internal structure of these clustered things?



What is the spatial distribution of points?

$\xi(r)$: The 2 point (correlation) function

The **two-point correlation function** $\xi(r)$ measures the "clumpiness" or clustering of objects.

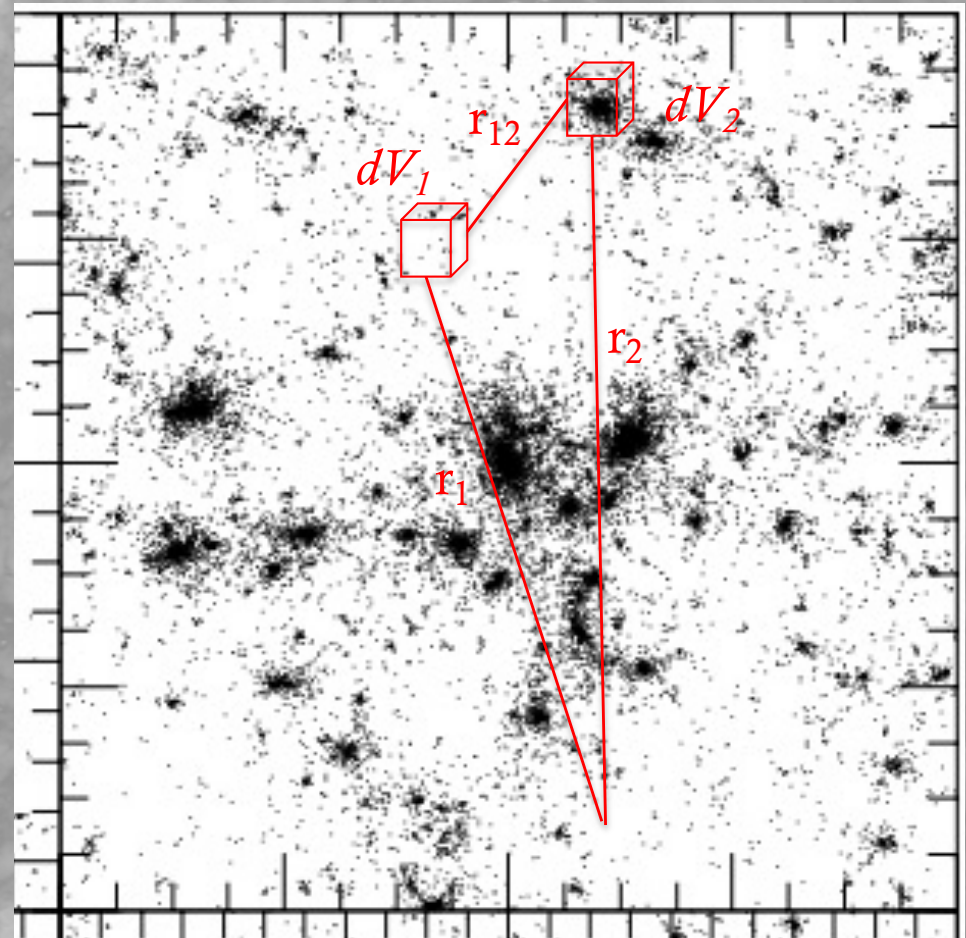
It quantifies the **excess probability** of finding a pair of objects at a certain separation, compared random (uniform)

$$dP = n^2 dV_1 dV_2 (1 + \xi(r_{12}))$$

$\xi(r)$ is the joint probability of finding a galaxy in two sub-volumes – it depends *ONLY* on separation (due to Copernican principle)

$$dP = n^2 dV_1 dV_2$$

For random the probabilities are independent



What is the spatial distribution of points?

$\xi(r)$: The 2 point (correlation) function

The **two-point correlation function**

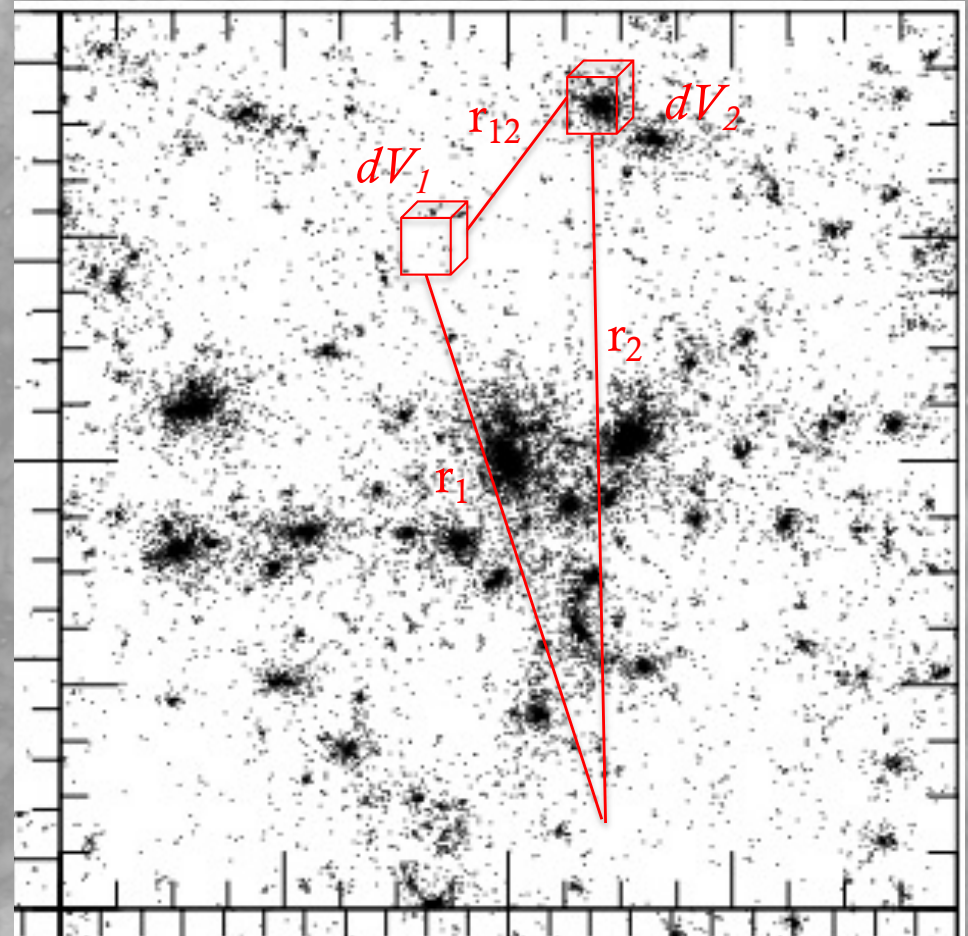
$\xi(r)$ measures the "clumpiness" or clustering of objects.

It quantifies the **excess probability** of finding a pair of objects at a certain separation, compared random (uniform)

$\xi(r) = 0$ un correlated

$\xi(r) > 0$ correlated

$\xi(r) < 0$ anti correlated



What is the spatial distribution of points?

$\xi(r)$: The 2 point (correlation) function

The **two-point correlation function**

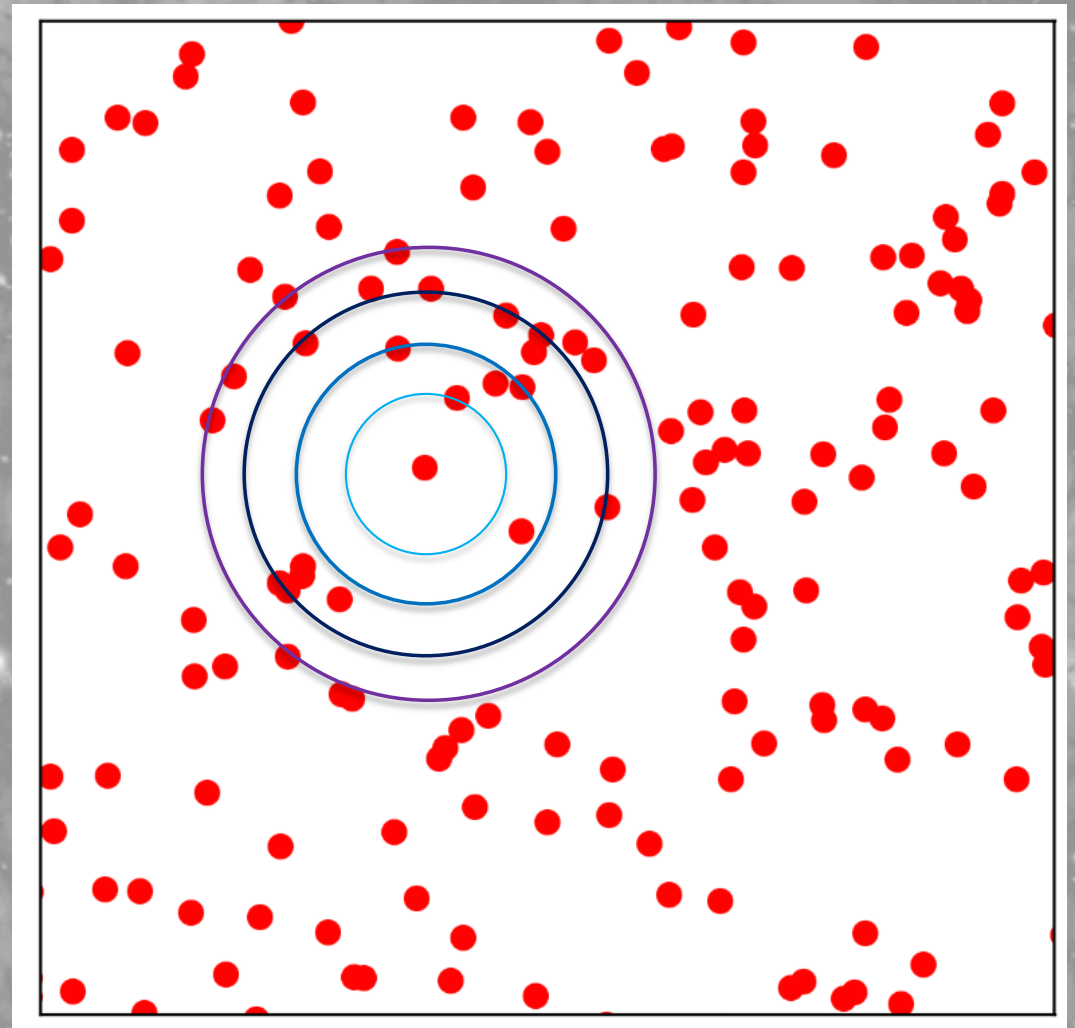
$\xi(r)$ measures the "clumpiness" or clustering of objects.

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On the clustering of particles in an expanding Universe



George Efstathiou

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J. W. Eastwood *Culham Laboratory, Abingdon, Oxfordshire OX1 3BD*

Received 1980 June 23; in original form 1980 February 20

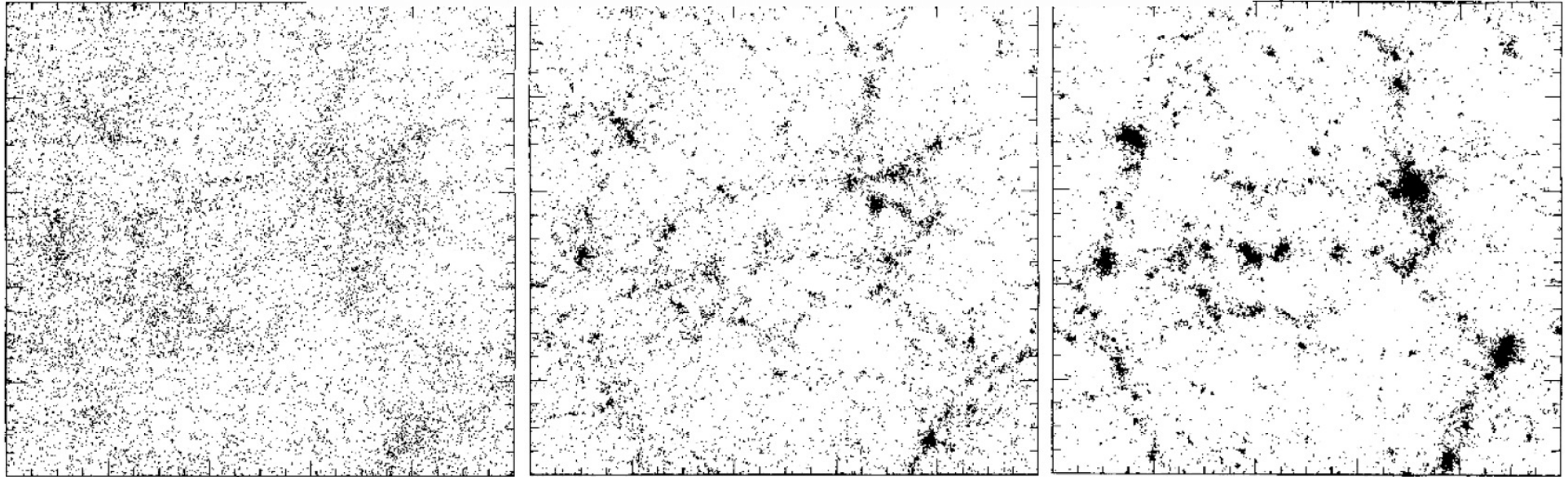
Summary. We investigate the clustering of particles in Friedmann models of the Universe using 1000- and 20 000-body numerical simulations. The results of these computations are analysed in terms of the two- and three-point correlation functions, the mean relative peculiar velocity between particle pairs $\langle v_{21} \rangle$, and the mean square peculiar velocity dispersion between pairs $\langle v_{21}^2 \rangle$. In the case of Einstein–de Sitter models we find that on scales corresponding to the transition region $\xi \sim 1$, $|\langle v_{21} \rangle| > Hr_{21}$ and this results in a non-power law form for $\xi(r)$, in rough agreement with simple analytic treatments based on the homogeneous spherical cluster models for the collapse of protoclusters. Our results are in conflict with the kinetic theory calculations of Davis & Peebles who studied the problem in the case of an Einstein–de Sitter Universe and found good agreement with observational data. These authors suggest that clusters develop substantial non-radial motions whilst they are still small density fluctuations, so that when a cluster fragments out of the general Hubble expansion, it is already virialized. This ‘previrialization’ effect does not appear to occur in the numerical models described here. We also examine the effects of particle discreteness and two-body relaxation, which are particularly important in the N -body models but neglected in the approach of Davis & Peebles. Because it is unclear as to whether these effects are important for galaxy clustering in the real Universe, it is difficult to assess the significance of our results. More observational and theoretical work is necessary in order to decide whether our approach is reasonable.

1 Introduction

The aim of this paper is to investigate whether gravitational instability can explain the observed forms of the low-order galaxy correlation functions (Peebles 1974a; Groth & Peebles 1977) under the assumption of some simple initial conditions.

On the clustering of particles in an expanding Universe

George Efstathiou[★] *Department of Physics, South Road, Durham DH1 3LE*
J. W. Eastwood *Culham Laboratory, Abingdon, Oxfordshire OX1 3BD*



The number of clustered galaxies within a radius $r_0 = 5h^{-1}$ Mpc (corresponding to $\xi(r_0) = 1$) is $\langle N \rangle \approx 30$ (taking the mean space density of bright galaxies as $0.02 h^{-3}$ Mpc). This number is comparable to the mean number of clustered particles within radius x_0 [$\xi(x_0) = 1$] of the particle distributions analysed in Section 4. Hence if we are justified in assuming the existence of some epoch z_* when galaxies were weakly clustered and act thereafter as the fundamental point particles, our approach may be applicable. We now explore the consequences of this hypothesis.

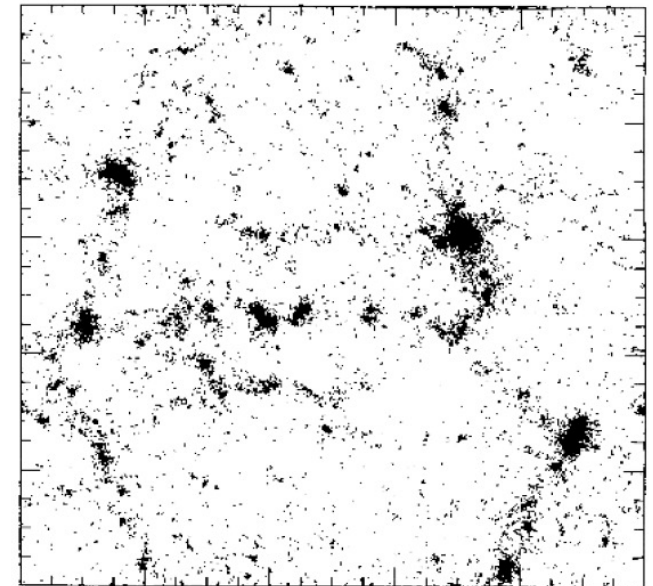
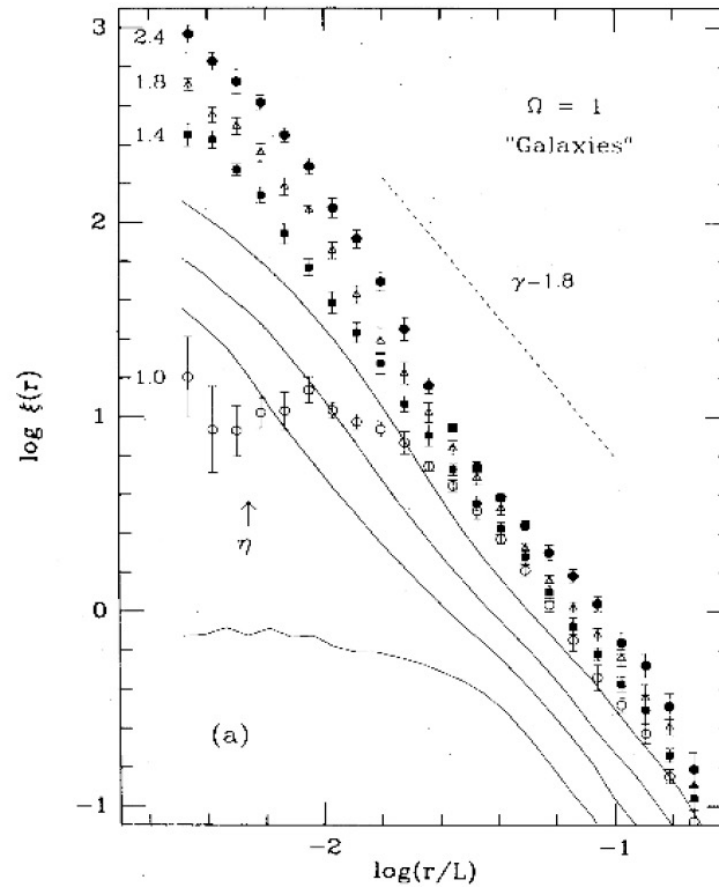
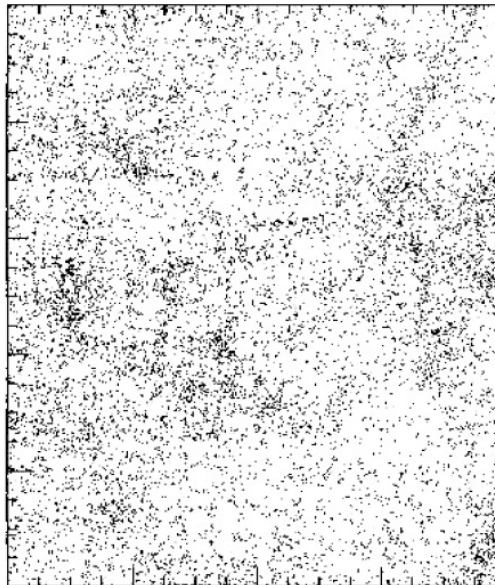
On the clustering of particles in an expanding Universe

$N = 32768$



George Efstathiou

George Efstathiou[★] *Department of Physics, South Road, Durham DH1 3LE*
J. W. Eastwood *Culham Laboratory, Abingdon, Oxfordshire OX1 3BD*

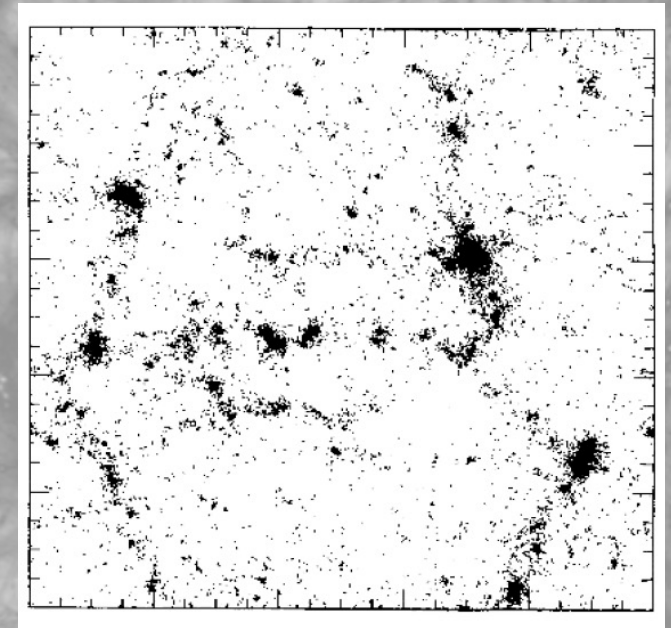
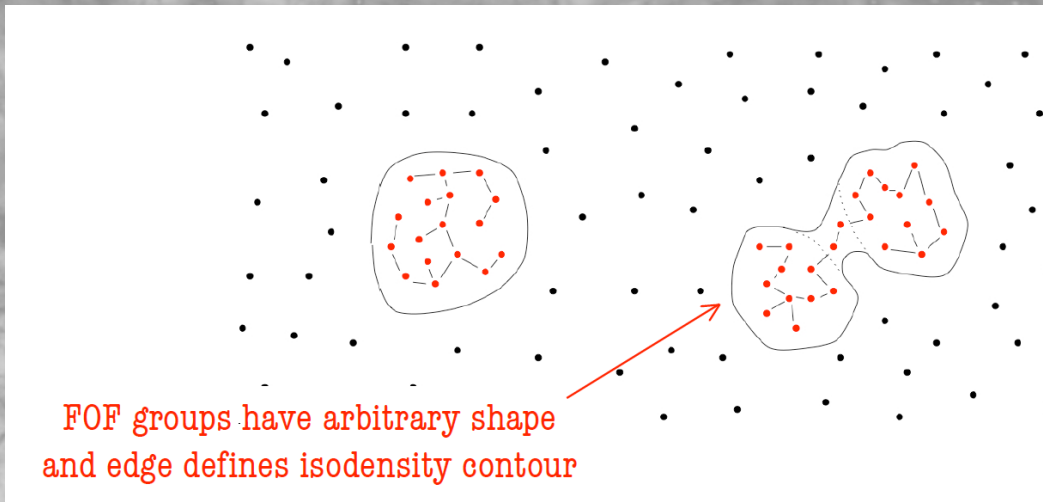


What is the distribution of these masses?

"Friends-of-friends" grouping:
Group together all particles that obey

$$\boxed{\boxed{|\vec{r}_i - \vec{r}_j| \leq b\bar{d}}}$$

$$\bar{d} = \frac{B}{\sqrt[3]{N}}$$



$b=0.2 \rightarrow$ isodensities ~ 200 mean

$$\frac{dn}{dM} dM = \sqrt{\frac{2}{\pi}} \frac{\langle \rho \rangle}{M} \frac{\delta_c}{\sigma_M} \left| \frac{d \ln \sigma_M}{d \ln M} \right| \exp \left(-\frac{\delta_c^2}{2\sigma_M^2} \right) \frac{dM}{M}$$

$\langle \rho \rangle$: mean density of Universe

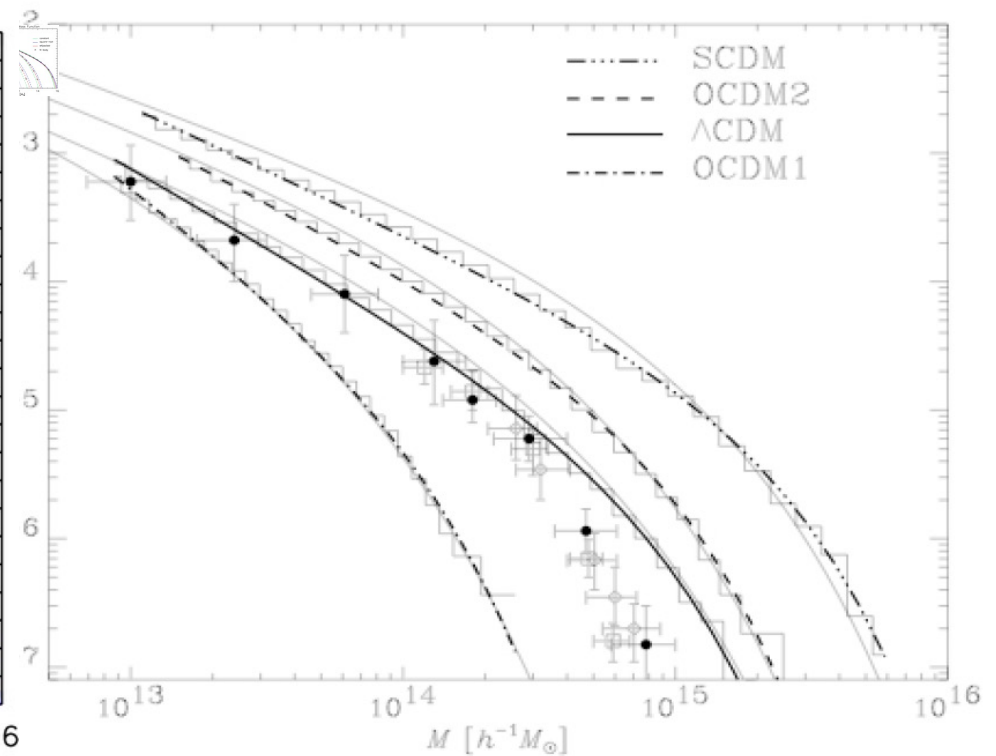
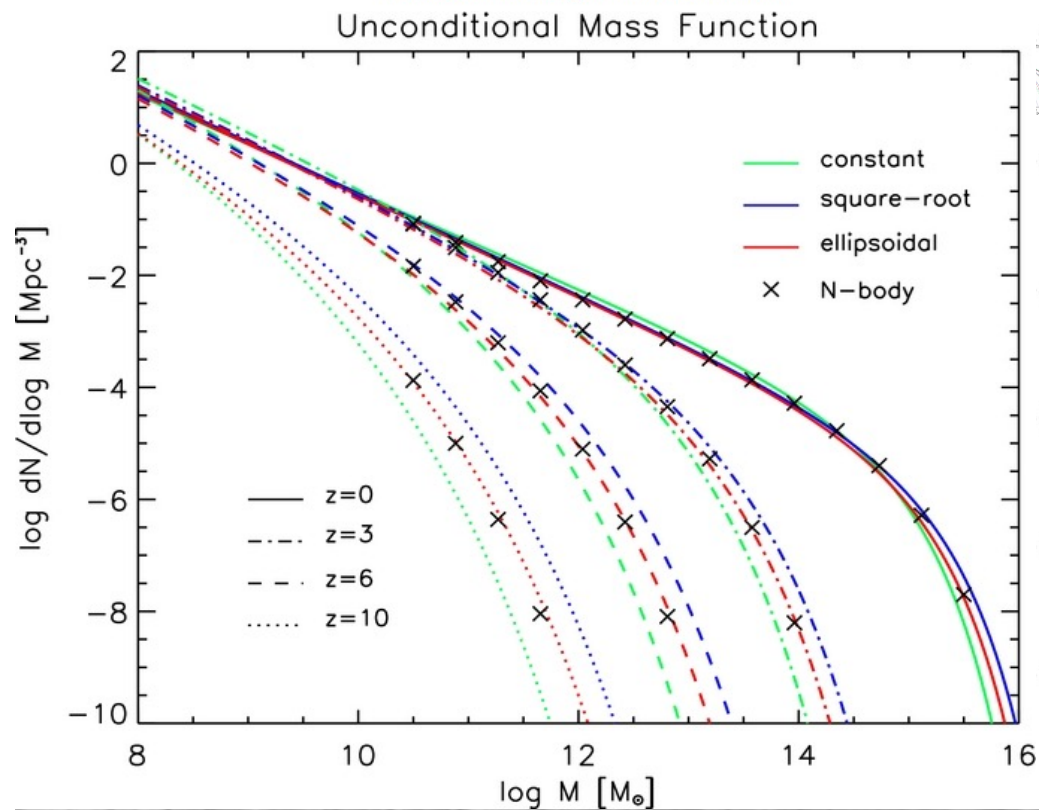
δ_c : density contrast of collapsed structure according to linear perturbation theory

$$\sigma_M^2(r) = \frac{1}{2\pi^2} \int_0^{+\infty} P(k) \hat{W}^2(kr) k^2 dk,$$

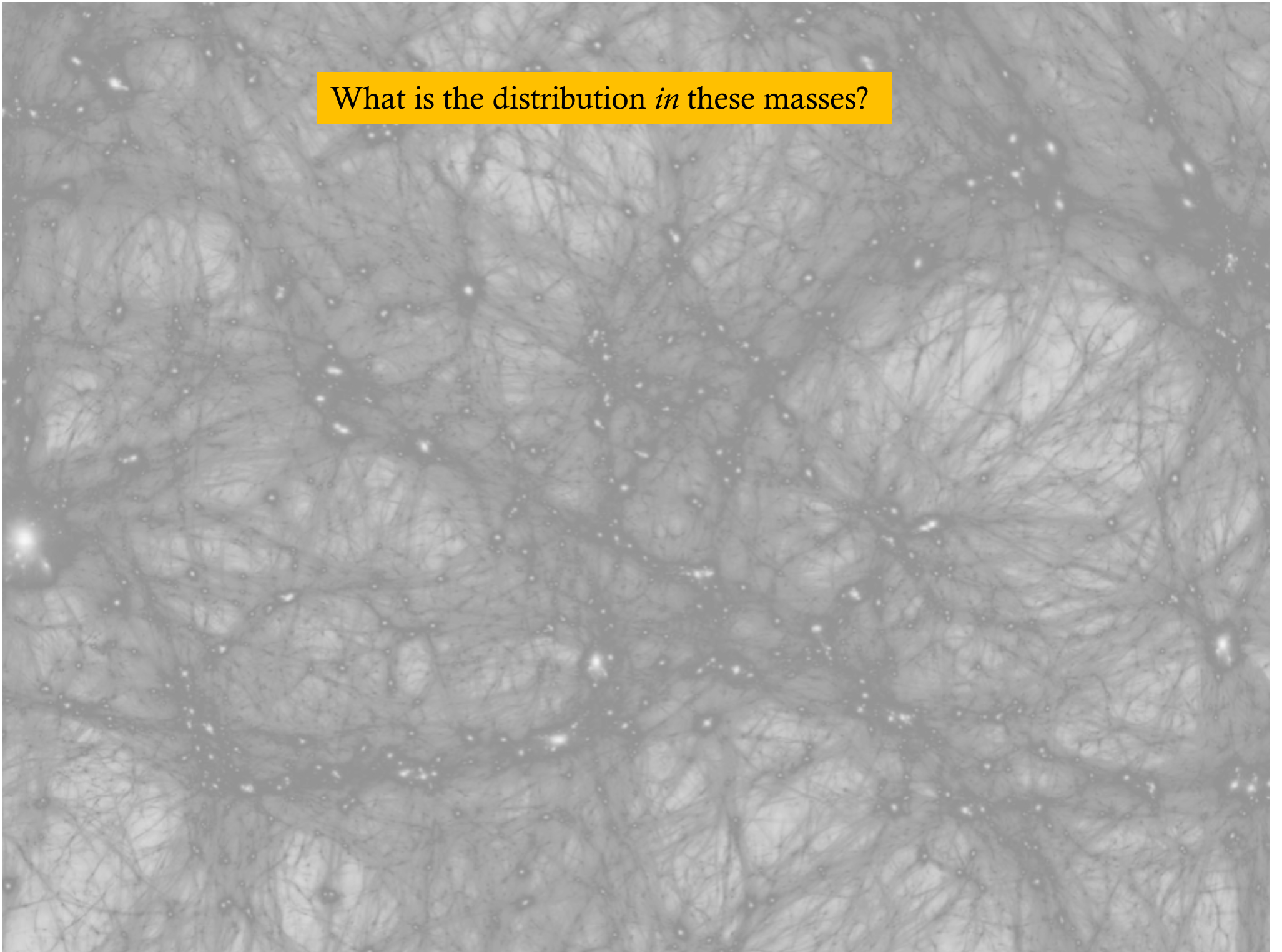
$$\hat{W}(x) = \frac{3}{x^3} (\sin x - x \cos x).$$

power spectrum of density fluctuations (more later...)

(Press)-Shechter function



What is the distribution *in* these masses?



What is the distribution *in* these masses?

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A UNIVERSAL DENSITY PROFILE FROM HIERARCHICAL CLUSTERING

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AND

SIMON D. M. WHITE

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Received 1996 November 13; accepted 1997 July 15

ABSTRACT

We use high-resolution N -body simulations to study the equilibrium density profiles of dark matter halos in hierarchically clustering universes. We find that all such profiles have the same shape, independent of the halo mass, the initial density fluctuation spectrum, and the values of the cosmological parameters. Spherically averaged equilibrium profiles are well fitted over two decades in radius by a simple formula originally proposed to describe the structure of galaxy clusters in a cold dark matter universe. In any particular cosmology, the two scale parameters of the fit, the halo mass and its characteristic density, are strongly correlated. Low-mass halos are significantly denser than more massive systems, a correlation that reflects the higher collapse redshift of small halos. The characteristic density of an equilibrium halo is proportional to the density of the universe at the time it was assembled. A suitable definition of this assembly time allows the same proportionality constant to be used for all the cosmologies that we have tested. We compare our results with previous work on halo density profiles and show that there is good agreement. We also provide a step-by-step analytic procedure, based on the Press-Schechter formalism, that allows accurate equilibrium profiles to be calculated as a function of mass in any hierarchical model.

Subject headings: cosmology: theory — dark matter — galaxies: halos — methods: numerical

What is the distribution *in* these masses?

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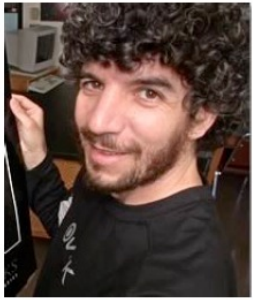
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Julio Navarro



Carlos Frenk

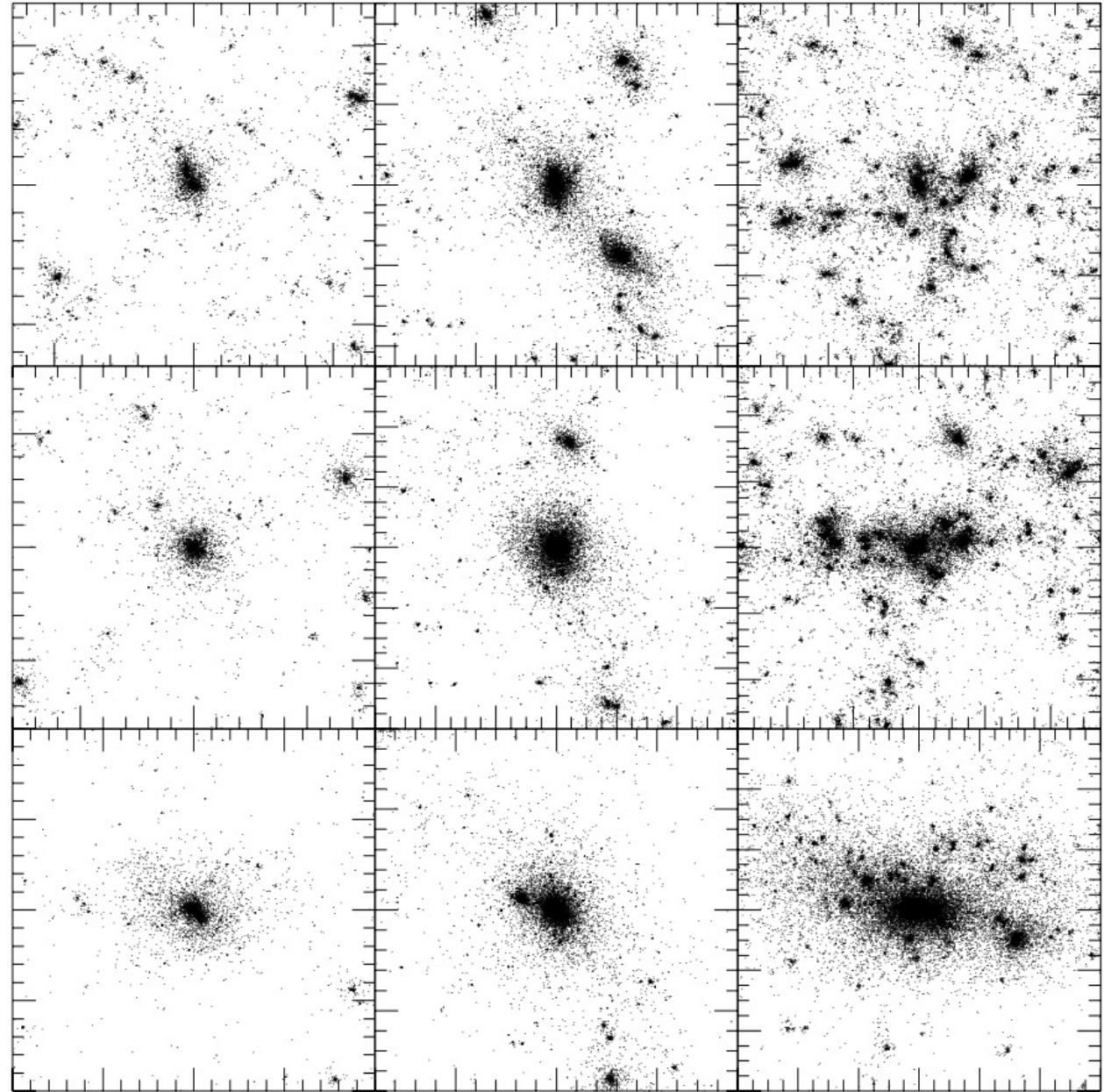


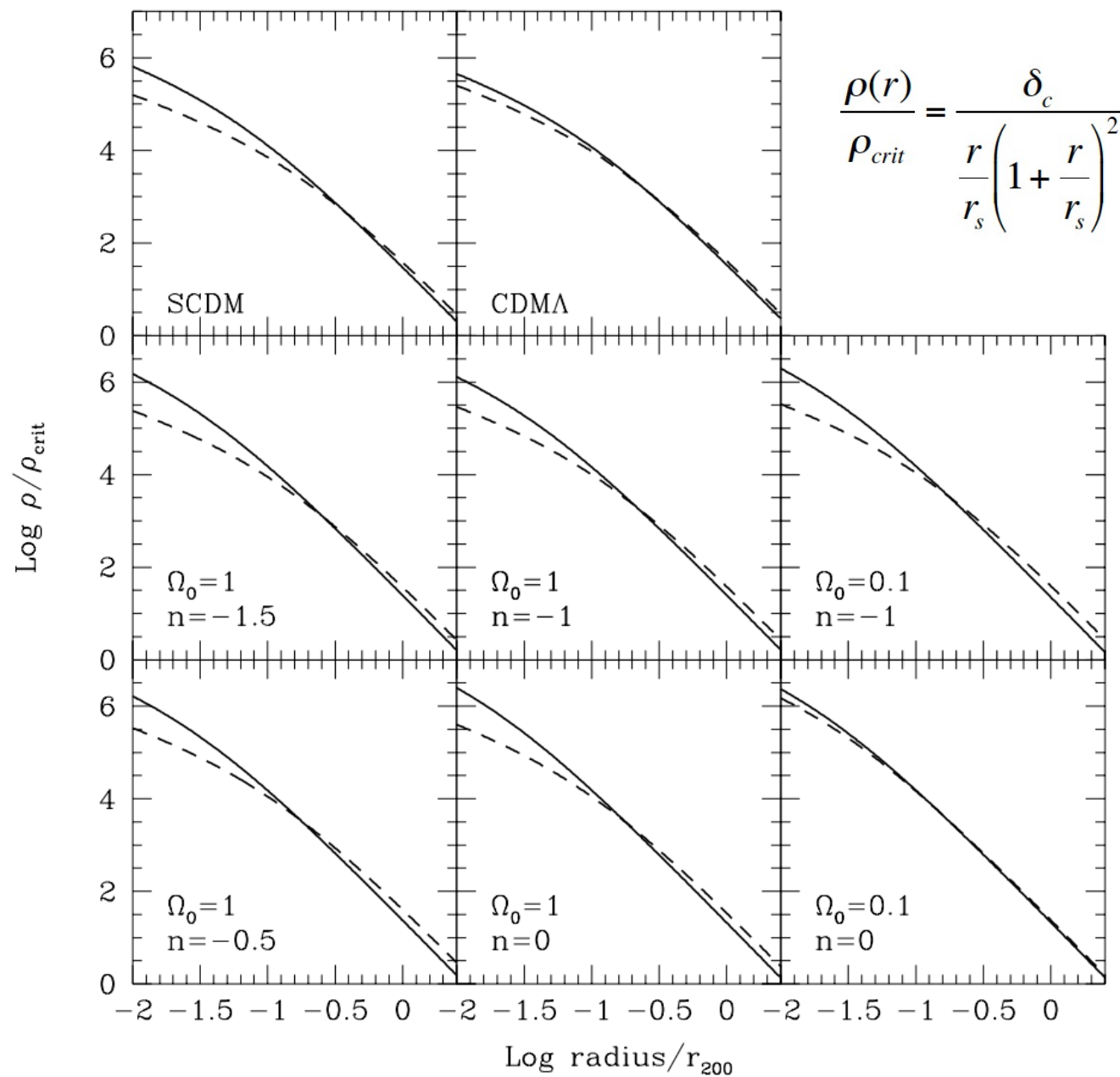
Simon White

z_2

z_1

z_0





But how can one even approach the idea of simulating the entire universe?

First: make some simplifying assumptions

1. Everything just gravitates
2. N-body simulations can model this
3. Identify halos (FOF)
 - a) Spatial distributions of haloes matches the 2PC of galaxies
 - b) Abundance of haloes of a given mass depends on power spectrum of fluctuation
 - c) Prediction for the density profile is universal (depends only on the gaussian nature of the initial perturbations)

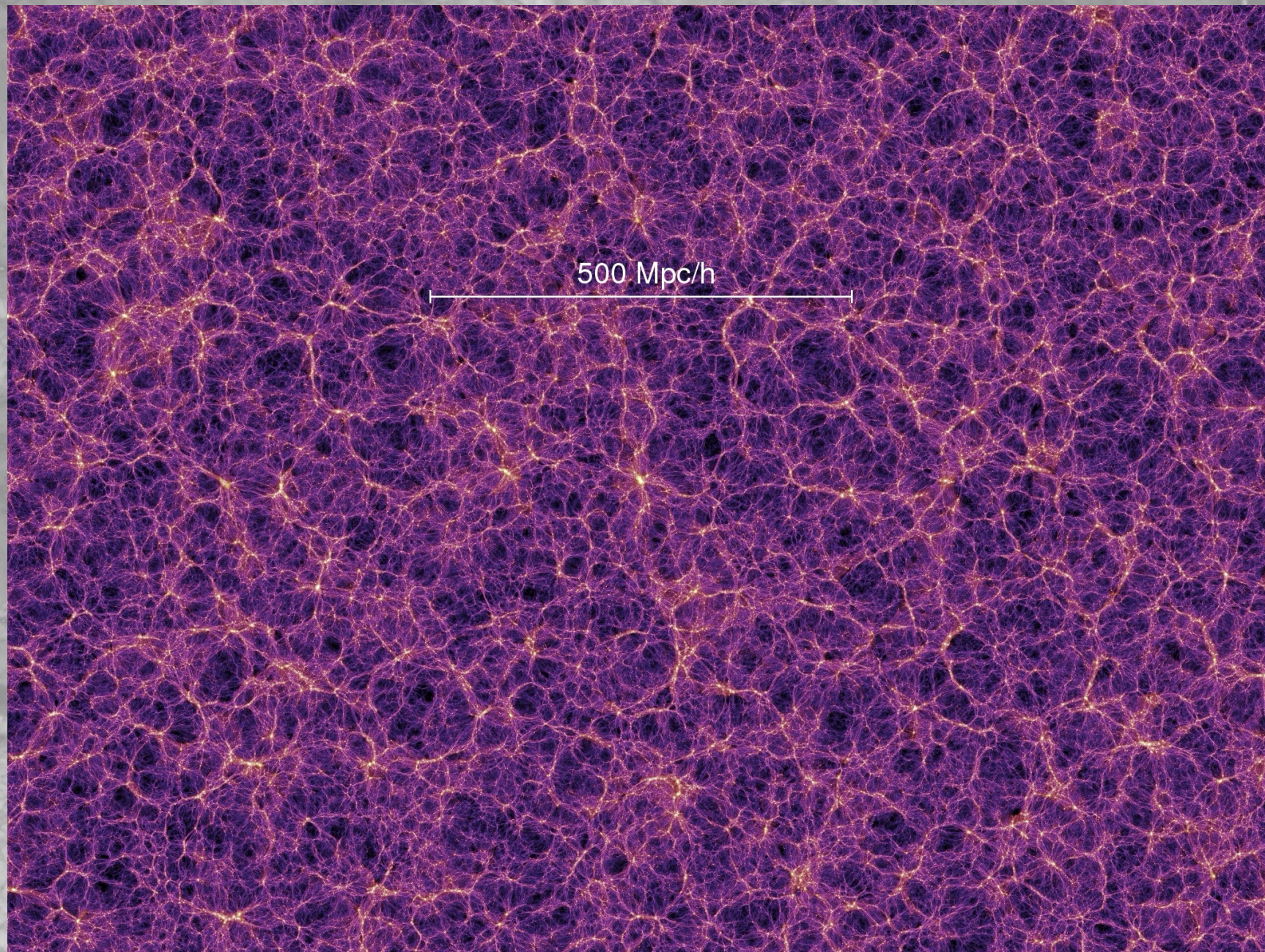
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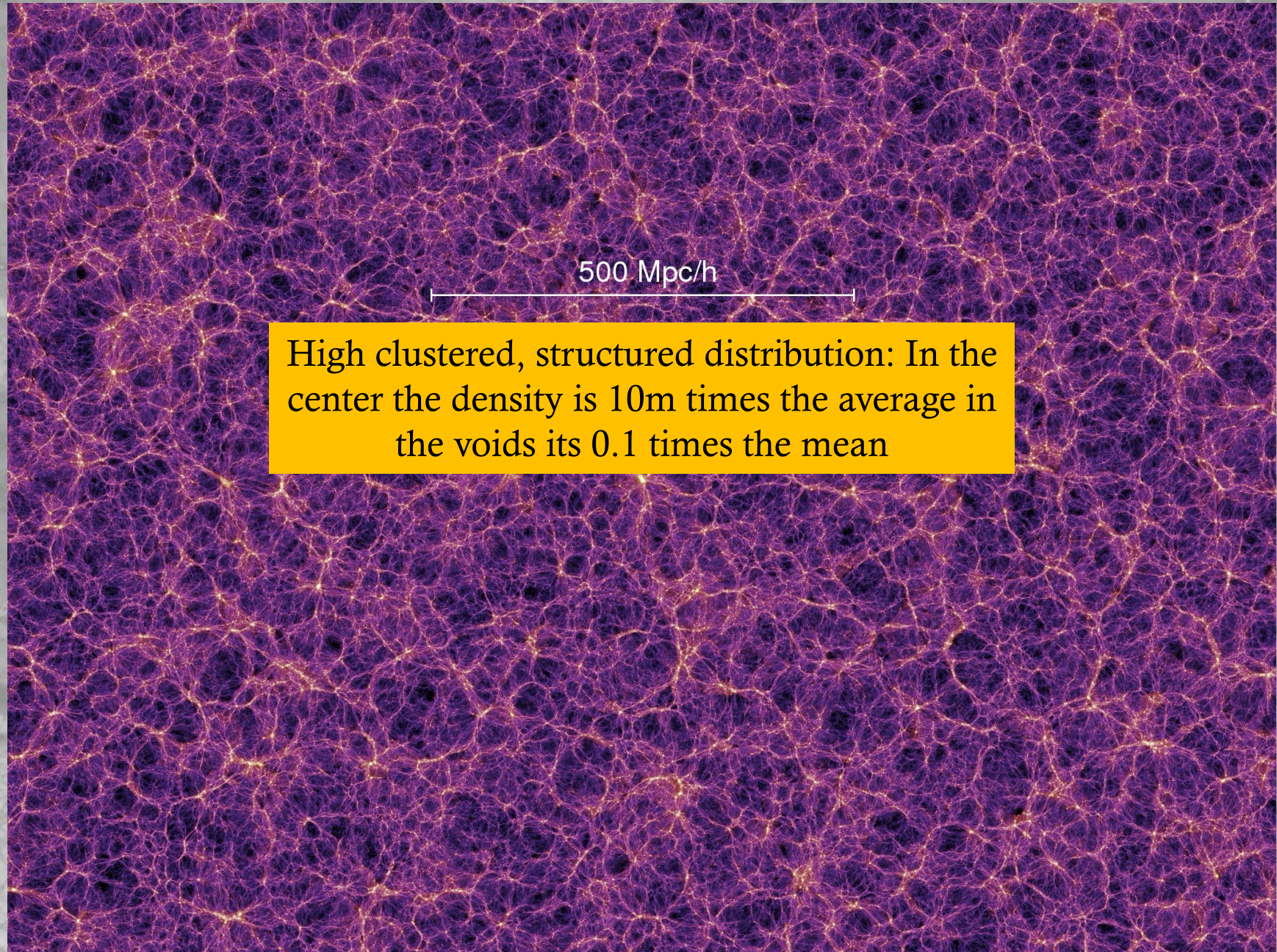
Second: use a huge computer



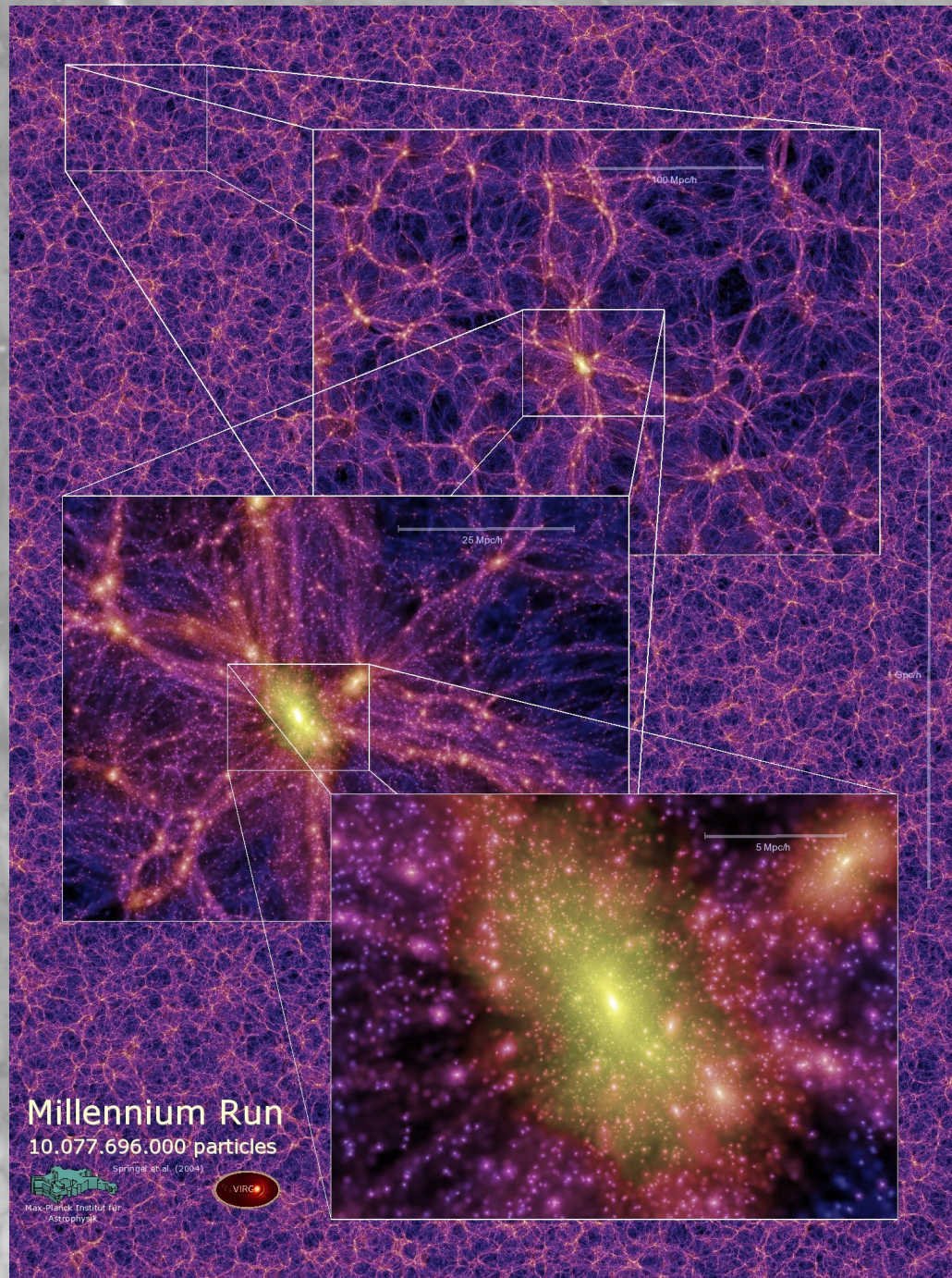
Large N -body cosmological simulations



Large N -body cosmological simulations



Large N -body cosmological simulations



We can use these simulations to
examine basic aspects of the
“collapsed” objects

Abundance of haloes as a function
of mass and time

Spatial distribution

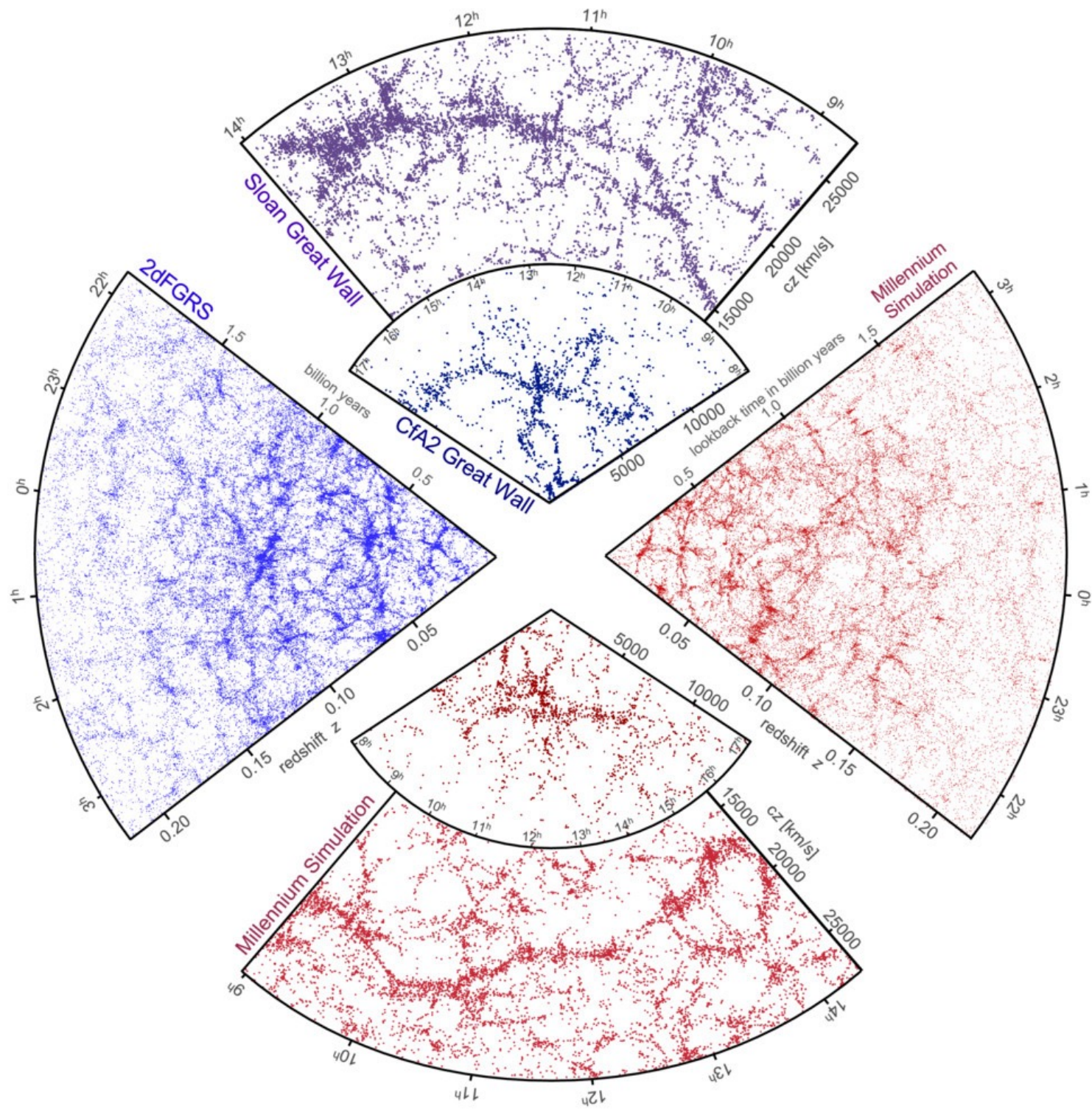
Internal structure (density prof)

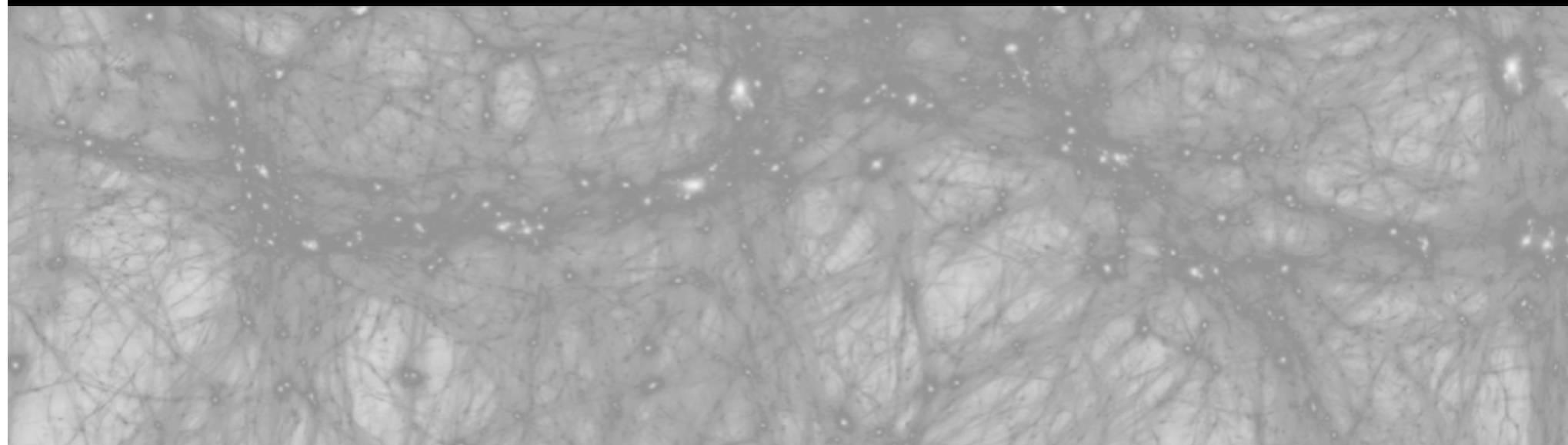
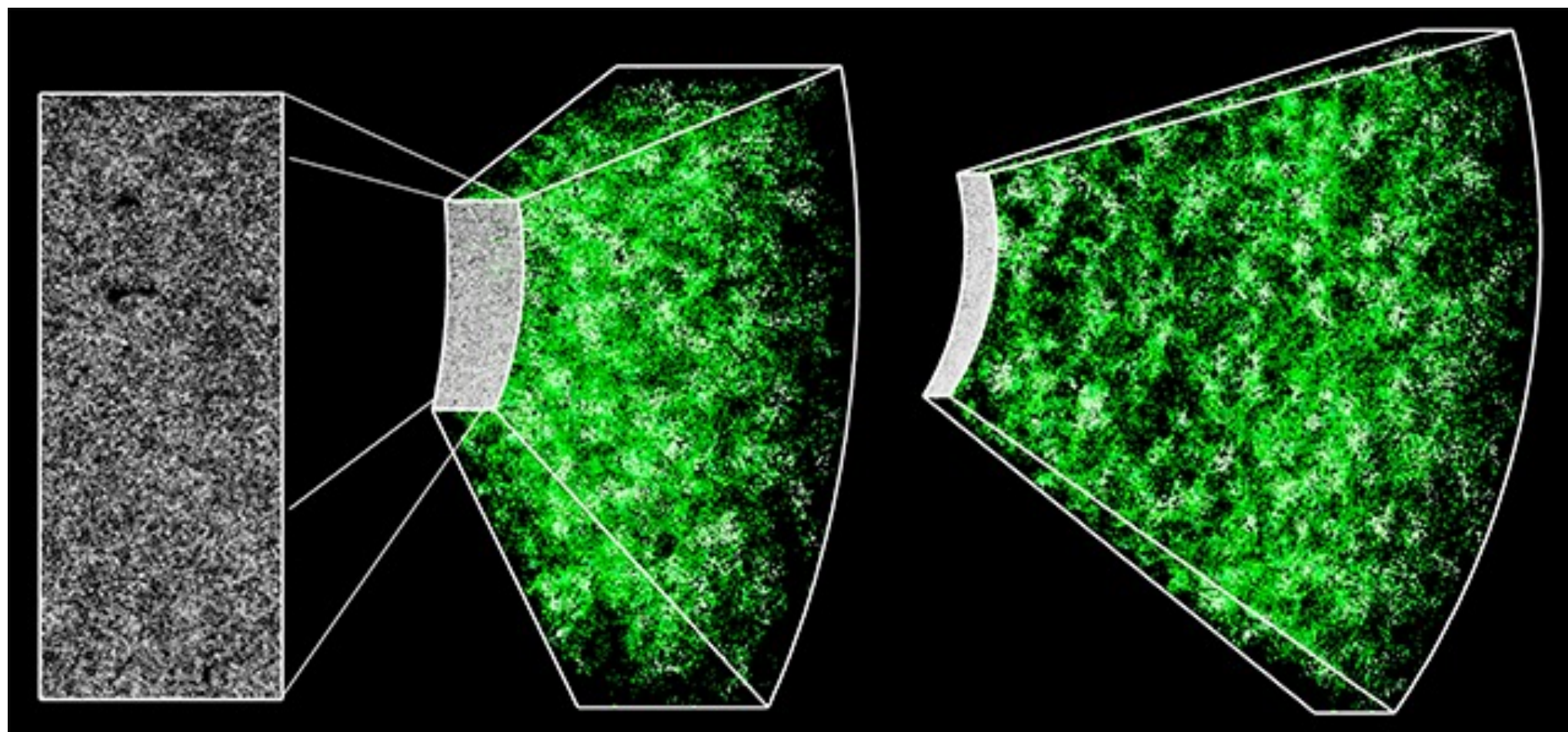
Sub structure

Merger rates as function of mass and
time

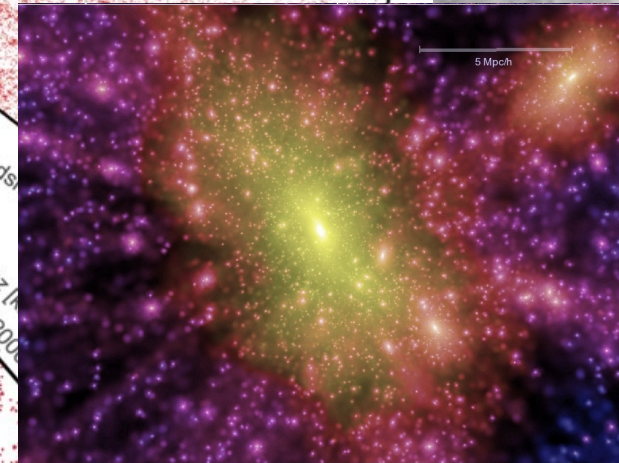
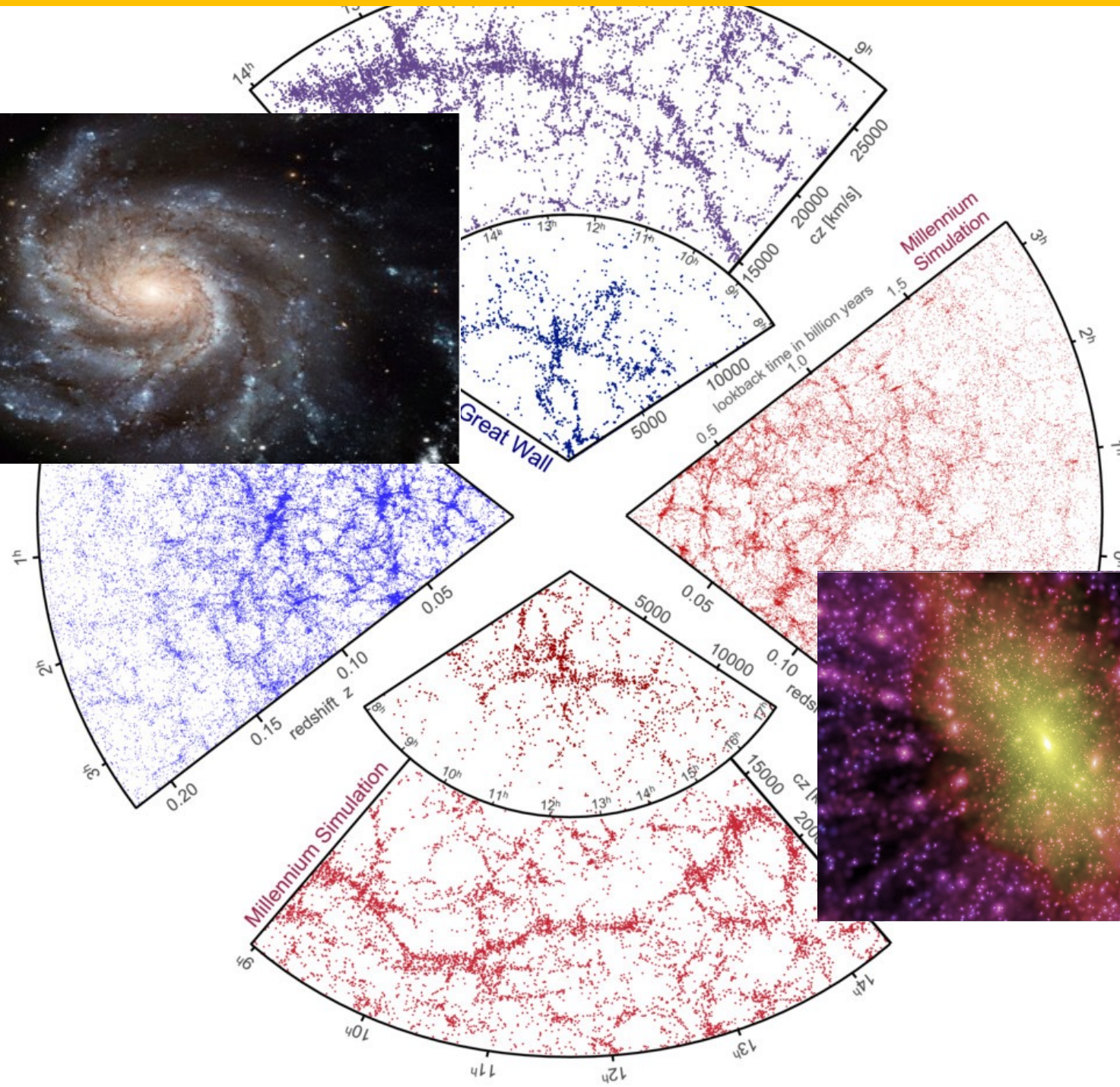
Formation epochs

Formation histories

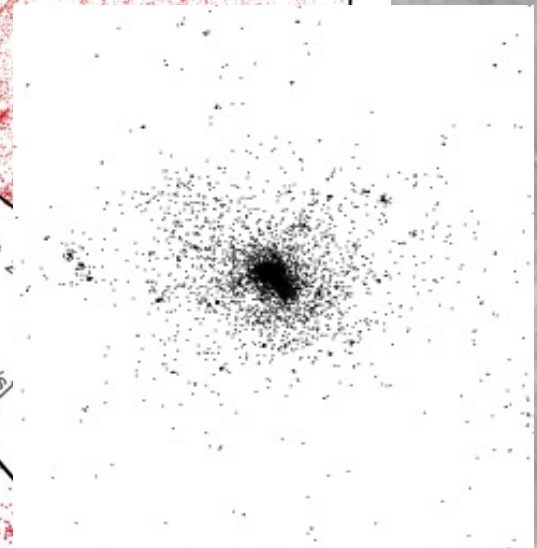
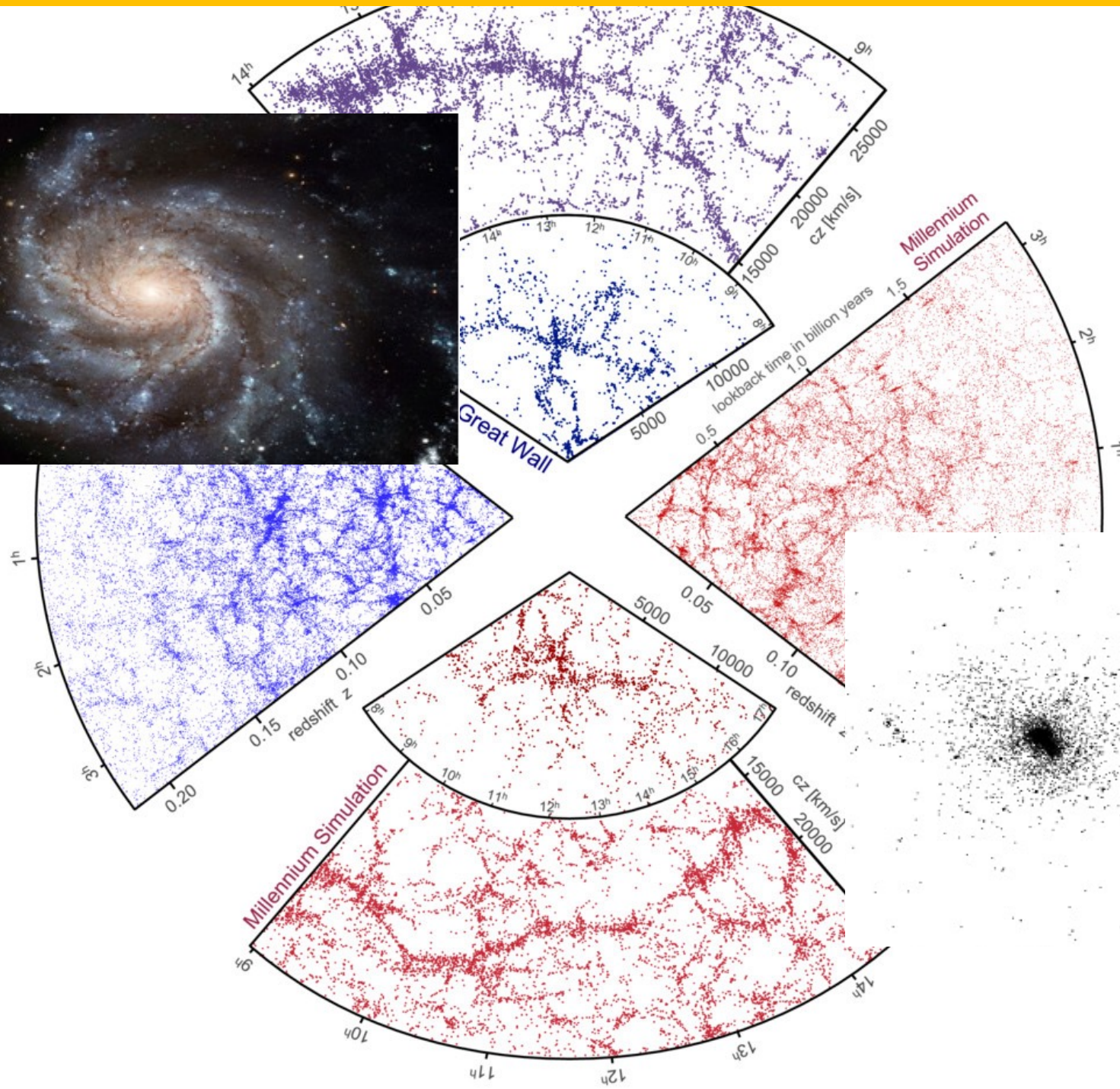




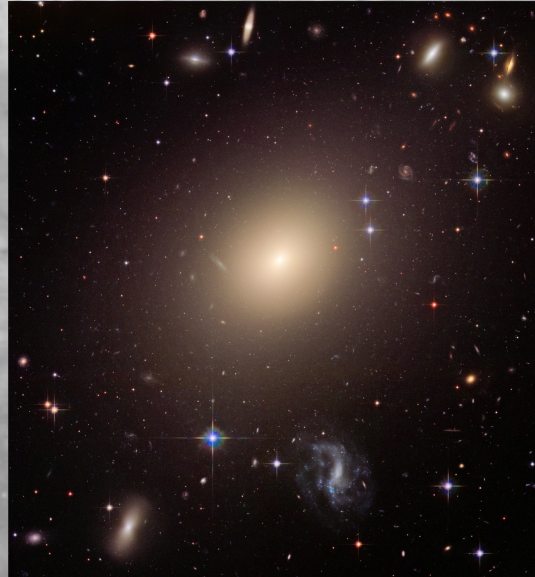
But where are the galaxies?



But where are the galaxies?

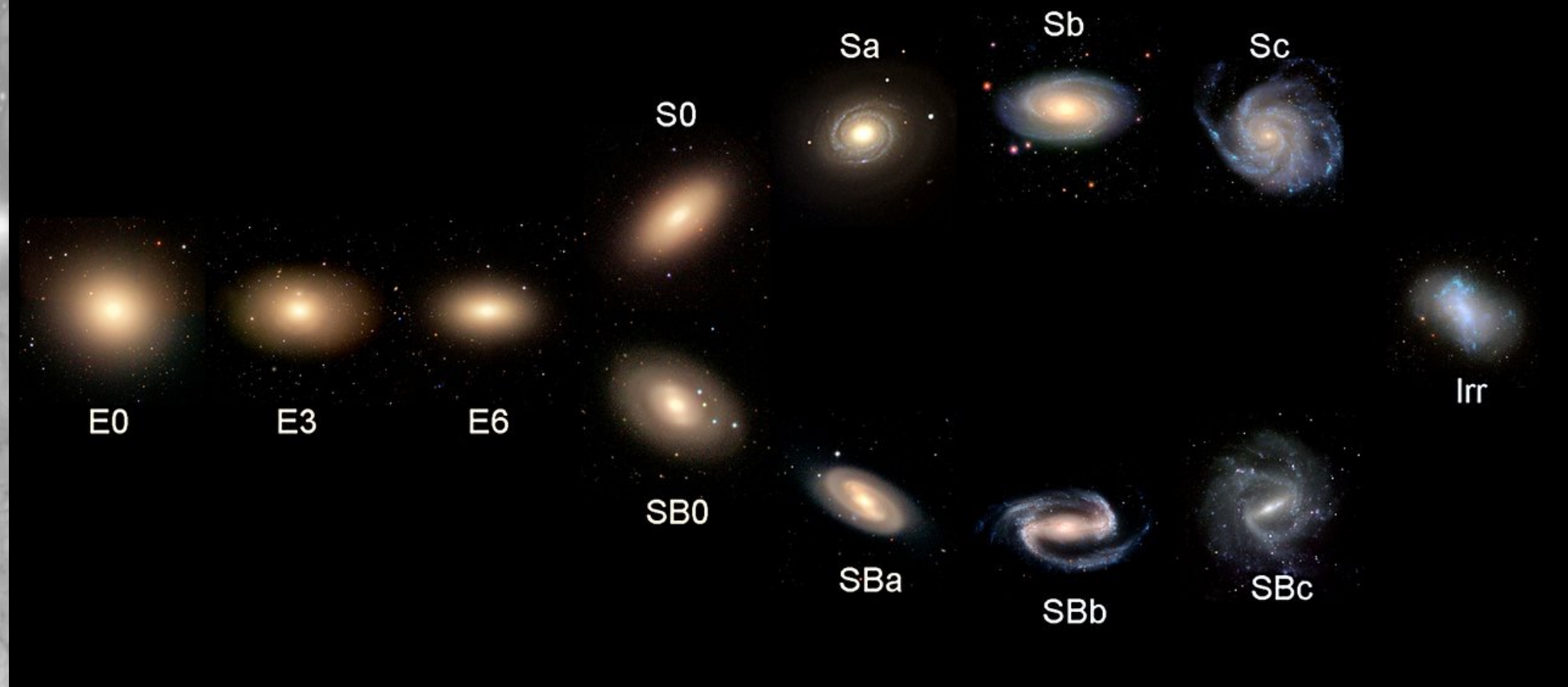


Enormous diversity in the galaxy population



Well known to astronomers for at least a century

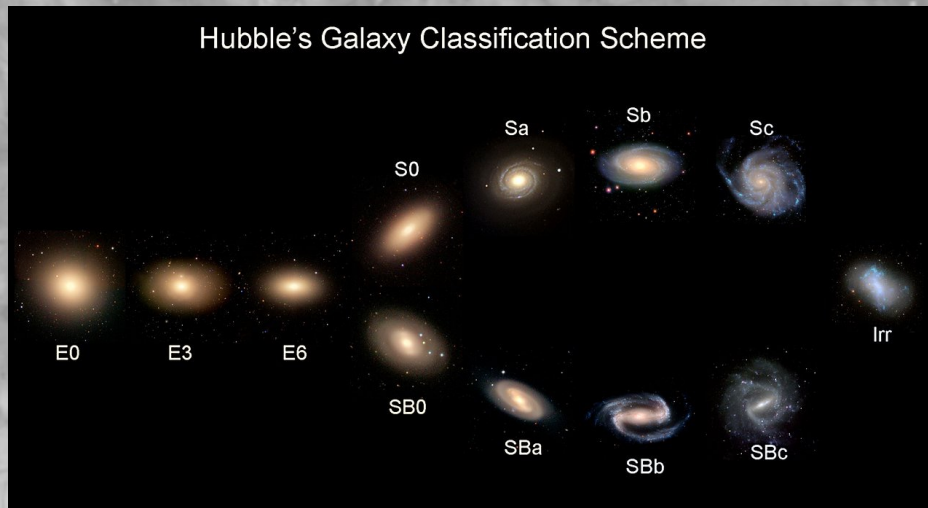
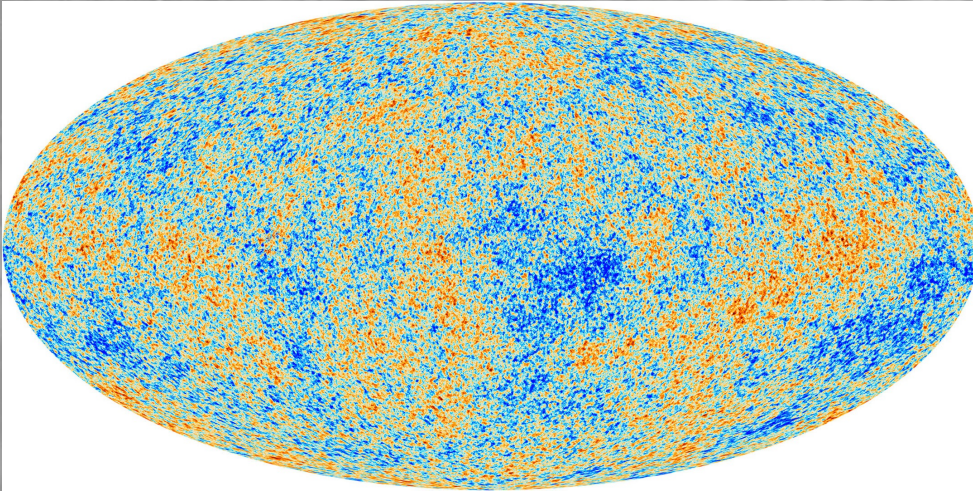
Hubble's Galaxy Classification Scheme



The central questions:

The central questions of cosmology:

*Given the initial conditions from the CMB,
how did structure in the universe form?*



The central questions:

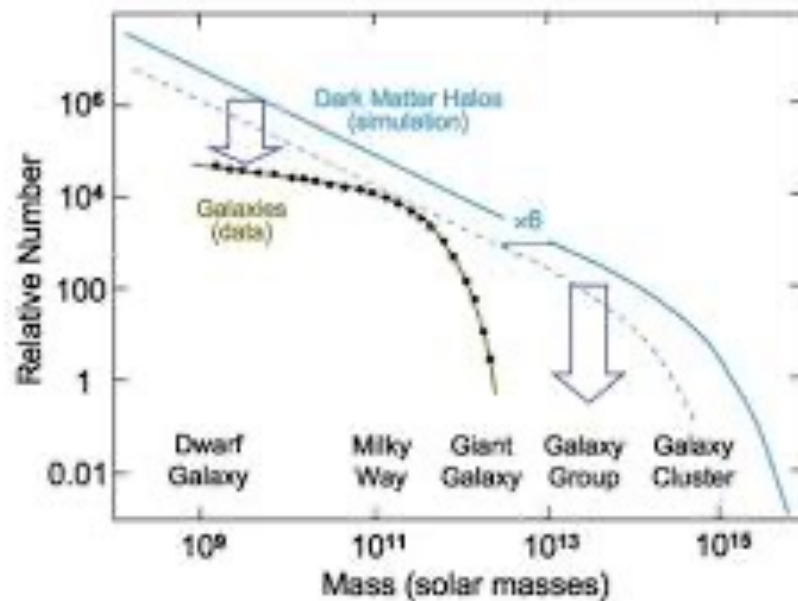
The central questions of cosmology:

Given the initial conditions from the CMB, how did structure in the universe form?

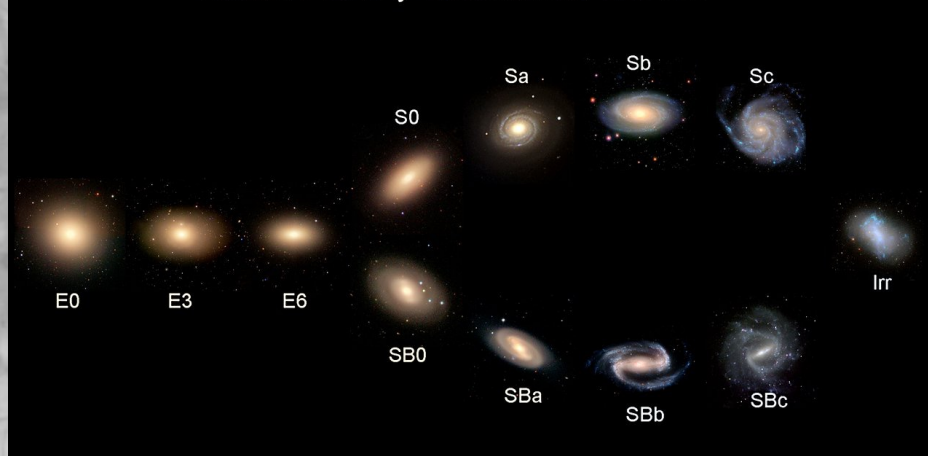
The Central question of galaxy formation

How do you turn the halo mass function into the galaxy luminosity function?

Halo and Galaxy Mass Distributions

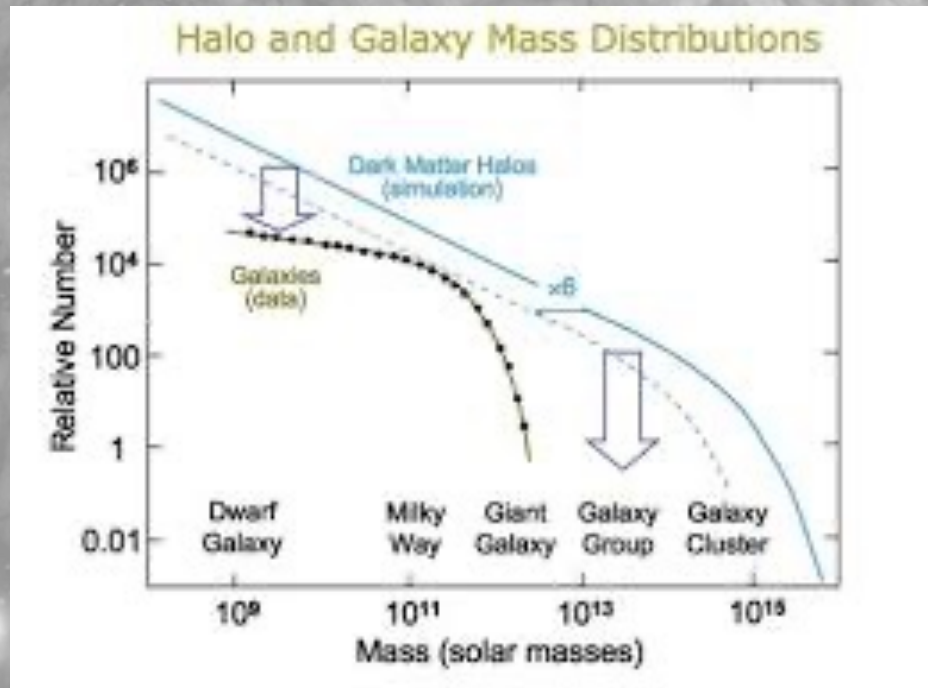


Hubble's Galaxy Classification Scheme



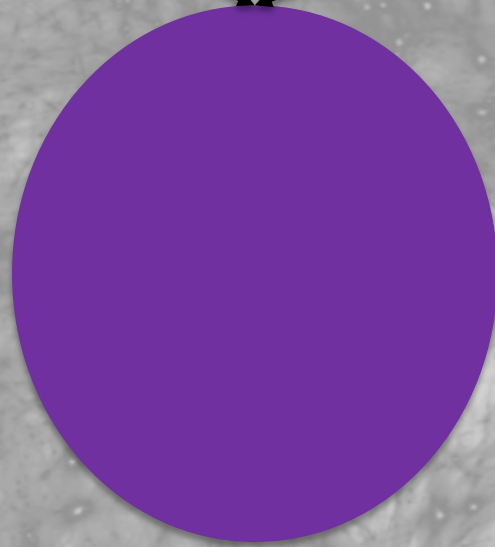
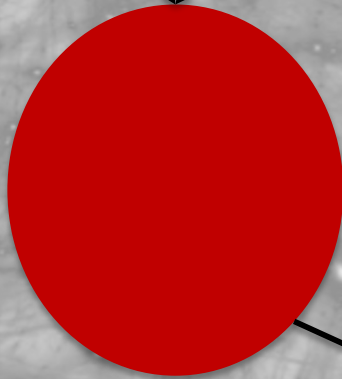
The central questions of cosmology:

N- body simulations allow us to trace the mass accretion history for each object.

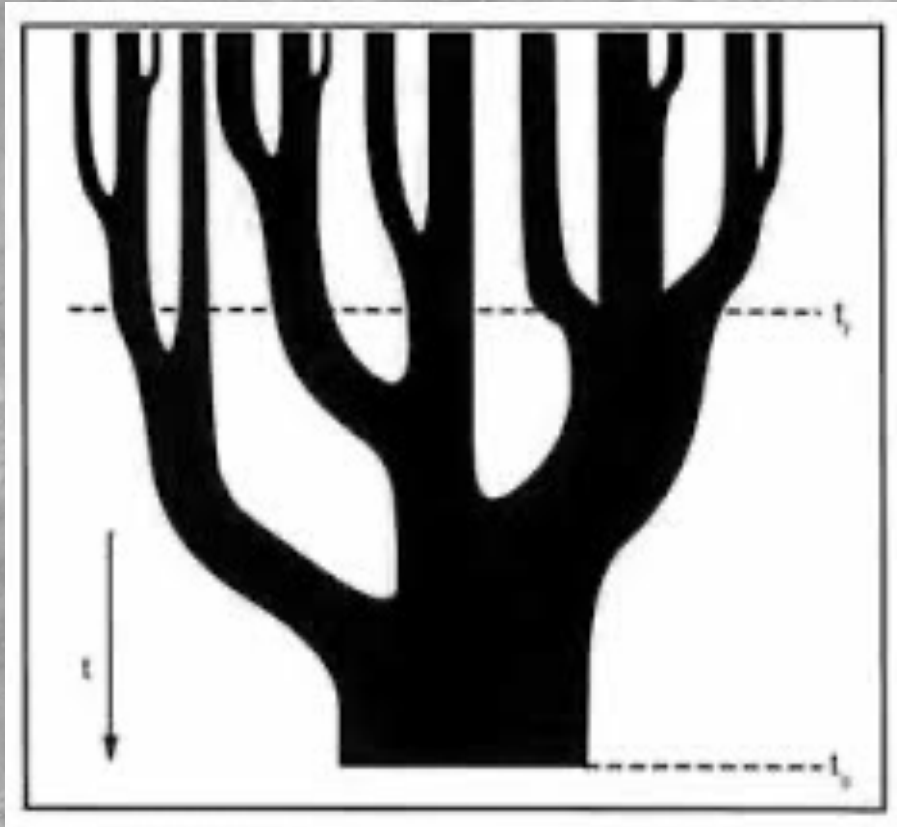




The Hierarchical model – Halo merger tree



The Hierarchical model – Halo merger tree



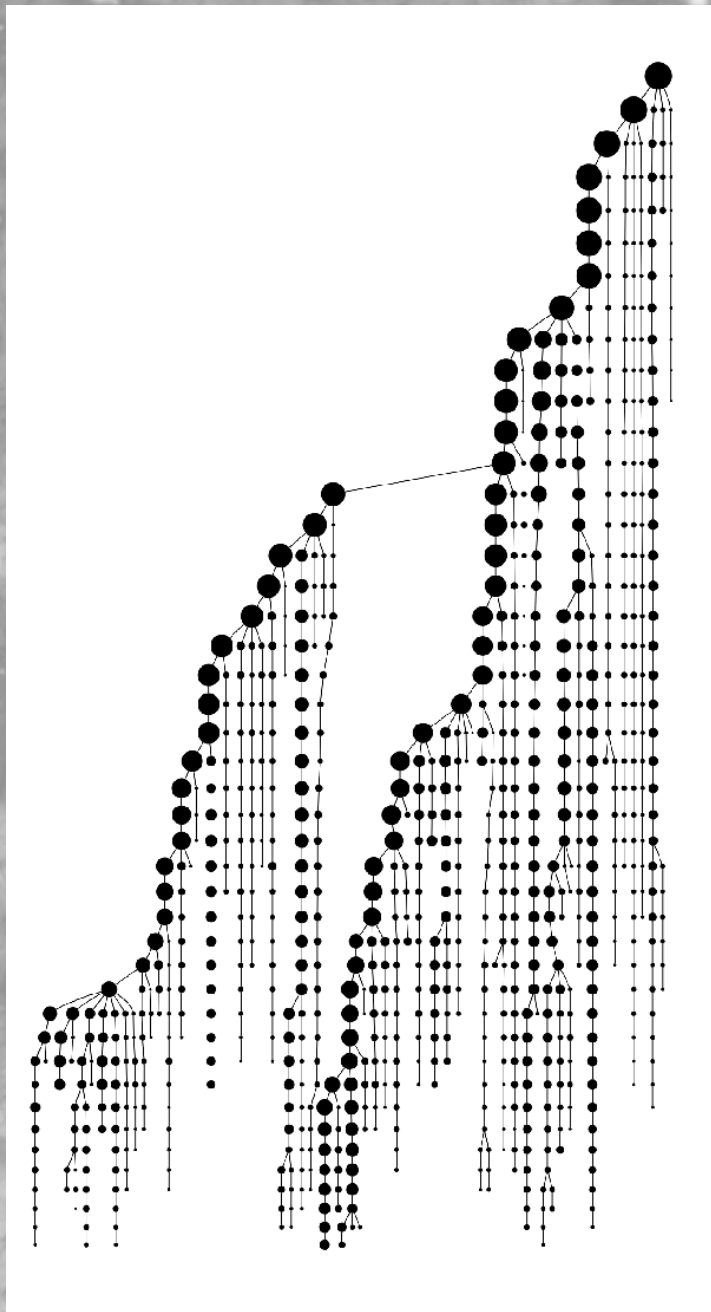
Cole et al 2000

Λ CDM is a model of mergers

First small things form
("dwarfs") which then merge to
create larger and larger objects
("clusters")

Clusters are dynamically
"young". (Yet they typically
have the reddest deadest
galaxies – cosmic downsizing)

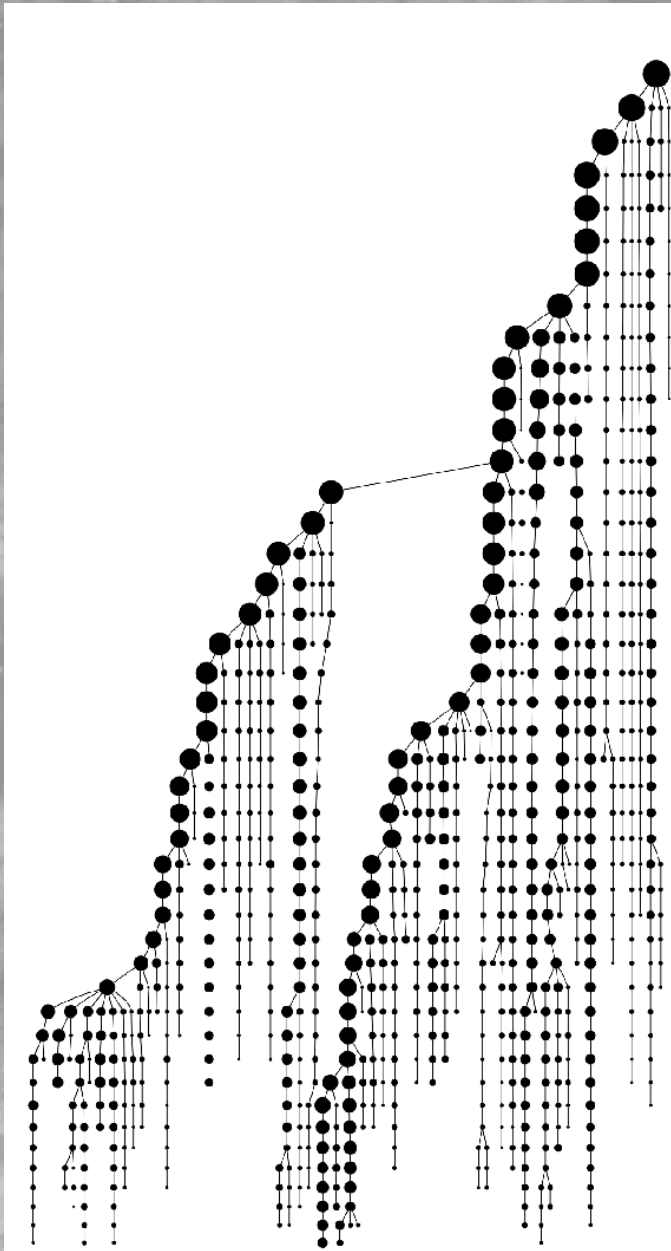
The Hierarchical model – Halo merger tree



time

Lacey & Cole 1993 described this analytically

The Hierarchical model – Halo merger tree



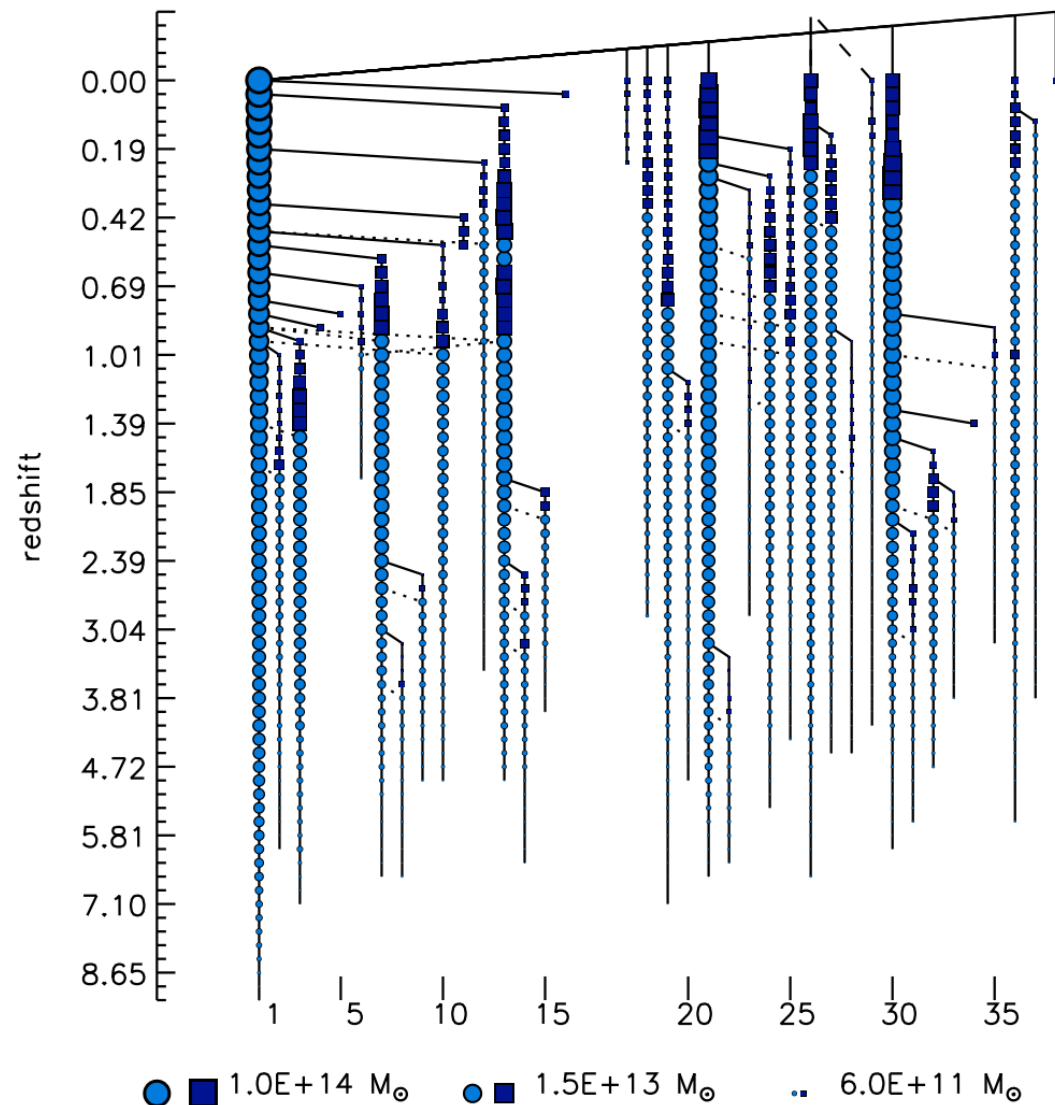
time

Lacey & Cole 1993 described this analytically

$$\begin{aligned}
 & \frac{d^2 p}{d \ln \Delta M dt} (M_1 \rightarrow M_2 | t) \\
 &= 2 \sigma(M_2) \left| \frac{d \sigma_2}{d M_2} \right| \Delta M \left| \frac{d \omega}{dt} \right| \frac{d^2 p}{d S_2 d \omega} (S_1 \rightarrow S_2 | \omega) \\
 &= \left(\frac{2}{\pi} \right)^{1/2} \frac{1}{t} \left| \frac{d \ln \delta_c}{d \ln t} \right| \left(\frac{\Delta M}{M_2} \right) \times \left| \frac{d \ln \sigma_2}{d \ln M_2} \right| \frac{\delta_c(t)}{\sigma_2} \frac{1}{(1 - \sigma_2^2 / \sigma_1^2)^{3/2}} \\
 &\times \exp \left[- \frac{\delta_c(t)^2}{2} \left(\frac{1}{\sigma_2^2} - \frac{1}{\sigma_1^2} \right) \right],
 \end{aligned} \tag{2.18}$$

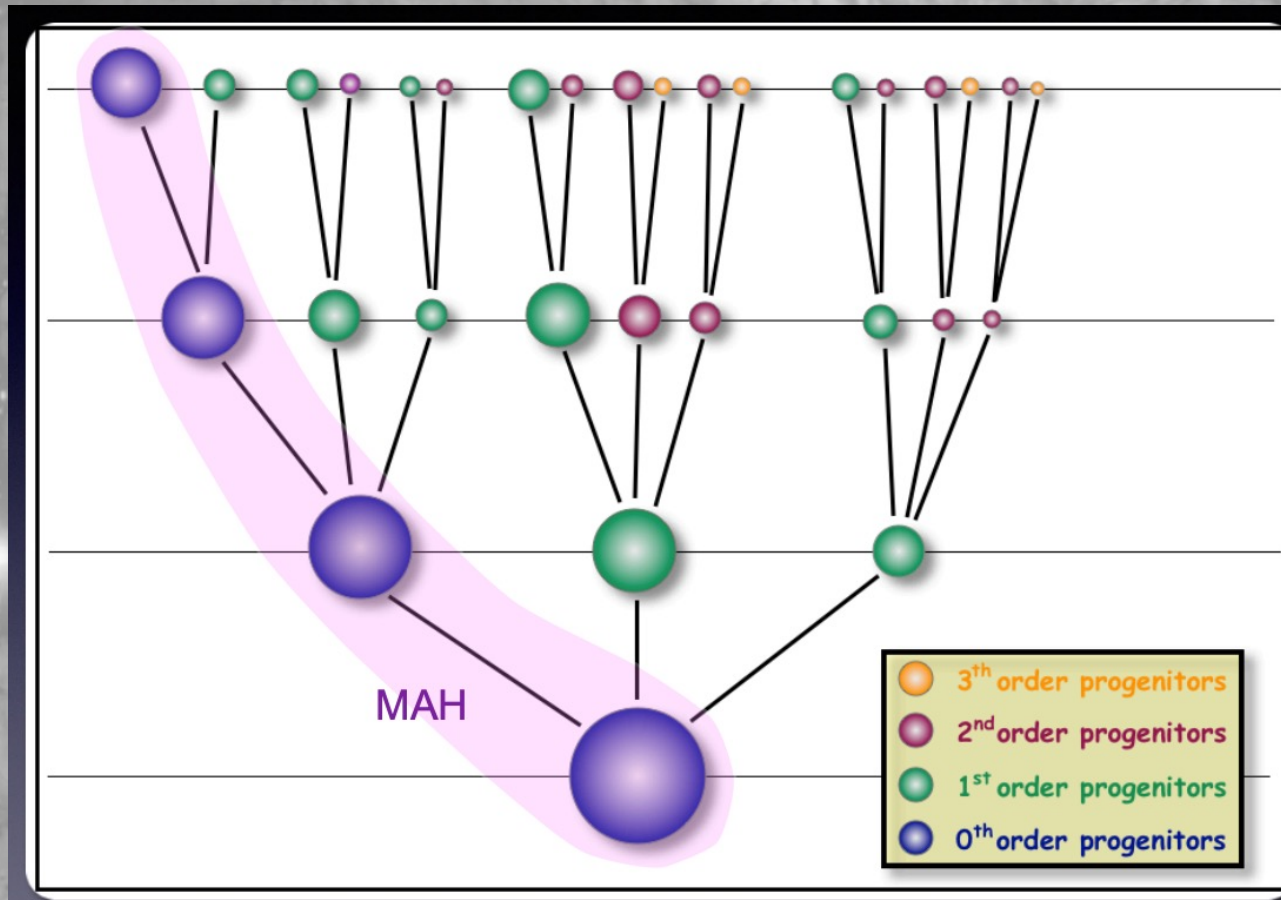
Depends on the power spectrum of perturbations (of course!) the matter content (Ω), the scale ...

The Hierarchical model – Halo merger tree



We can extract the halo merger trees from the simulations by linking haloes at one snap shot (via the identity of the particles in it) to its “progenitor” at earlier times

The Hierarchical model – Halo merger tree

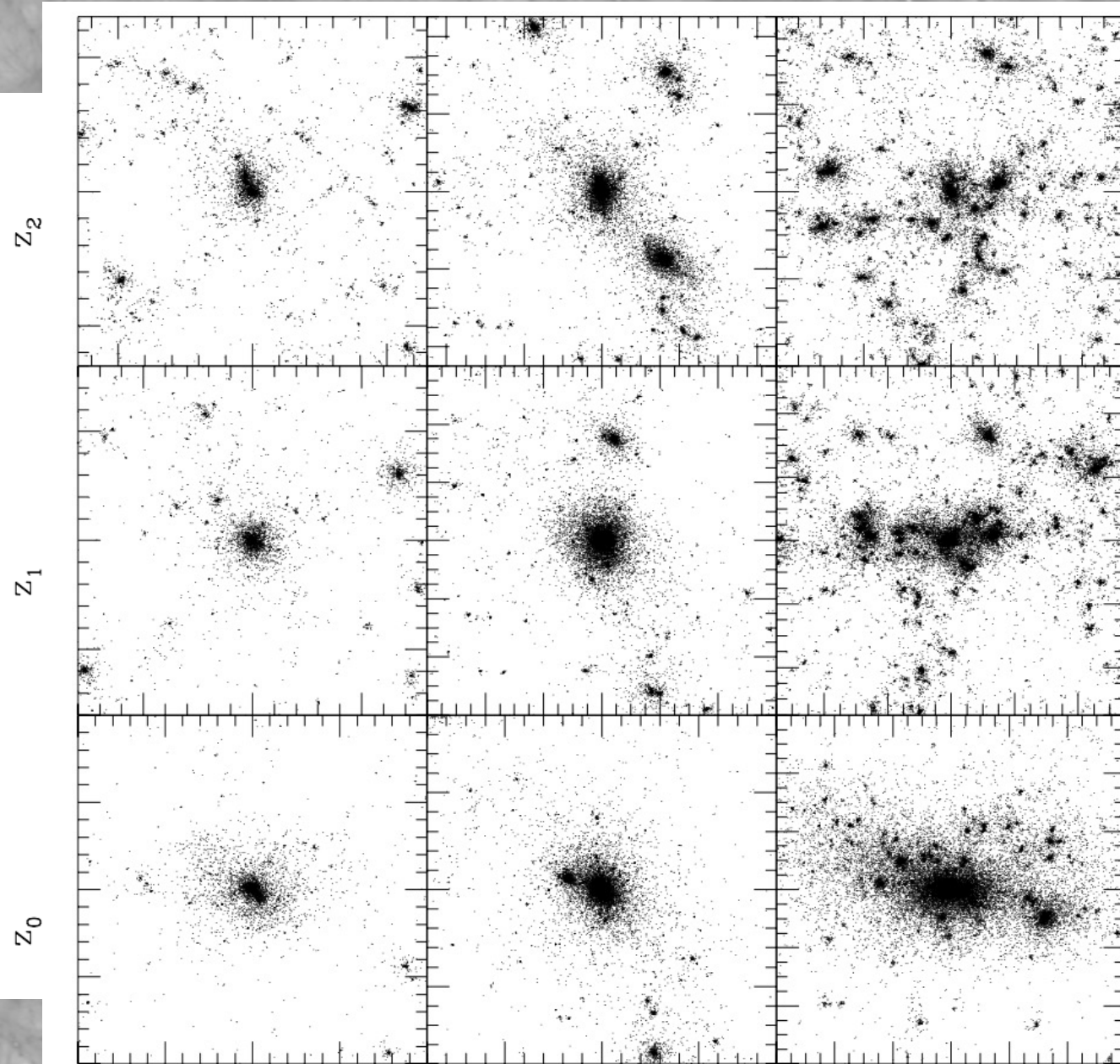


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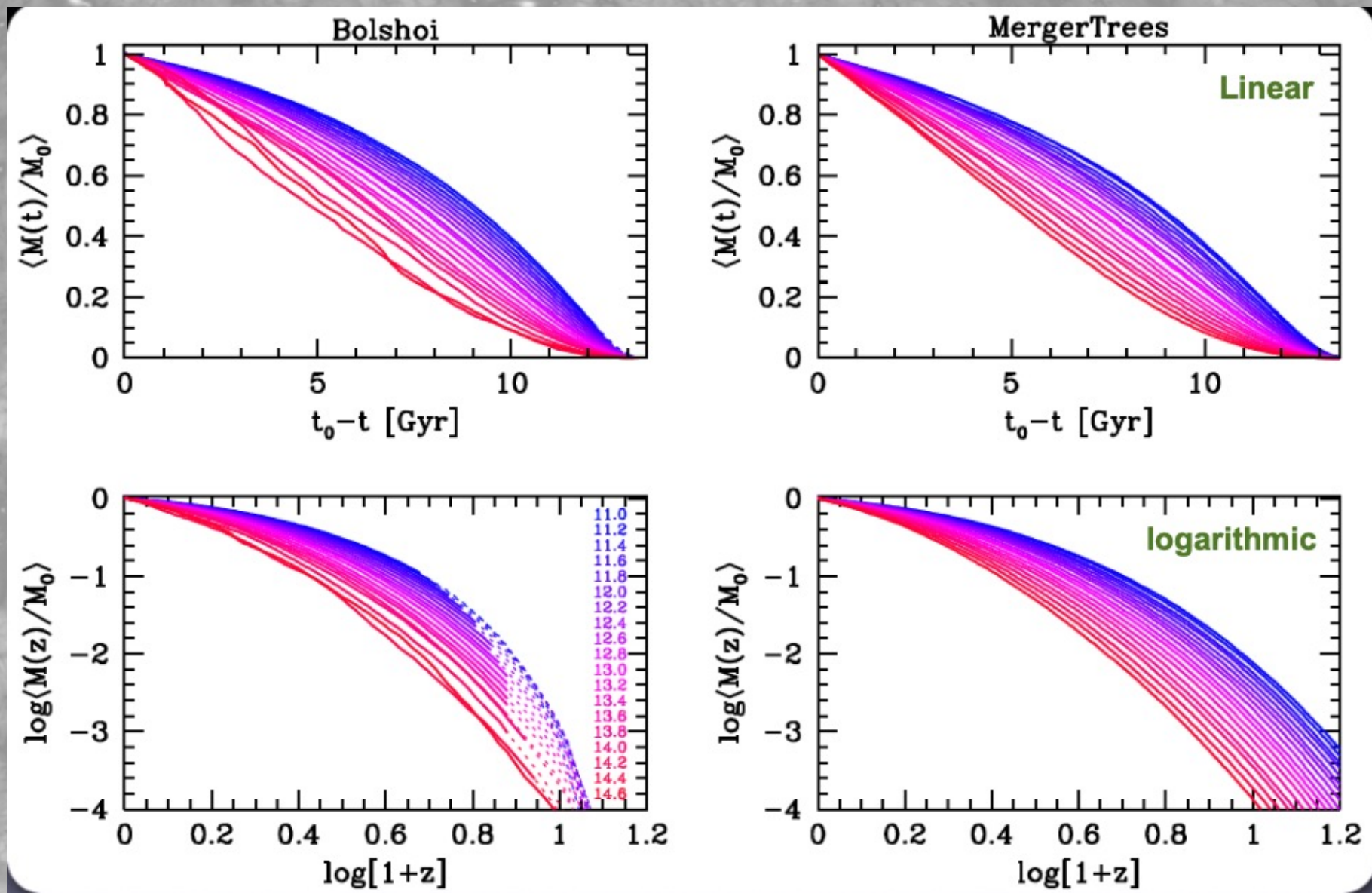
The mass accretion history of a halo is the 0th order progenitor

Basic prediction:

If objects get
bigger over time,
large things are
(dynamically)
younger – they
form later



The Hierarchical model – Halo merger tree

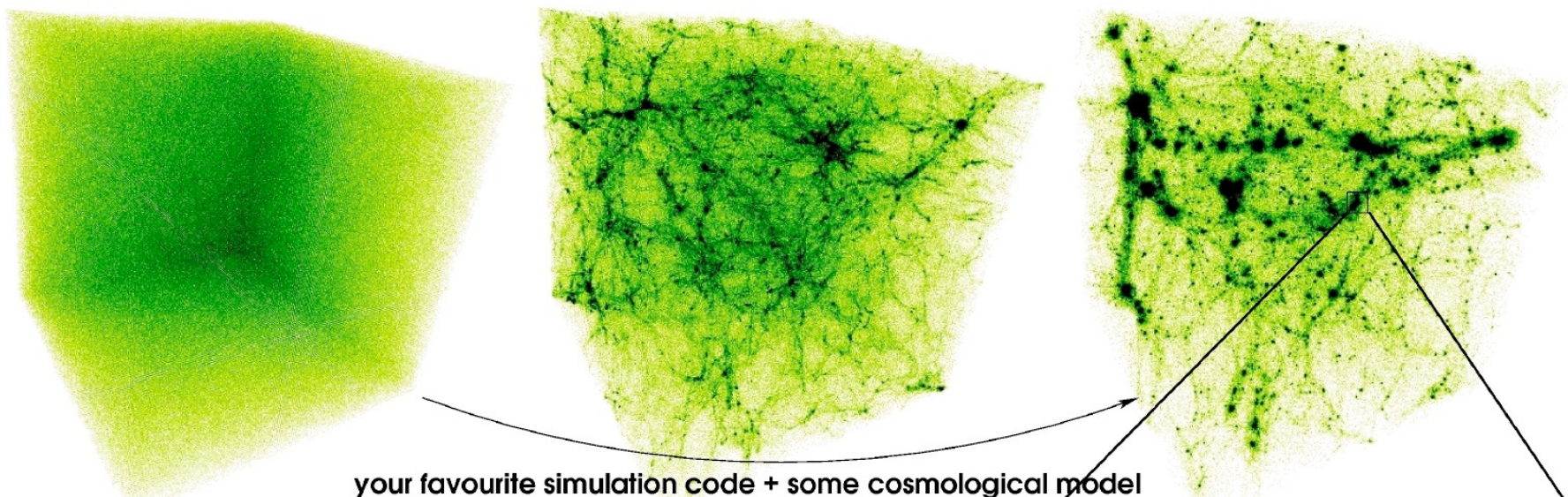


Hierarchical growth – big things form later

Semi-analytical modelling

Assuming a cosmology (power spectrum + parameters), we know “everything” namely the halo mass function (at any z) and the merger history.

We can “paint” the galaxies into the haloes by making physically motivated assumptions about how gas behaves



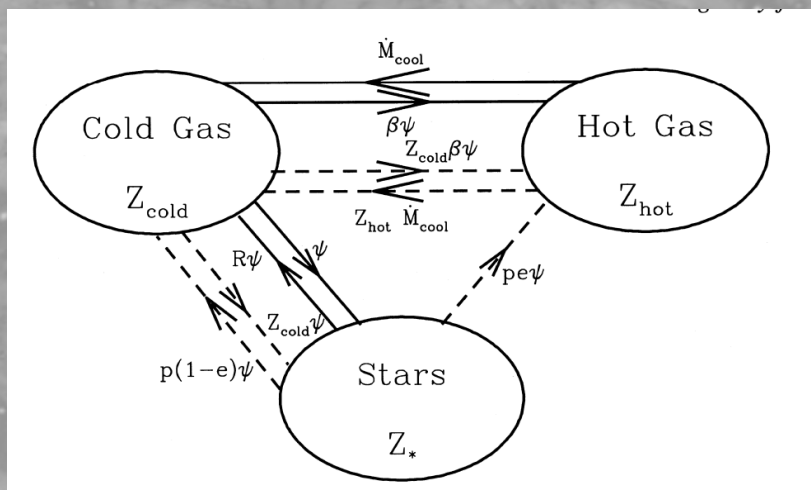
Hierarchical galaxy formation

Shaun Cole,^{1★} Cedric G. Lacey,^{1,2,3★} Carlton M. Baugh^{1★} and Carlos S. Frenk^{1★}

¹*Department of Physics, University of Durham, Science Laboratories, South Road, Durham DH1 3LE*

²*Theoretical Astrophysics Center, Juliane Maries Vej 30, DK-2100 Copenhagen, Denmark*

³*SISSA, via Beirut 2–4, 34014 Trieste, Italy*



$$\dot{M}_* = (1 - R)\psi \quad (4.6)$$

$$\dot{M}_{\text{hot}} = -\dot{M}_{\text{cool}} + \beta\psi \quad (4.7)$$

$$\dot{M}_{\text{cold}} = \dot{M}_{\text{cool}} - (1 - R + \beta)\psi \quad (4.8)$$

$$\dot{M}_*^Z = (1 - R)Z_{\text{cold}}\psi \quad (4.9)$$

$$\dot{M}_{\text{hot}}^Z = -\dot{M}_{\text{cool}}Z_{\text{hot}} + (pe + \beta Z_{\text{cold}})\psi \quad (4.10)$$

$$\dot{M}_{\text{cold}}^Z = \dot{M}_{\text{cool}}Z_{\text{hot}} + [p(1 - e) - (1 + \beta - R)Z_{\text{cold}}]\psi, \quad (4.11)$$

Fill your DM haloes with gas

Compute expected cooling and star formation rates

Make assumptions regarding feedback reheating

Make assumptions about how mergers turn disks into ellipticals

Compute observables (colors, etc) and compare

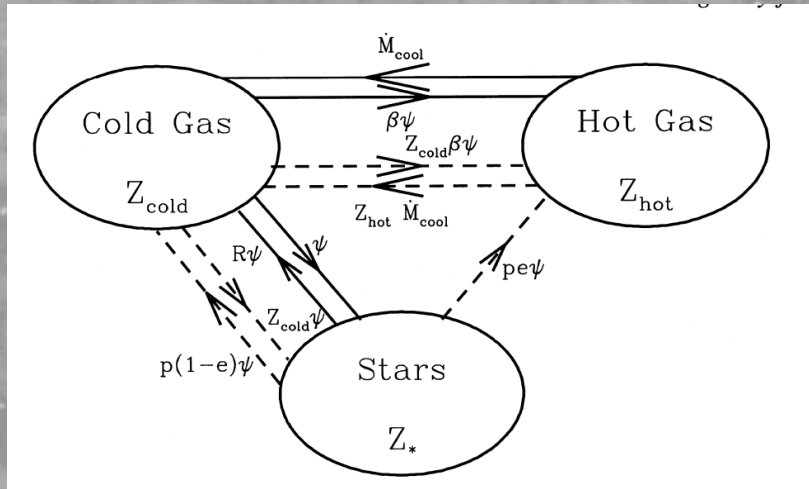
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R – recycled fraction
 ψ – instantaneous SFR
 \dot{M}' – cooling rate
 Z – metallicity
 β, e – Feedback efficiency
 p – yield

$$\dot{M}_* = (1 - R)\psi \quad (4.6)$$

$$\dot{M}_{\text{hot}} = -\dot{M}_{\text{cool}} + \beta\psi \quad (4.7)$$

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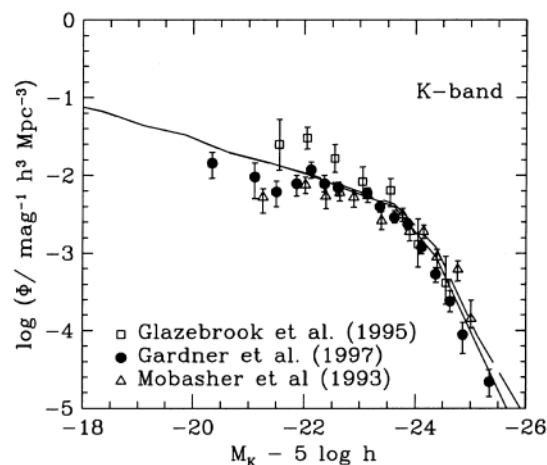
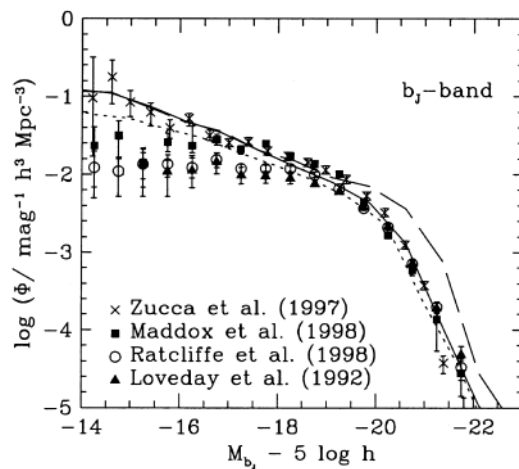
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Fill your DM haloes with gas

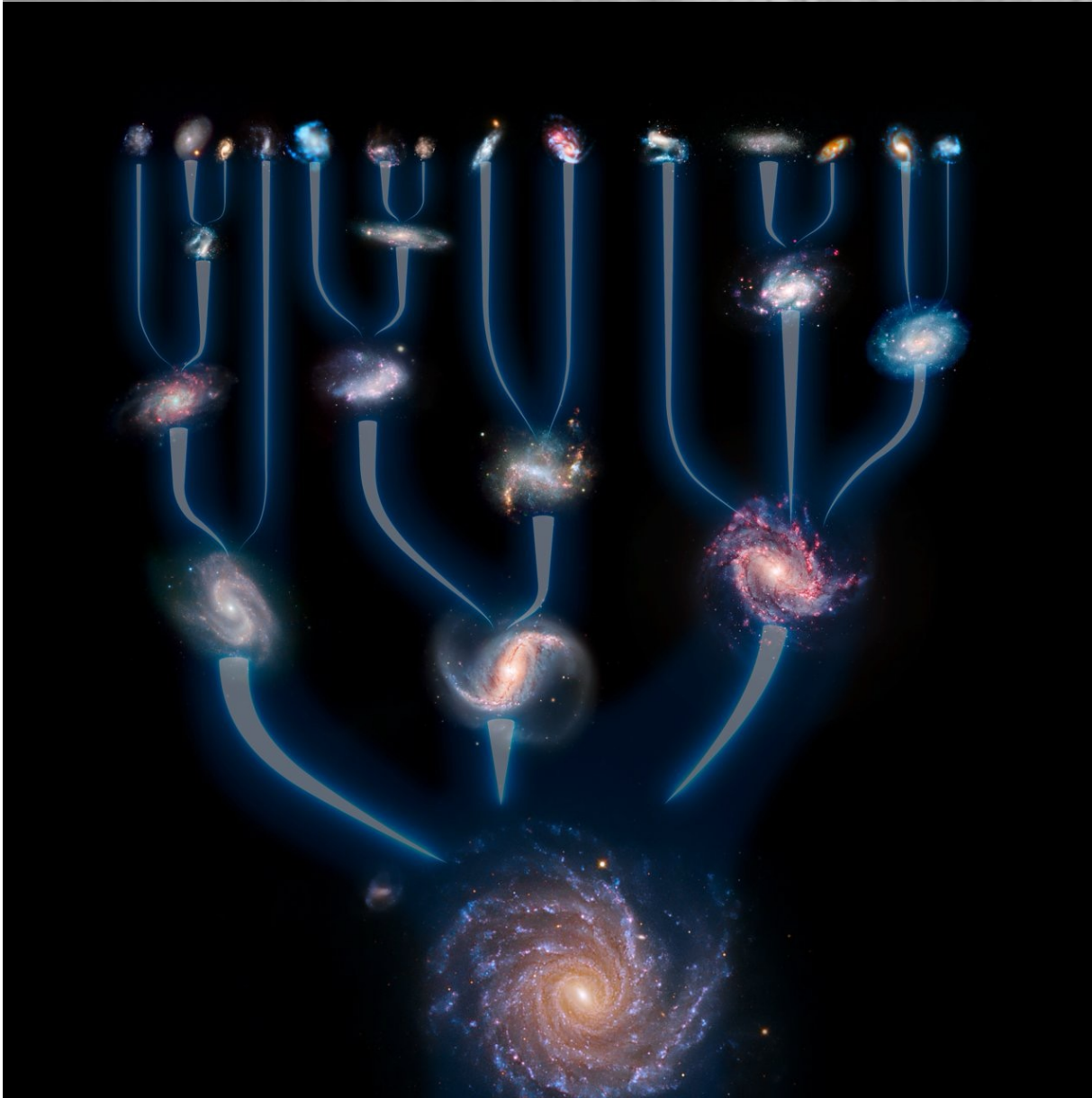
Compute expected cooling and star formation rates

Make assumptions regarding feedback reheating

Make assumptions about how mergers turn disks into ellipticals

Compute observables (colors, etc) and compare

Semi-analytical modelling versus hydro



Allows all properties of the galaxy population at any given time to be computed

Semi-analytical modelling versus hydro

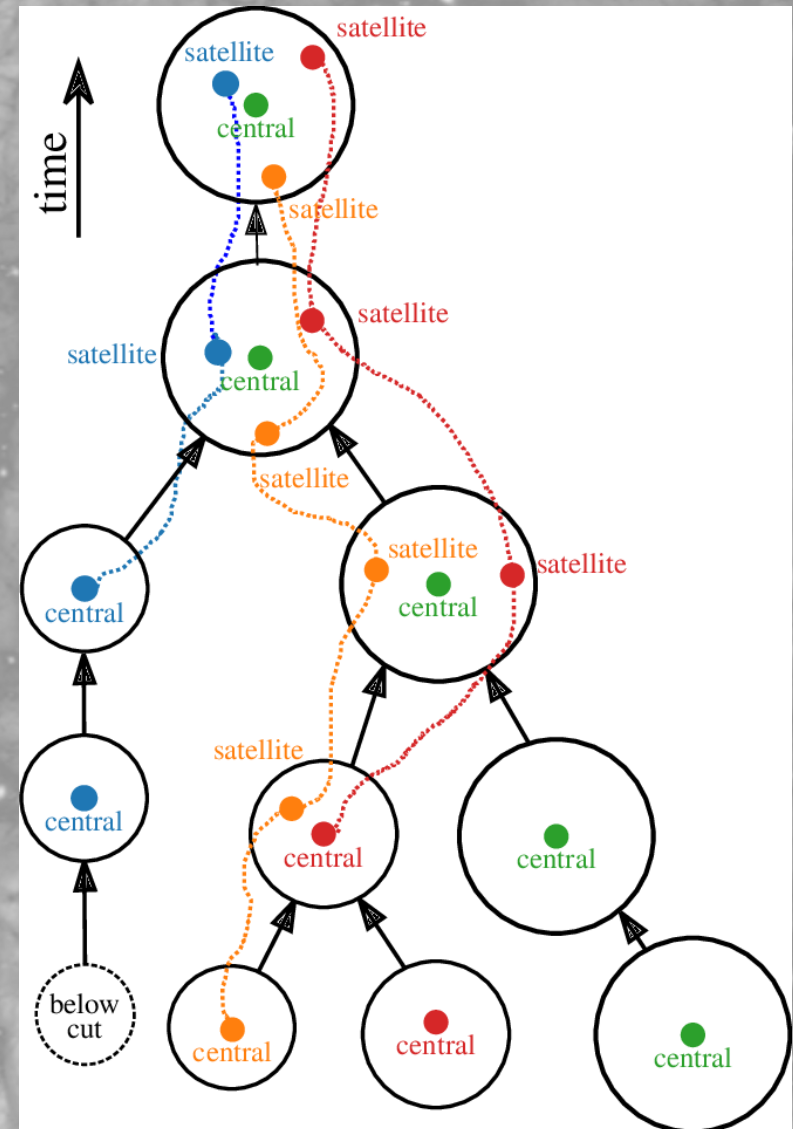
SAMs are powerful ways to test out ideas of galaxy formation – the IMF, cooling etc.

Yet they have huge numbers of parameter that need to be “fine tuned” and which are not necessarily physical

- + resolution
- + time steps

They lack spatial information inside the halo (substructures or black holes)

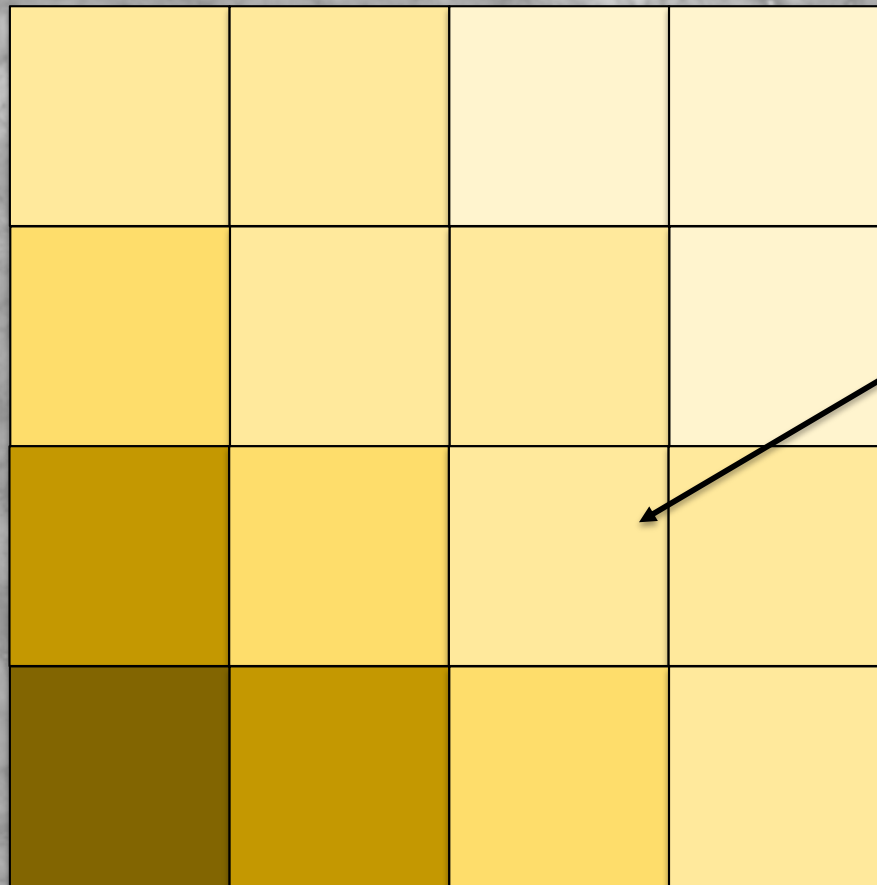
Cant say much of the IGM, or stripped material, or indeed anything outside of the halo



Hydrodynamical simulations

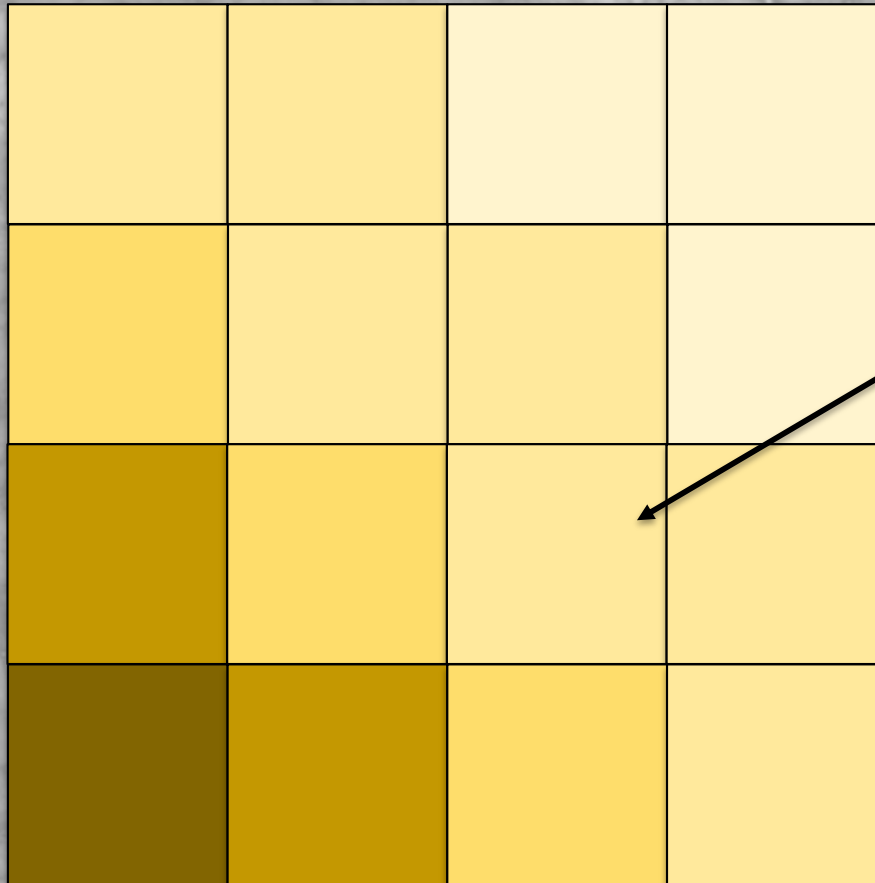


Hydrodynamical simulations



“Resolution element”

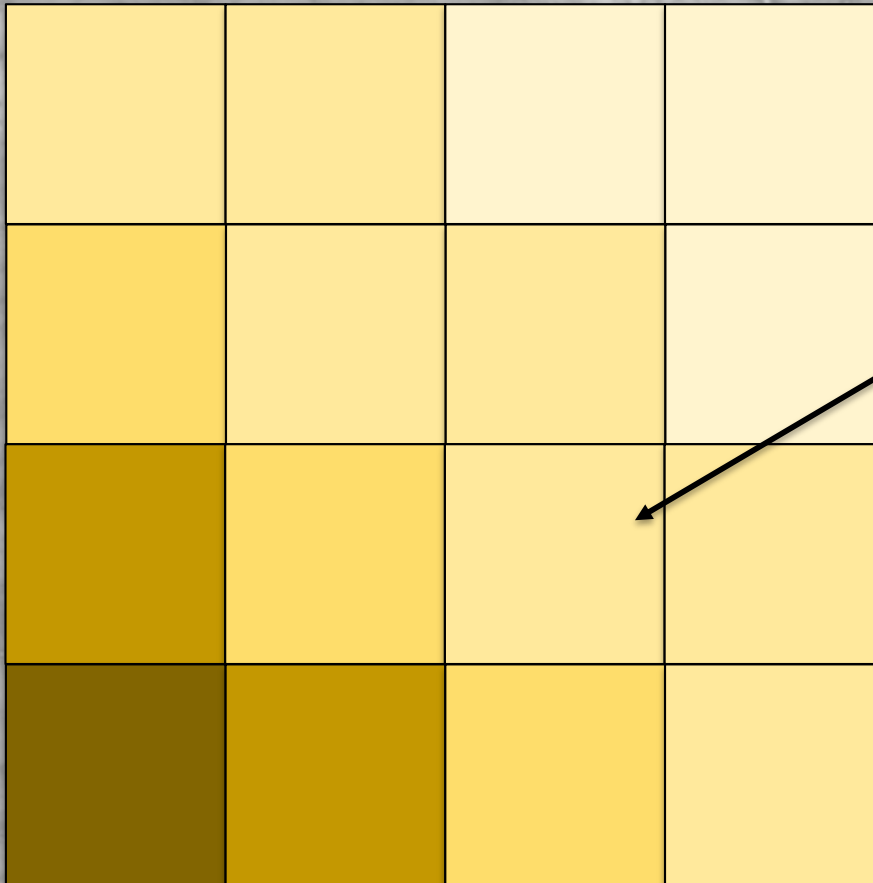
Hydrodynamical simulations



“Resolution element”

- Mass, density (dM/dt , $d\rho/dt$)
- Temperature (heating/cooling)
- Pressure (dP/dt)
- Momentum (dv/dt)

Hydrodynamical simulations



“Resolution element”

- Mass, density (dM/dt , dp/dt)
- Temperature (heating/cooling)
- Pressure (dP/dt)
- Momentum (dv/dt)

▪ governing equations (non-relativistic fluid with pressure)

- Poisson's equation

$$\Delta\Psi = 4\pi G\rho$$

- continuity equation

$$\frac{\partial\rho}{\partial t} + \nabla \cdot (\rho\vec{v}) = 0$$

- conservation of momentum

$$\frac{\partial\vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\nabla\Psi - \frac{\nabla p}{\rho}$$

- equation of state

$$p = c_s^2\rho$$

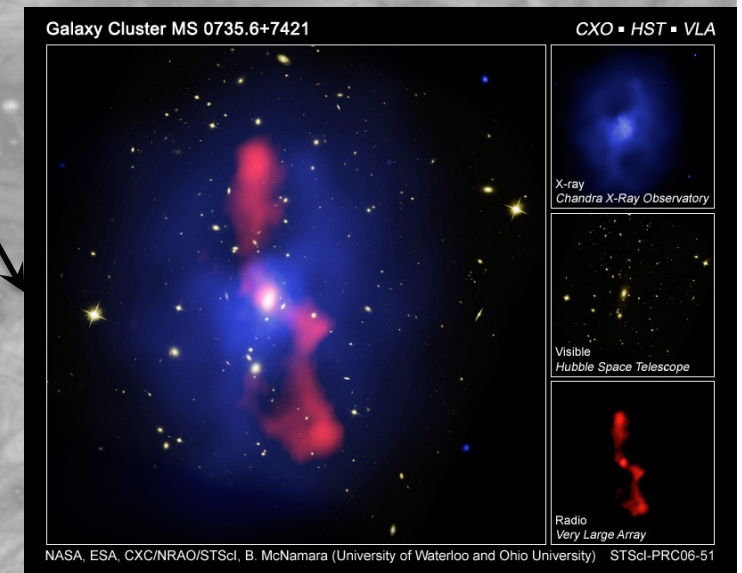
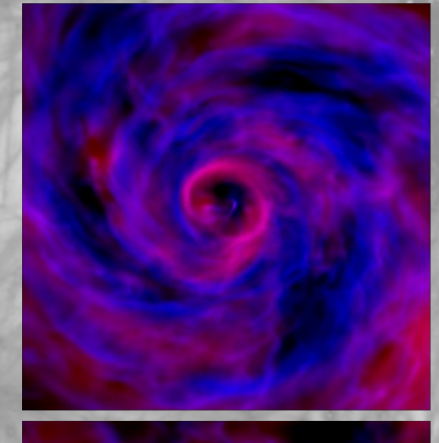
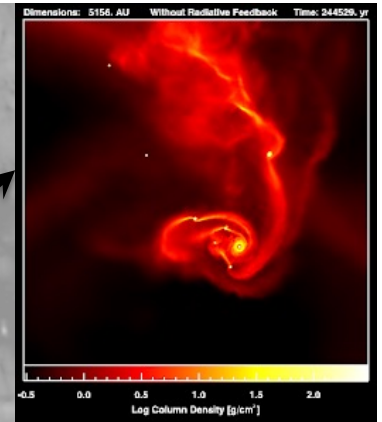
(c_s : sound speed)

The Kelvin-Helmholtz instability



Hydrodynamical simulations

| Object | Scale (m) | Scale (Mpc) |
|------------------------|-----------|-------------|
| Stars | 10^8 | 10^{-14} |
| Black hole | 10^{10} | 10^{-12} |
| Solar system | 10^{13} | 10^{-9} |
| Interstellar distances | 10^{16} | 10^{-6} |
| Small galaxies | 10^{20} | 0.01 |
| Milky Way halo | 10^{21} | 0.1 |
| Local Group distances | 10^{22} | 1 |
| Cluster | 10^{23} | 10 |
| Large-scale structures | 10^{24} | 100 |



Hydrodynamical simulations

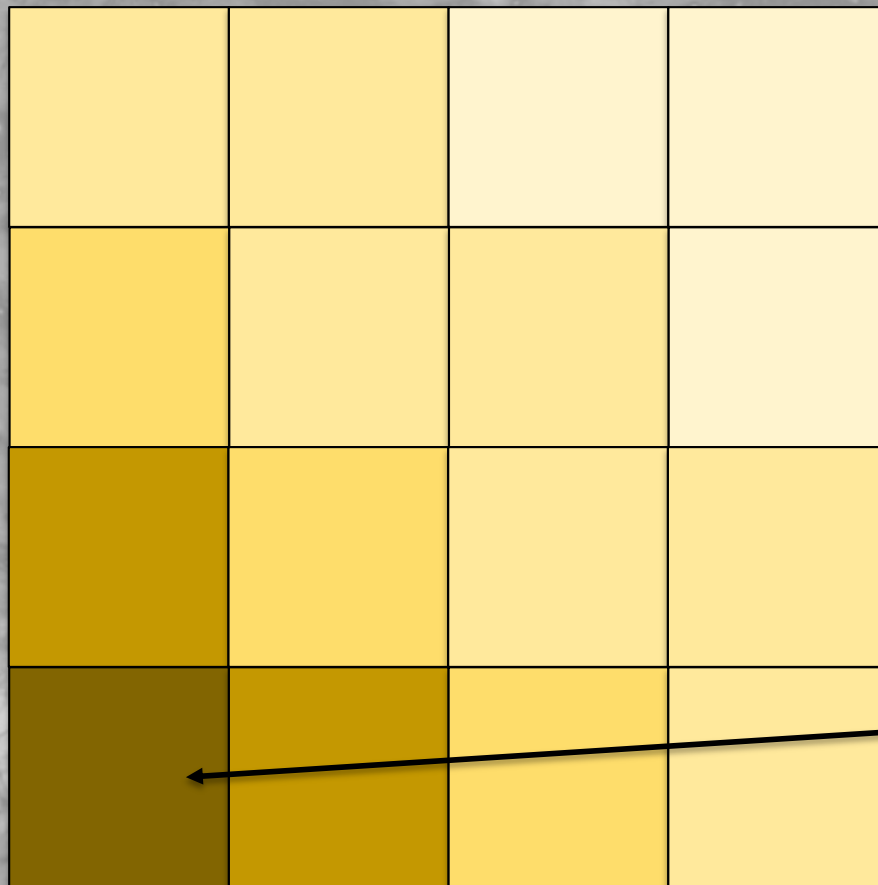
Although Hydro sims can resolve the thermodynamical properties of the gas, they can NOT resolve star formation in cosmological simulations.

Thus we need “sub grid” physics - an analogous method to SAMs

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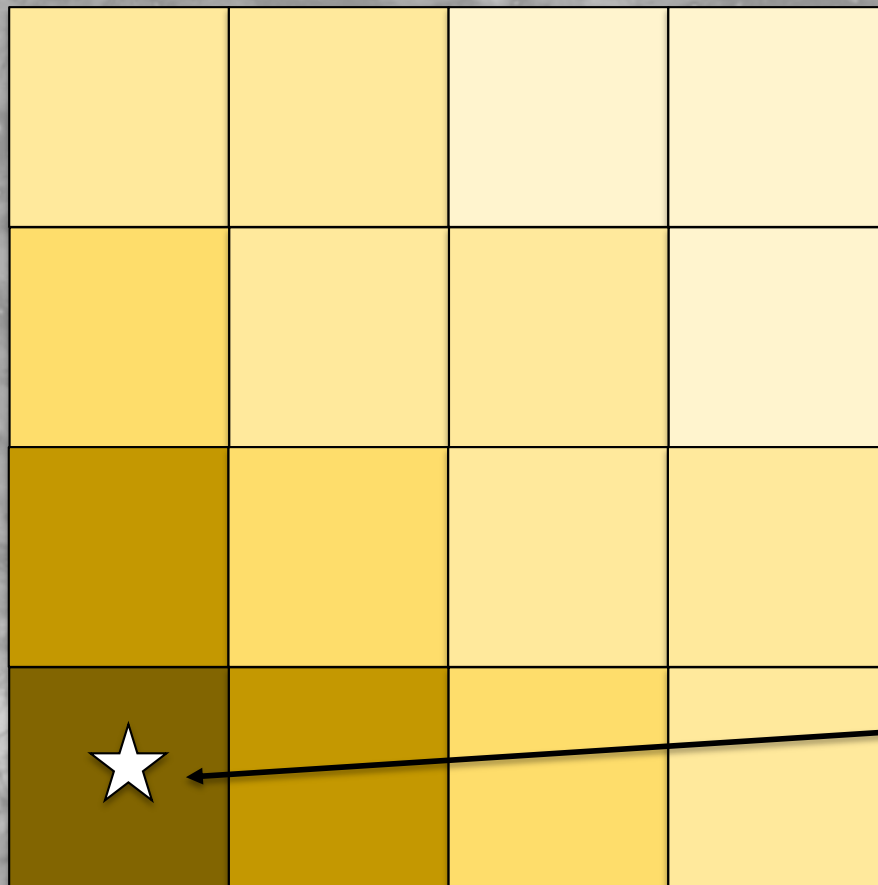


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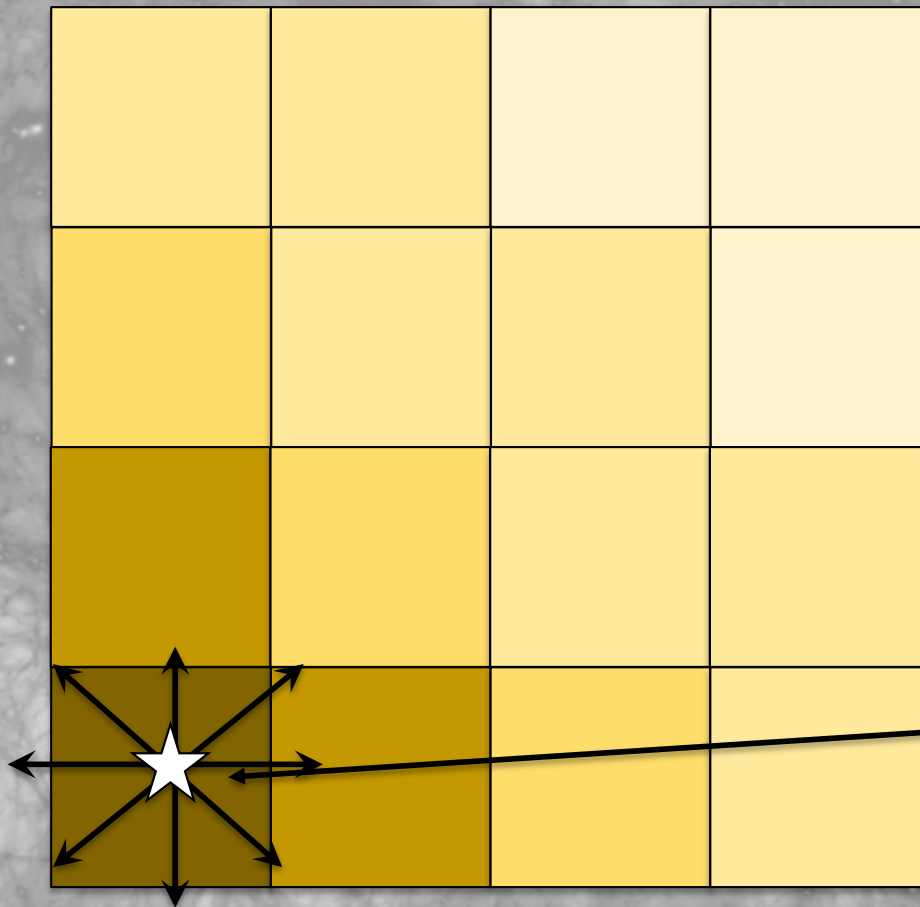


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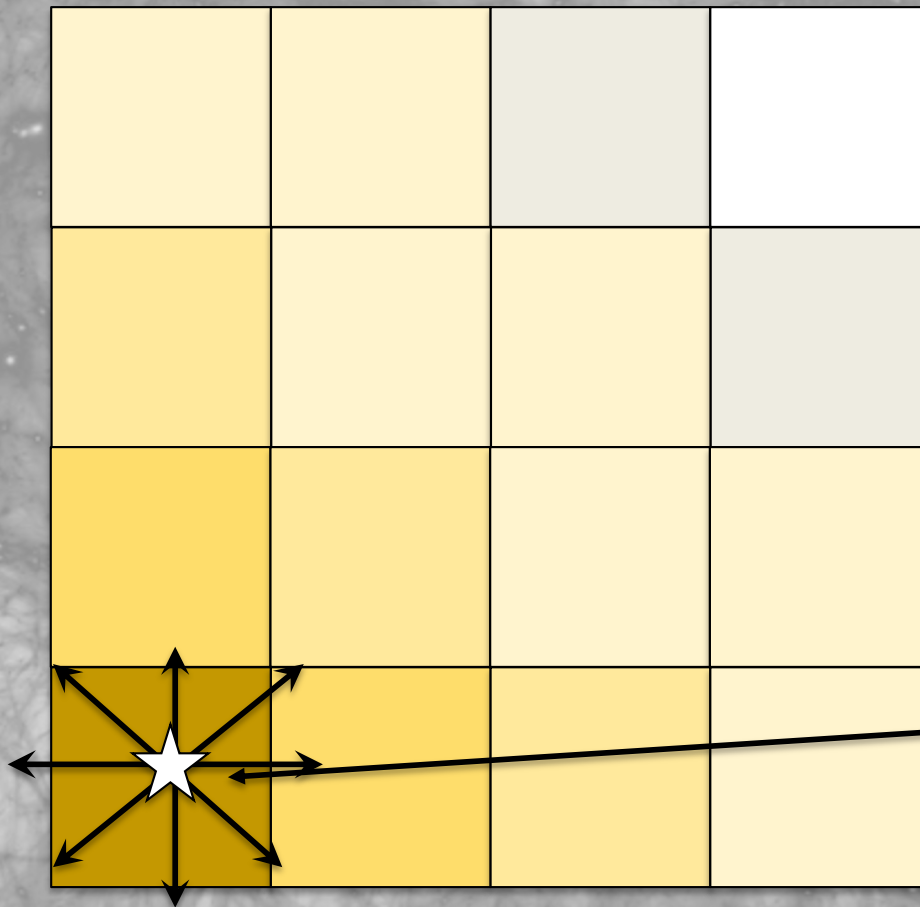


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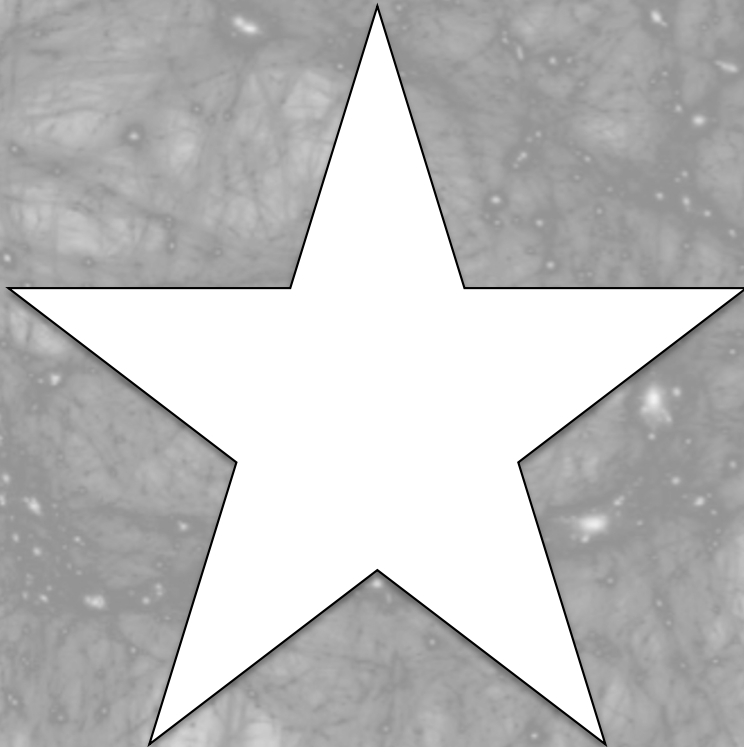


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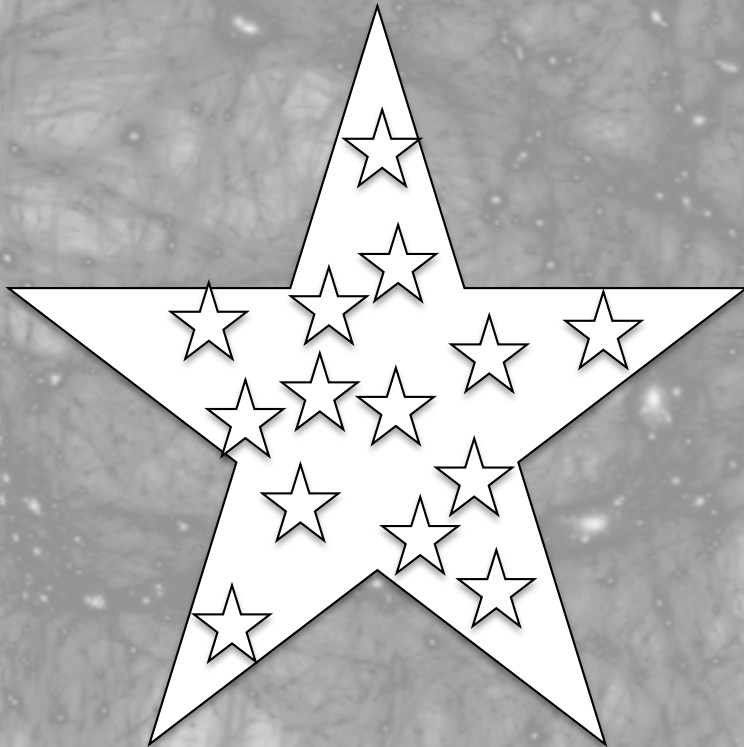


This star “particle” represents a stellar population usually of 10^4 - 10^6 Solar masses

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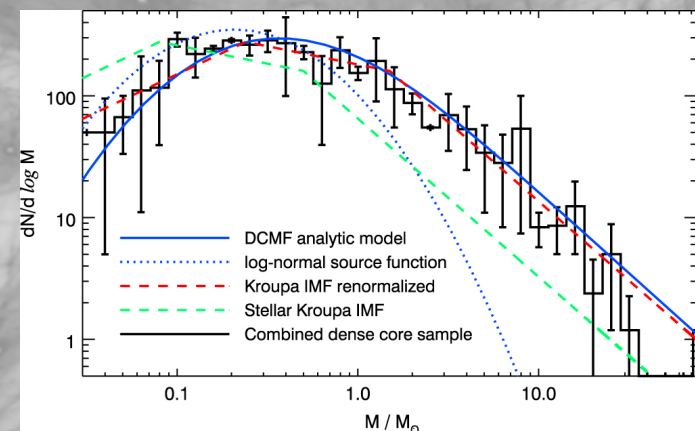
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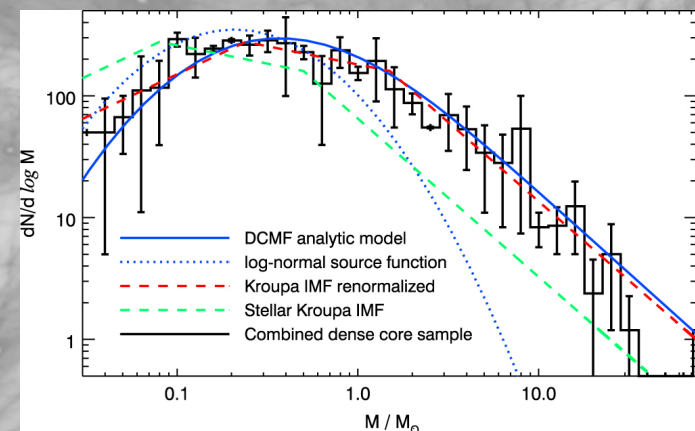
Thus we need “sub grid” physics - an analogous method to SAMs

- Colors
- Which stars explode (age)
- Metallicity recycled
- Energy injected

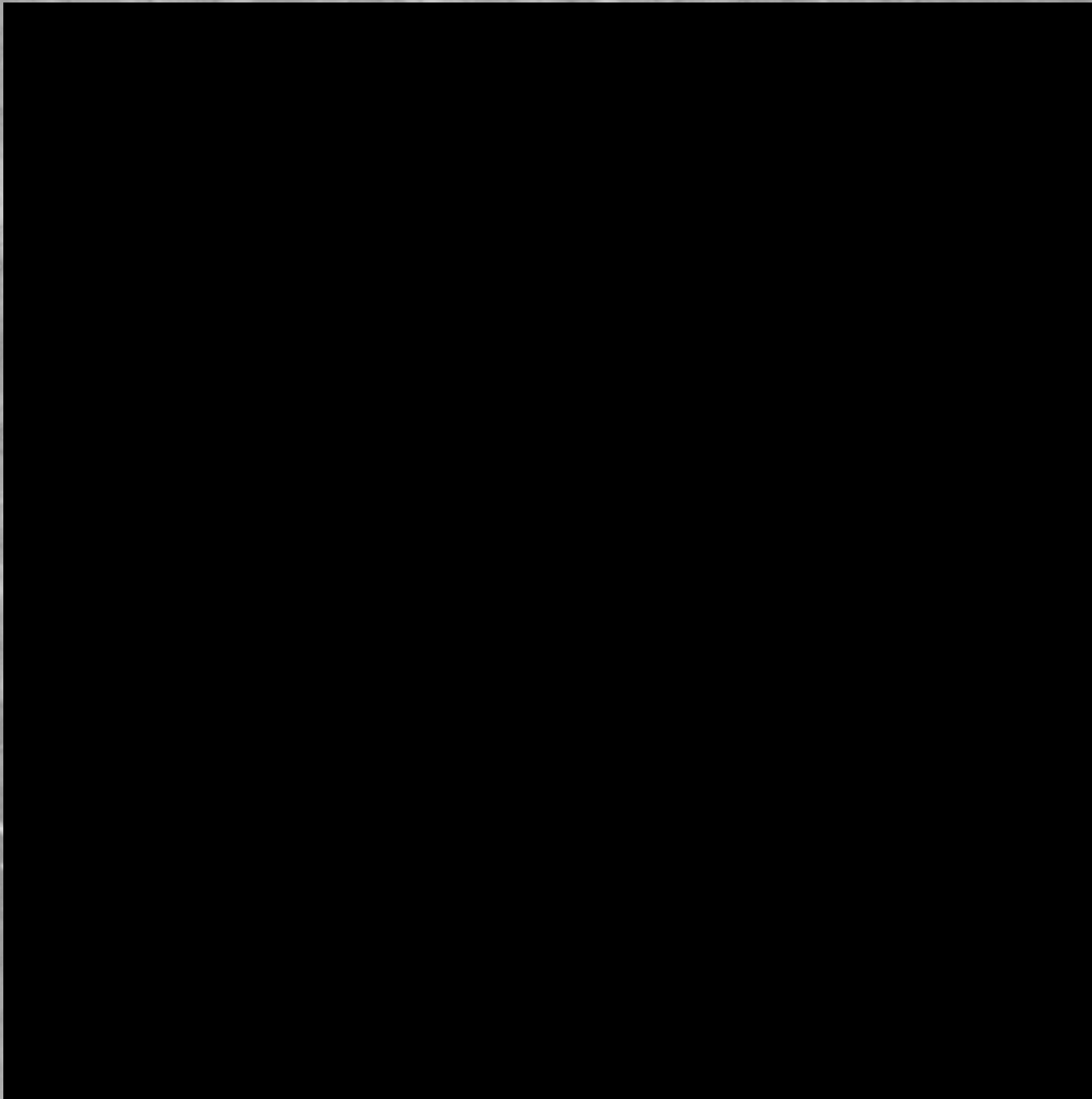


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Hydrodynamical simulations





$z=30.0$

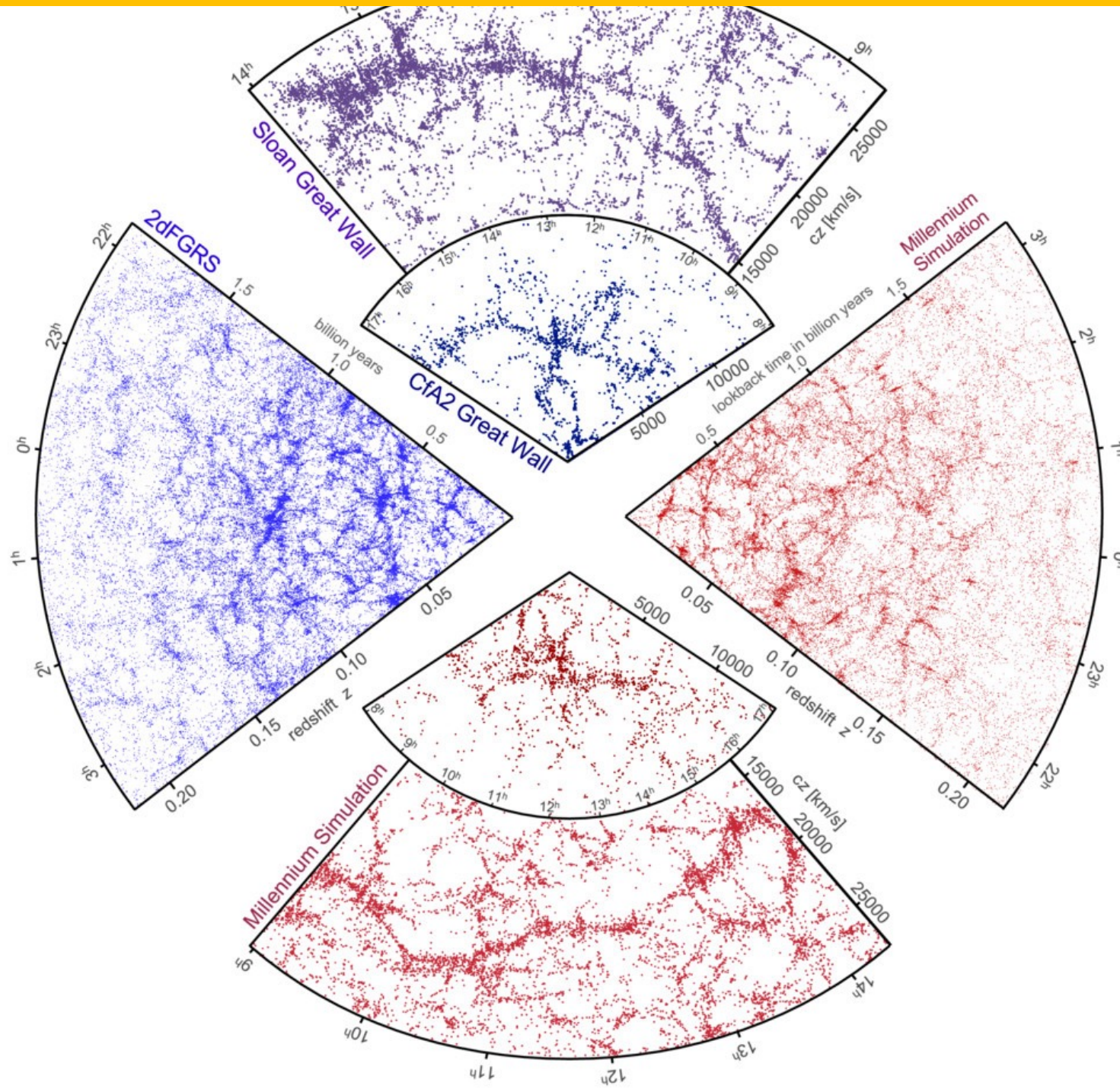


Latte simulation

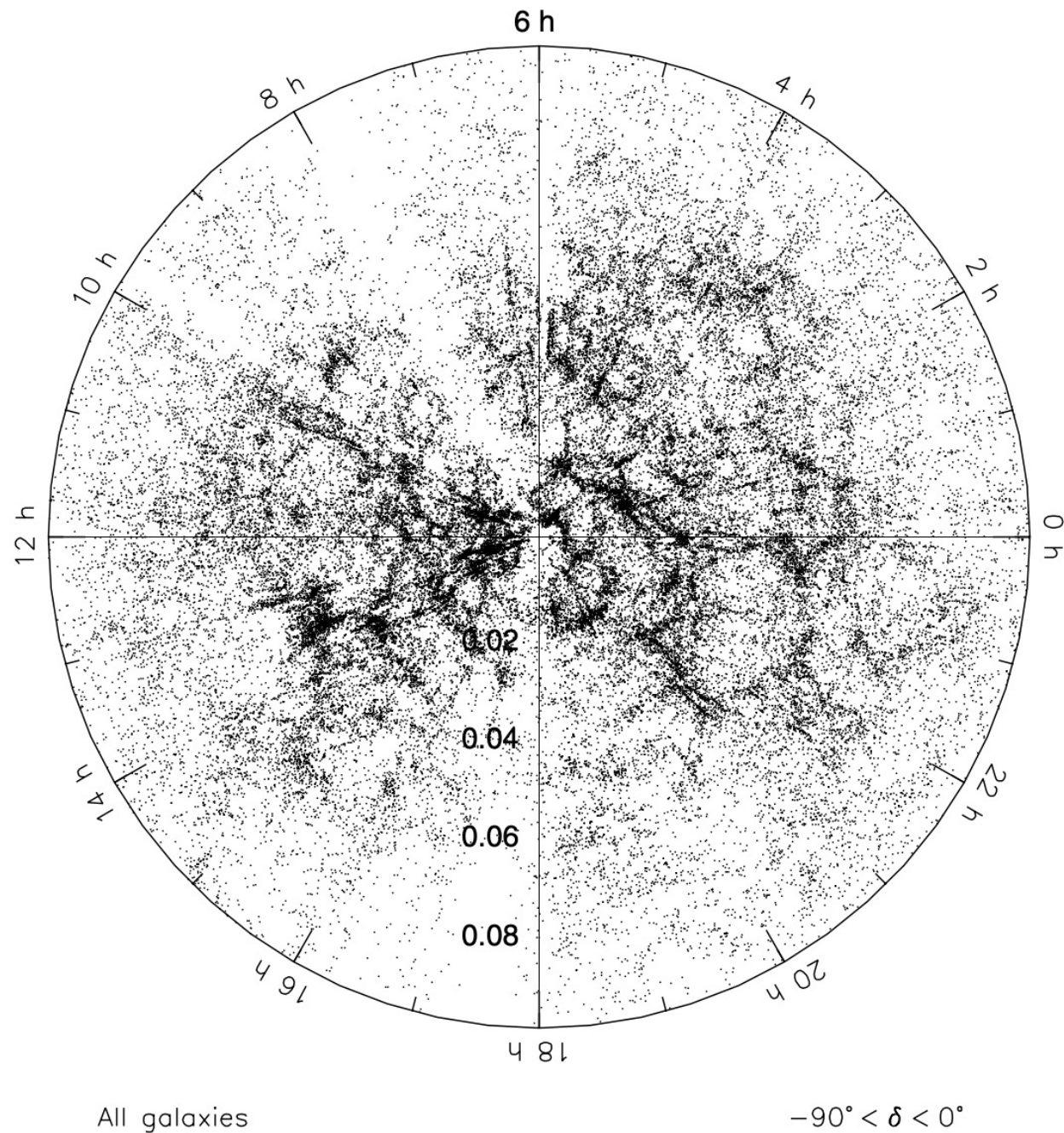


ESO-420-G013 (HST)

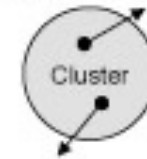
Mapping the universe - with redshifts



Fingers of god – distortion in redshift surveys



non-linear
structure



Actual
shape

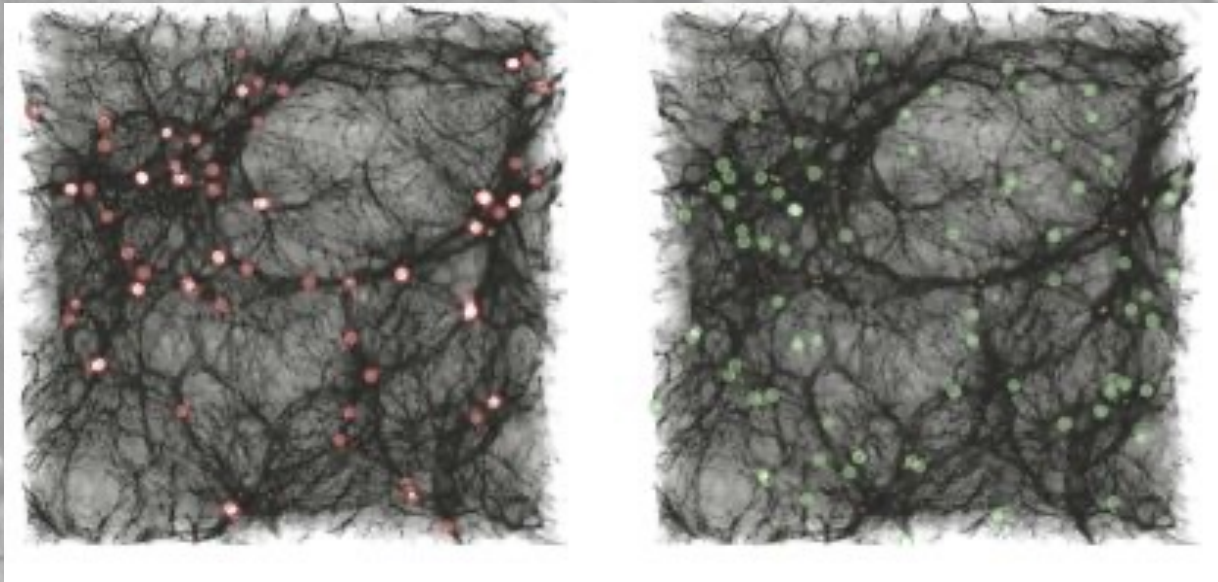


Apparent
shape
(viewed from
below)



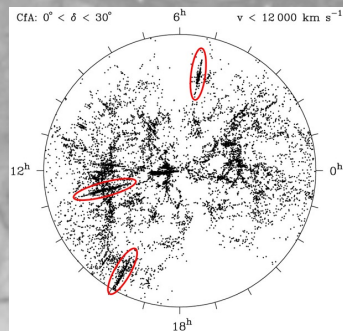
A summary of the challenges faced when trying to map the Universe

Galaxy bias – light doesn't trace matter



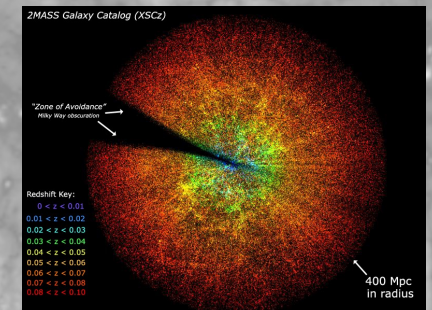
“Malmquist bias” – you only see the brightest galaxies at any given distance, given your telescope sensitivity

Selection bias and obstructions – incomplete sky coverage, Zone of Avoidance, dust, etc

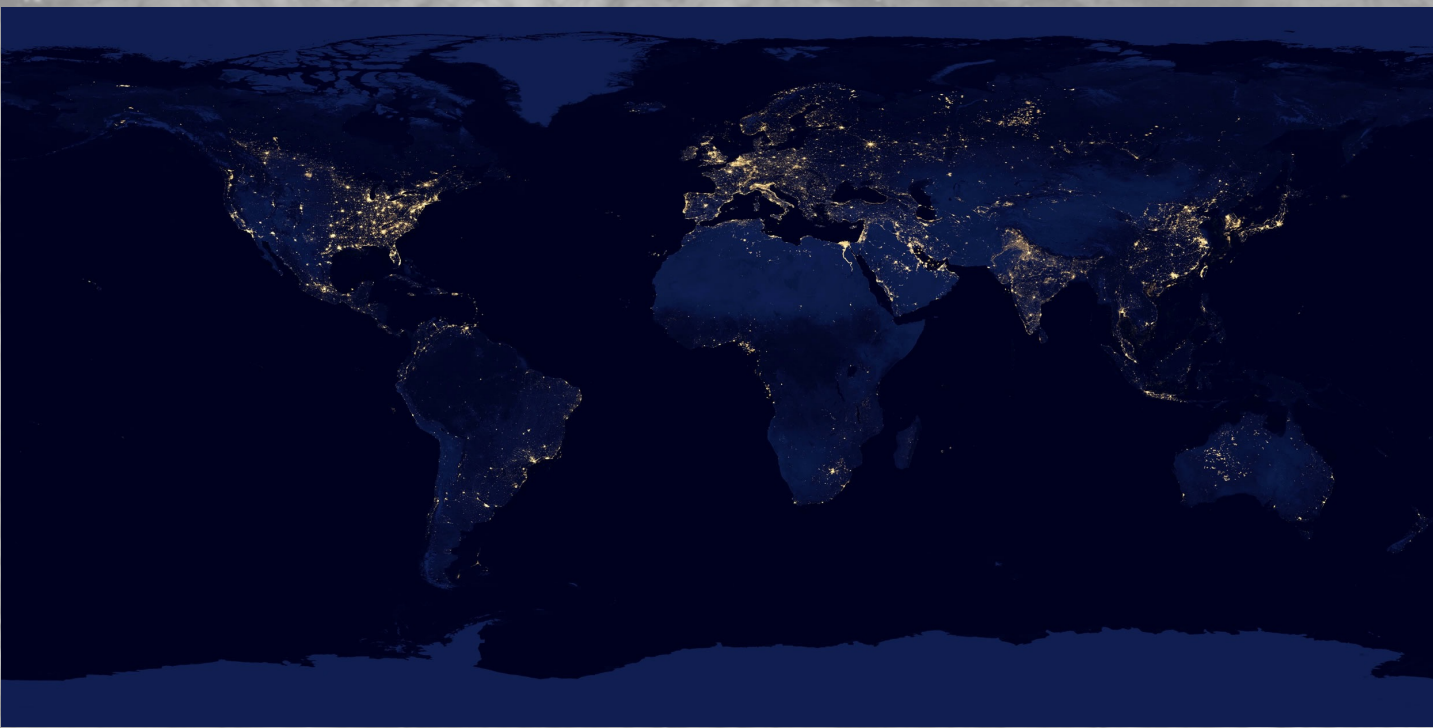


Red shift space distortions: “fingers-of-god” and the Kaiser effect

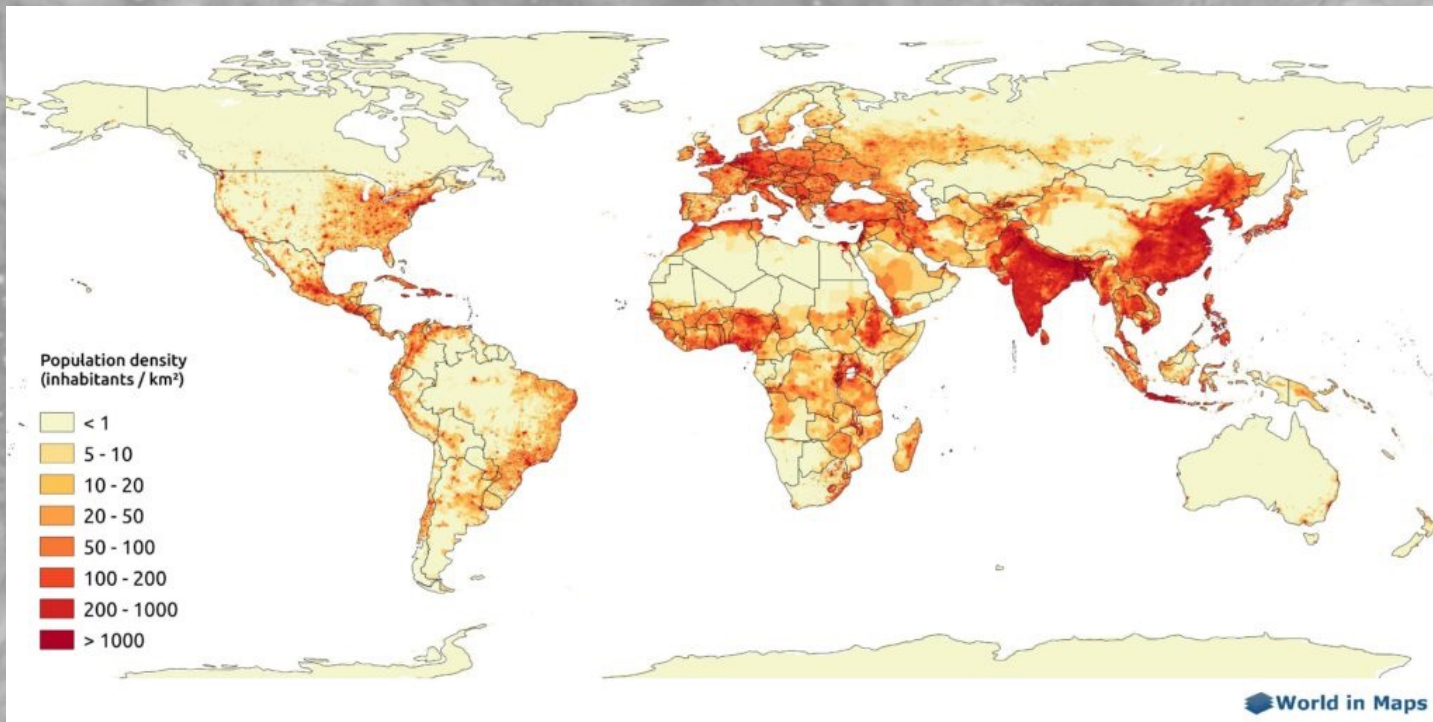
Dynamic evolving matter field, changed by competing forces – gravity and expansion



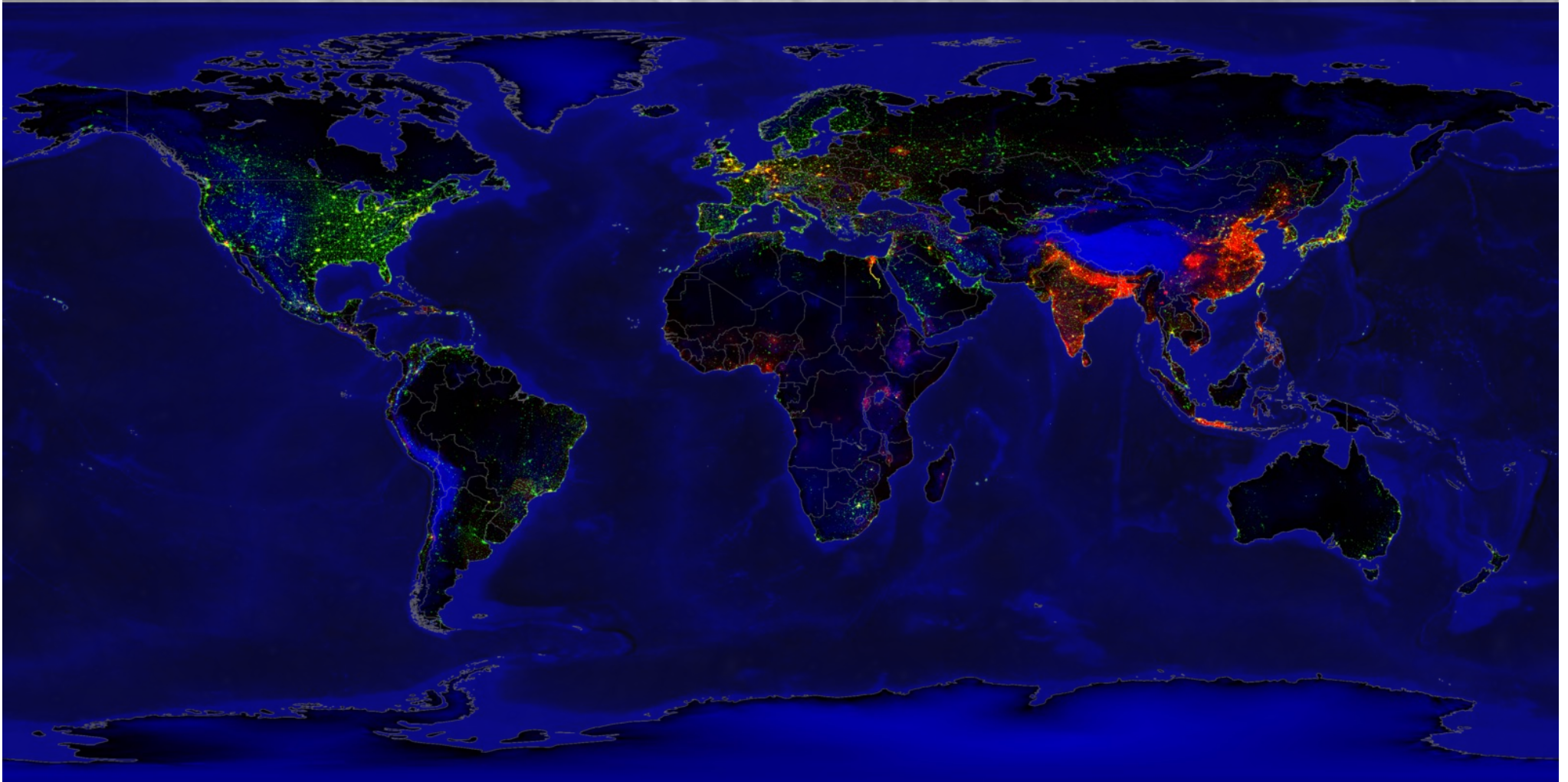
Light from the Earth at night



Population density



Bias – light does not trace density



Red – high mass to light

Green – low mass to light

What does trace the density?

The gravitational velocity –
mistakenly called the peculiar
velocity

$$\nabla^2\Phi = 4\pi G\rho$$

$$\delta = -\nabla \cdot \mathbf{v} / H_0 f(\Omega_m),$$



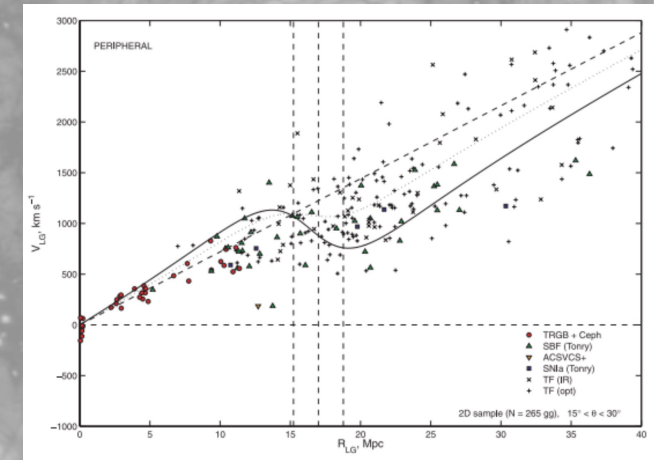
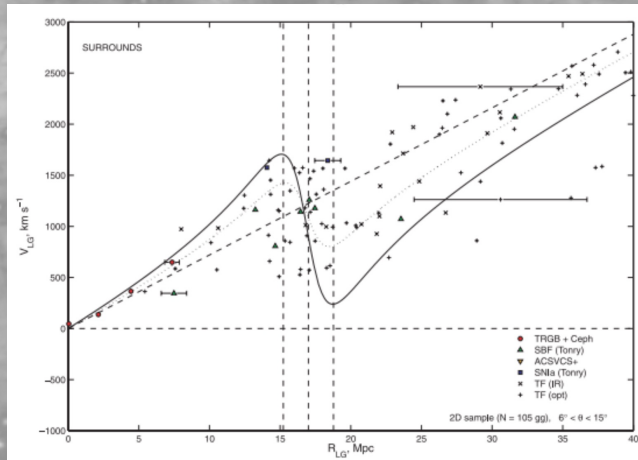
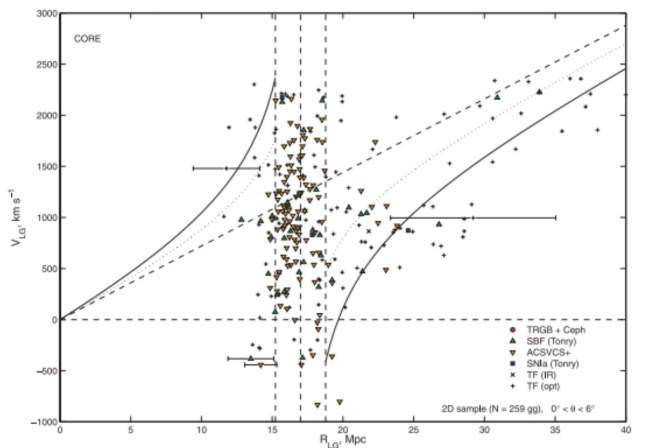
One of the best examples of this is the so-called “backside in fall” of the Virgo cluster

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Karatchensev et al 2012

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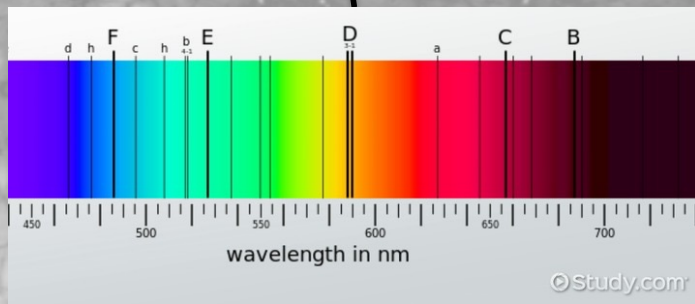
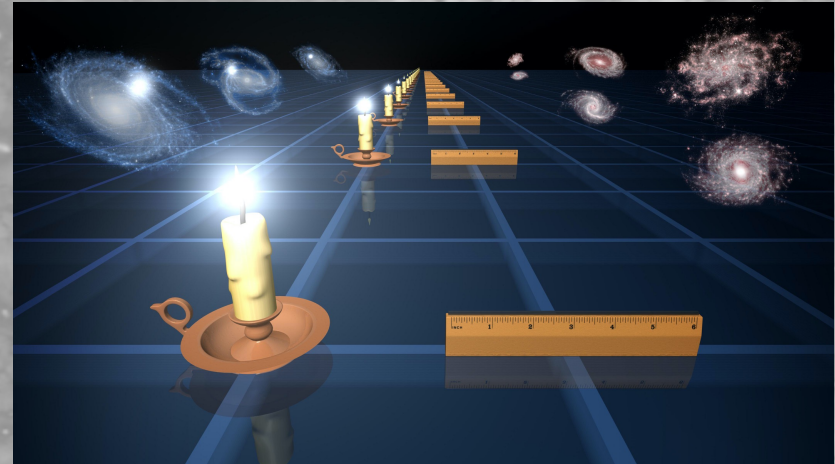
How do you get a peculiar velocity?

Measure the distance + redshift

$$cz = v_{exp} + v_{pec}$$

$$v_{exp} = H_0 d$$

$$v_{pec} = cz - H_0 d$$



How do you get a peculiar velocity?

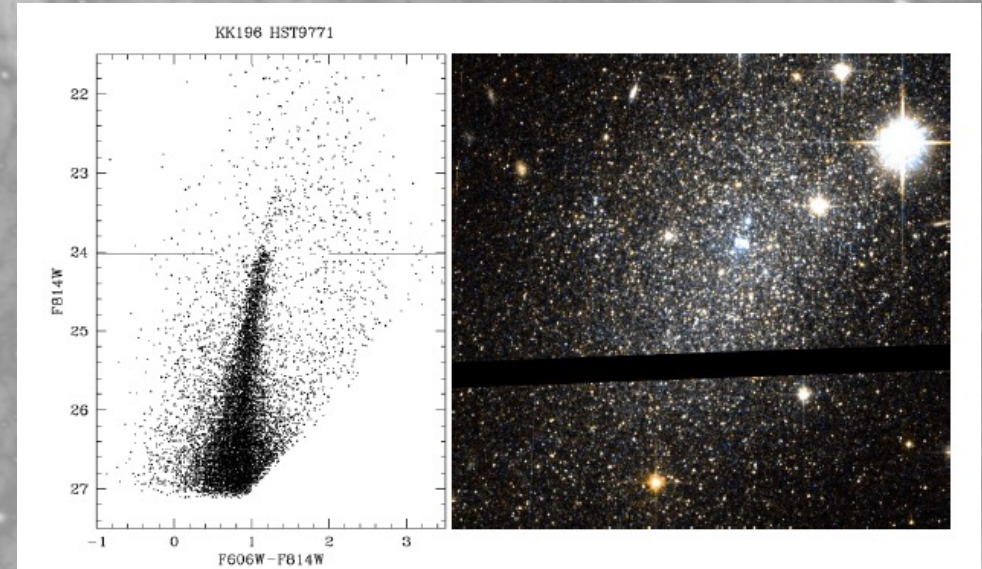
Measure the distance + redshift

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$$v_{exp} = H_0 d$$

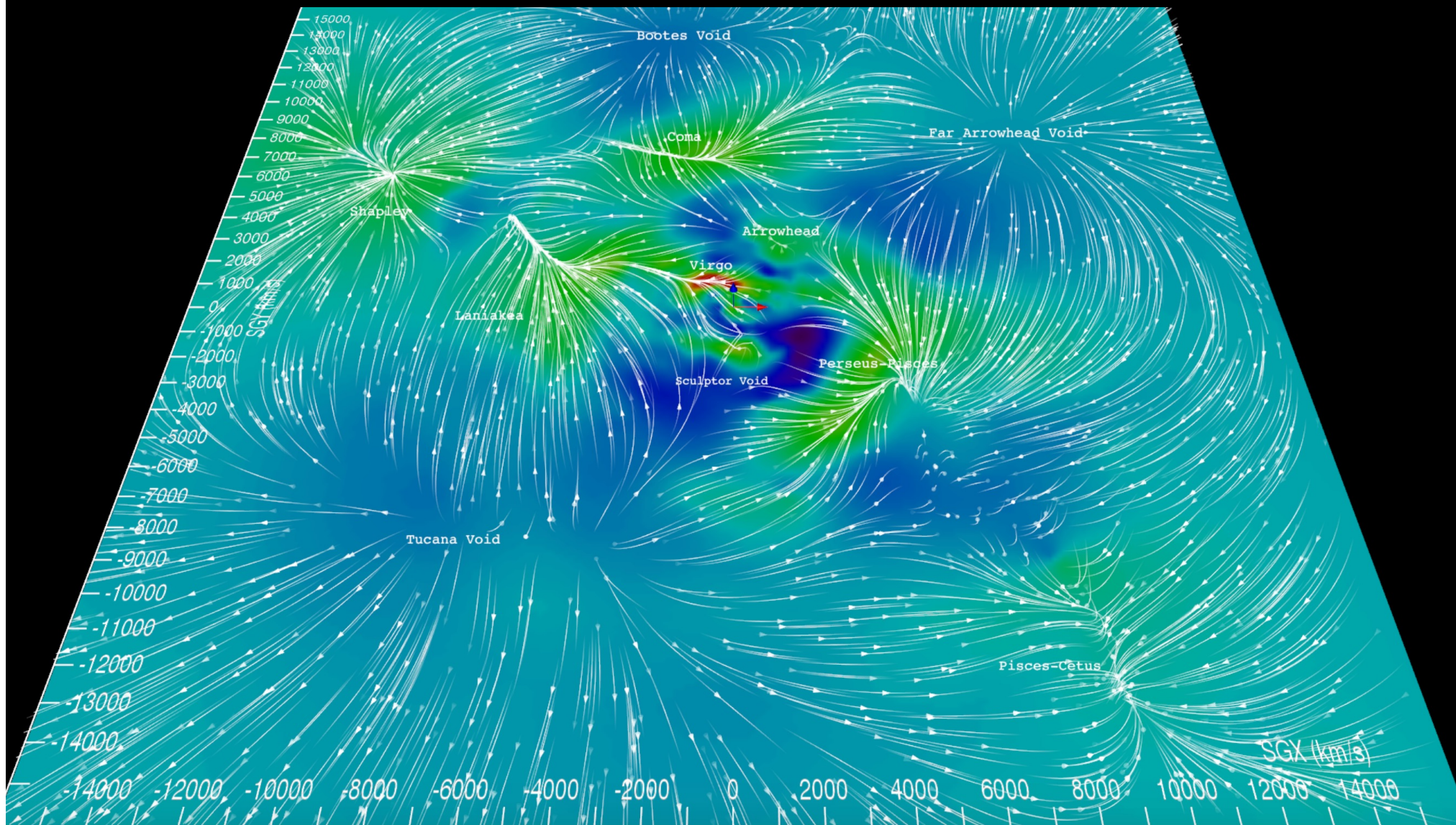
$$v_{pec} = cz - H_0 d$$

| Method | error | distance | Common? |
|-------------------|--------|----------|---------|
| SN | 1-5% | far | no |
| TRGB | ~5% | local | yes |
| SBF | ~5% | local | yes |
| Scaling relations | 10-20% | far | yes |



Standard candles such as Super Novae, TRGB, SBF, Cepheids, etc give distances

This allows us to separate the peculiar velocity from the Hubble expansion



Major Rivers and River Basins of Europe
CCM River and Catchment Database, Version 2

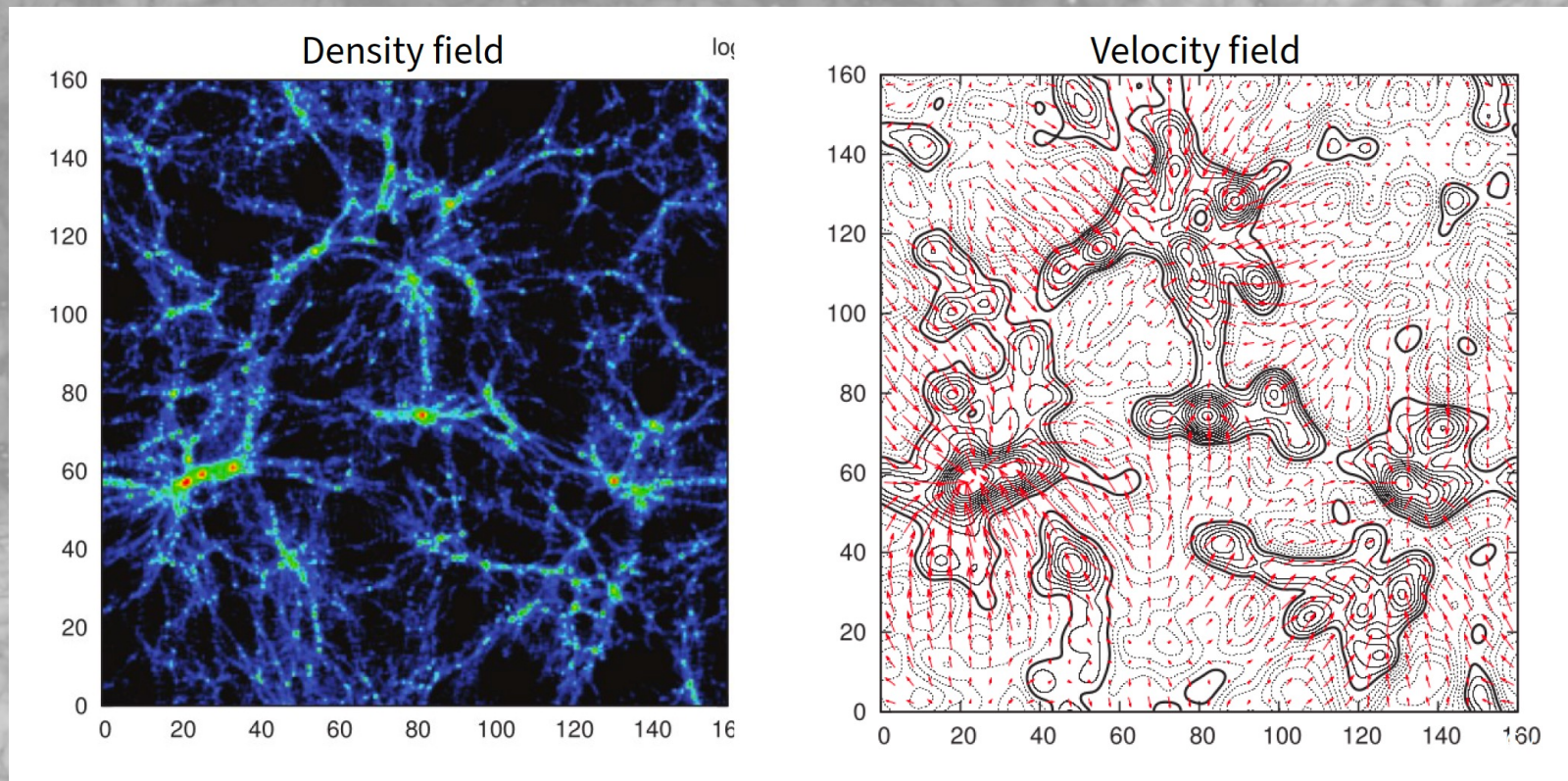
(c) European Commission - Joint Research Centre, March 2007



Reconstructing the underlying matter distribution of the Local universe

$$\delta = -H_0 f \nabla \cdot v$$

Its all based on the laminar flow, linear relationship between velocity and over-density

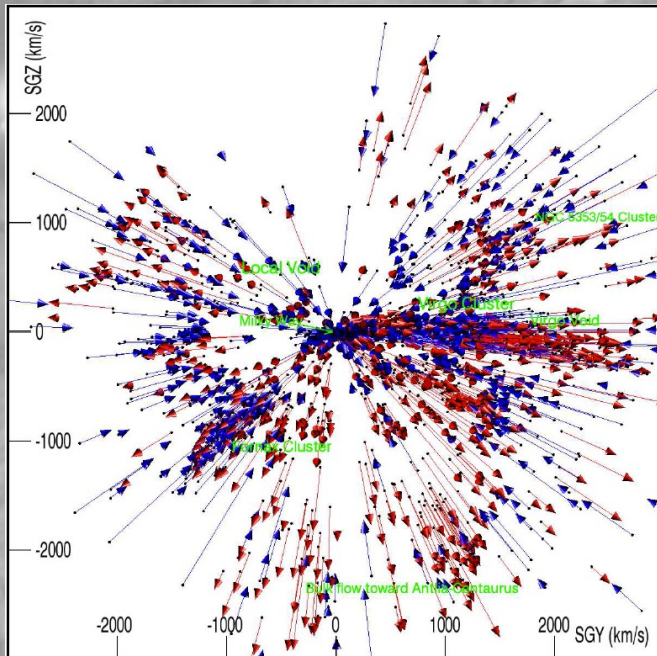


Reconstructing the underlying matter distribution of the Local universe

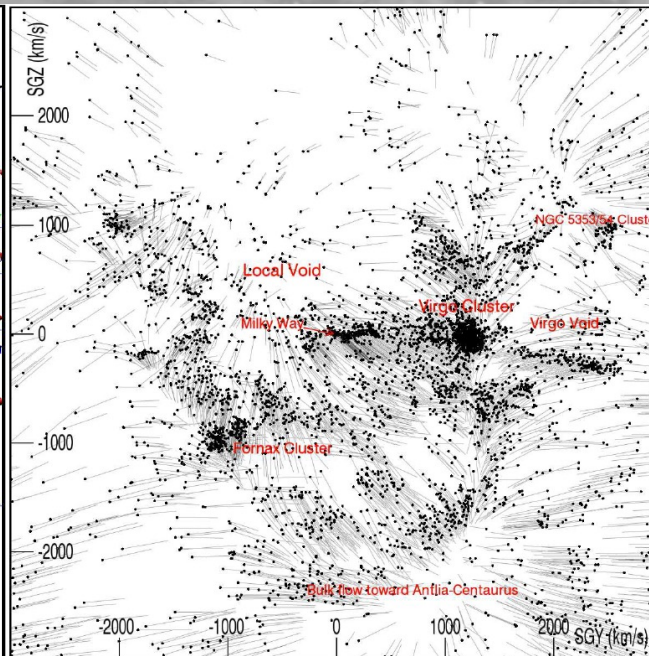
$$\rho \propto -\vec{\nabla} \cdot \vec{v}$$

In the linear regime there is a very simple relationship between density and peculiar velocity

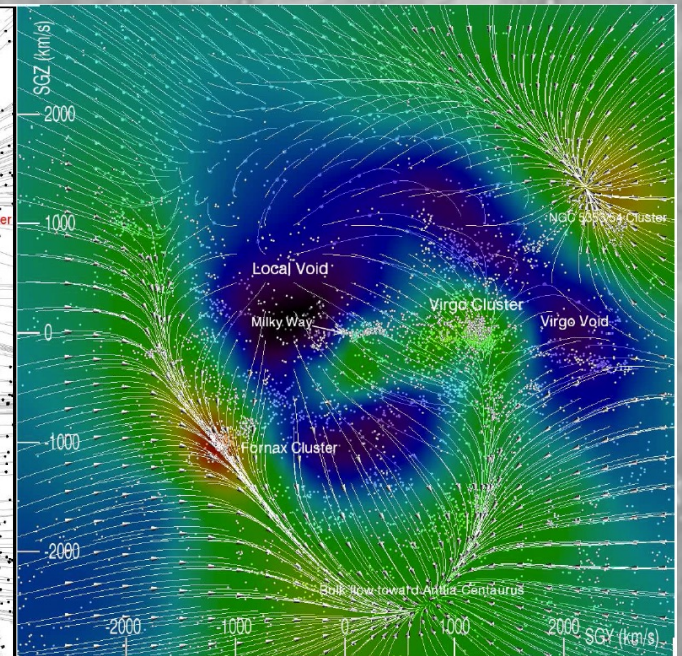
radial peculiar velocity



reconstructed 3D peculiar velocity

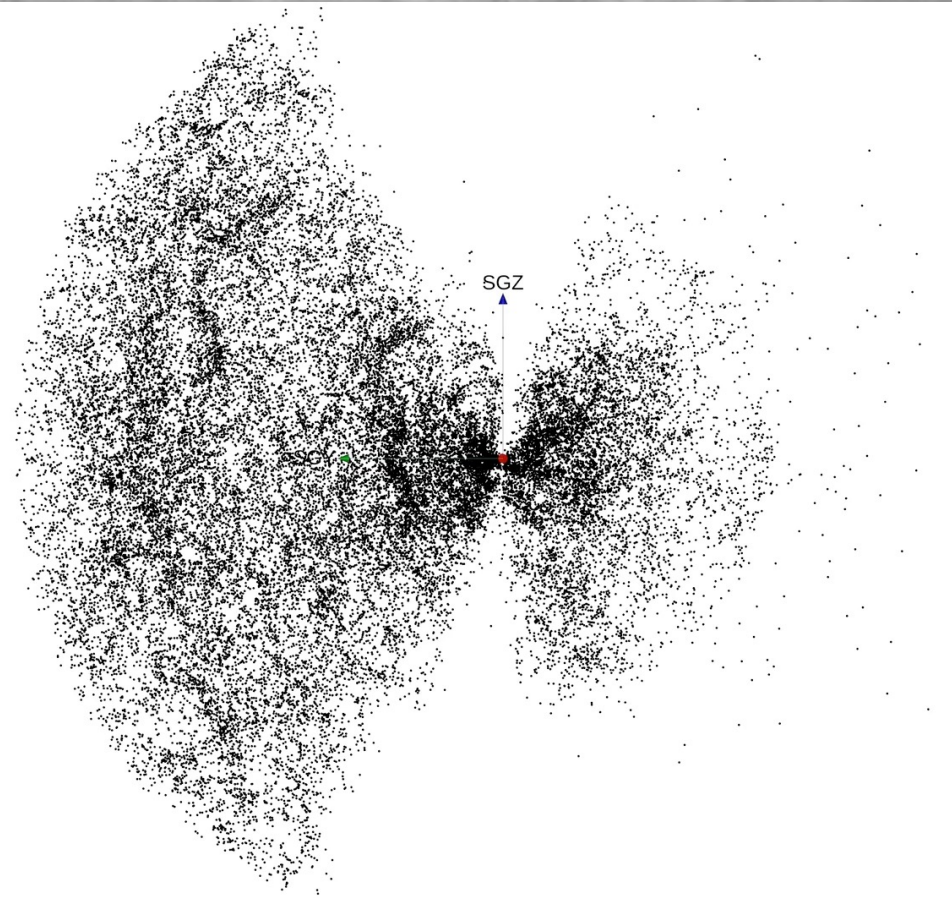


Corresponding 3D density field



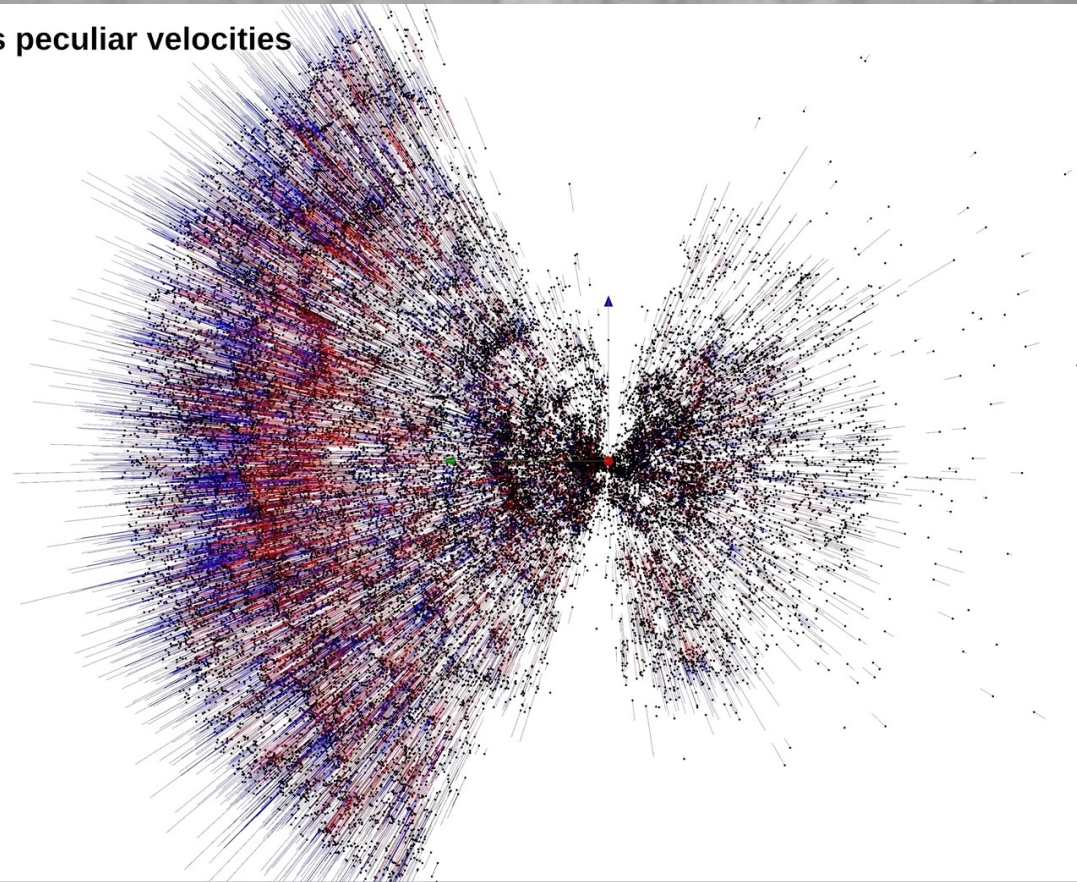
Our starting point is the CF4 set of 50,000 data points, grouped into ~38,000 groups

CF4 Groups



Peculiar Velocity measurements done by standard candles

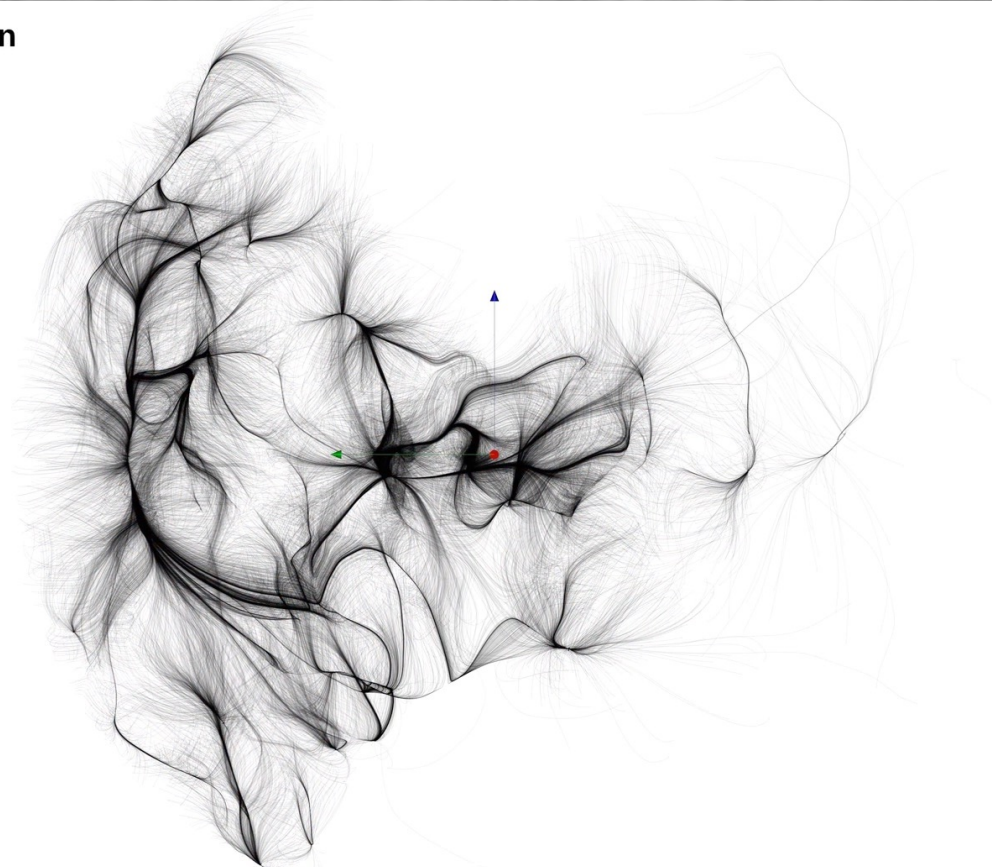
CF4 Groups peculiar velocities



Radial peculiar velocity vectors associated with groups; red outward and blue inward

3D Flow lines

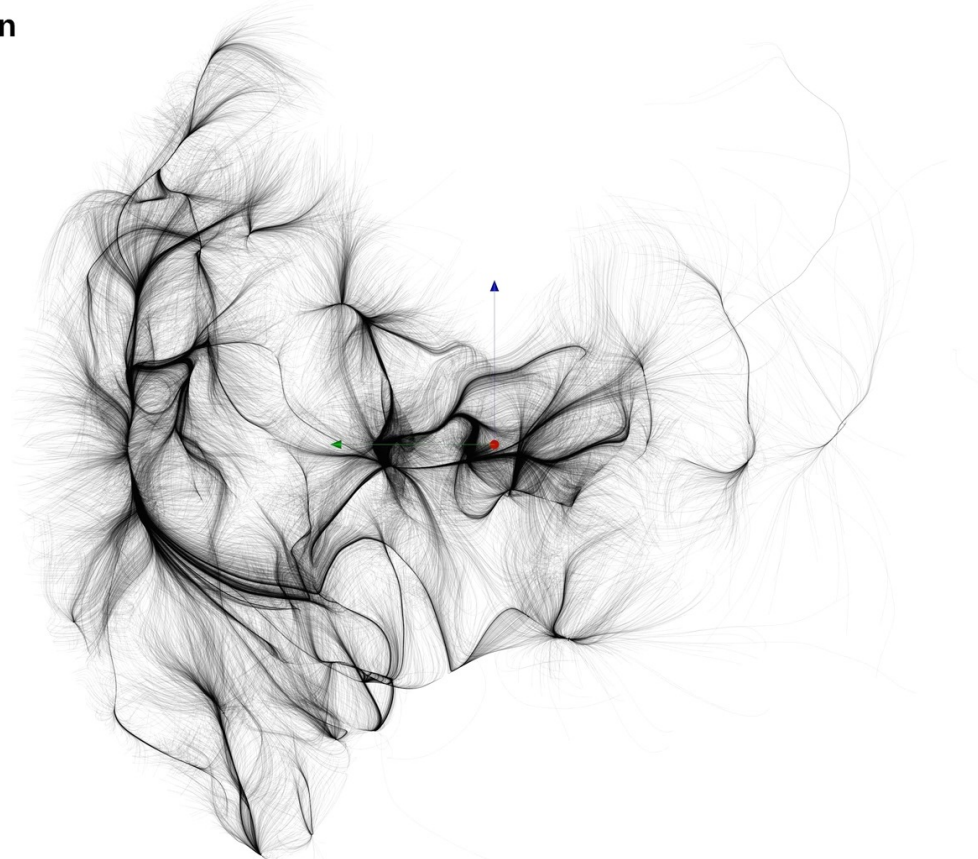
CF4 Reconstruction



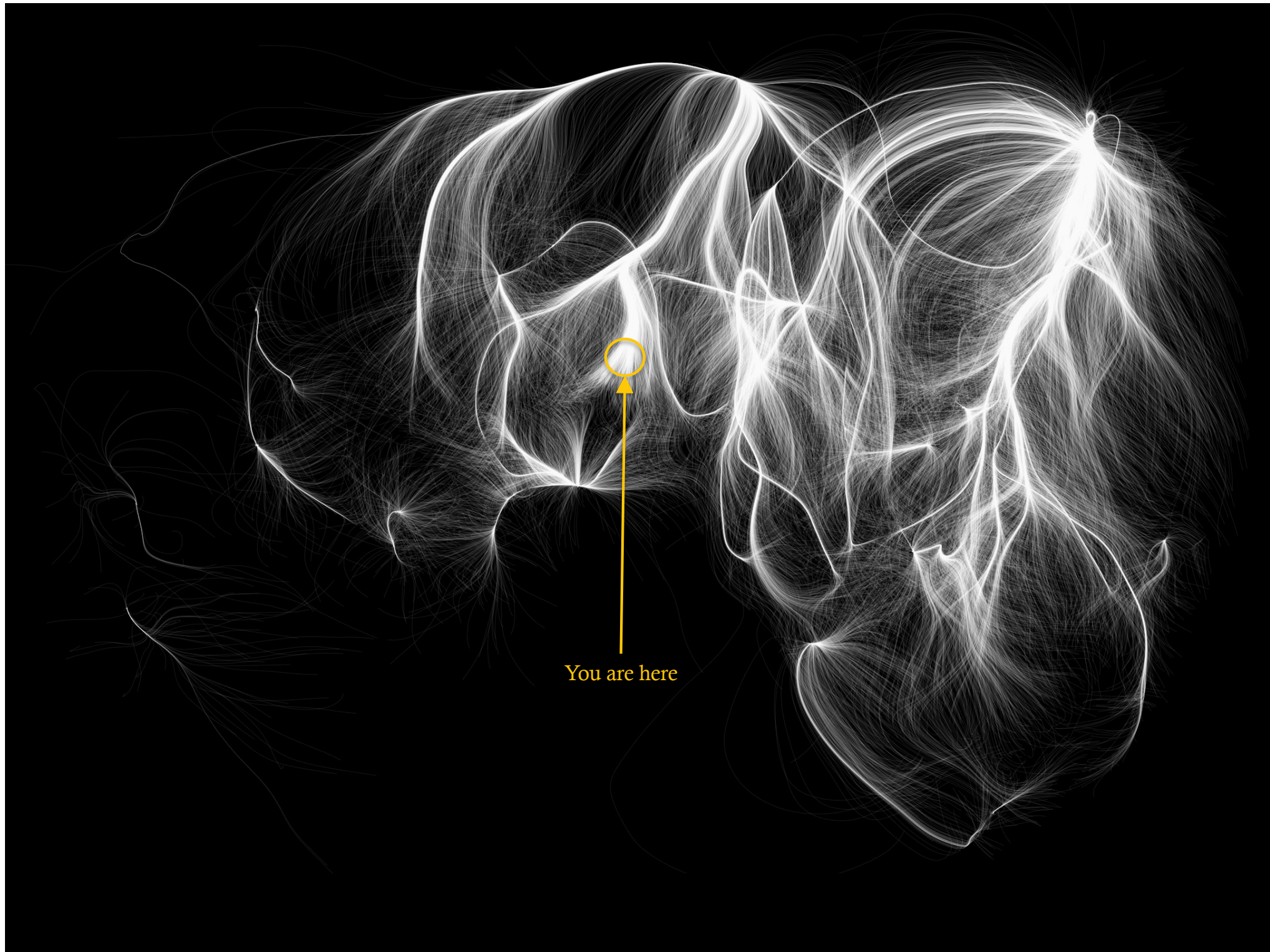
Velocity streamlines from model of the peculiar velocity field

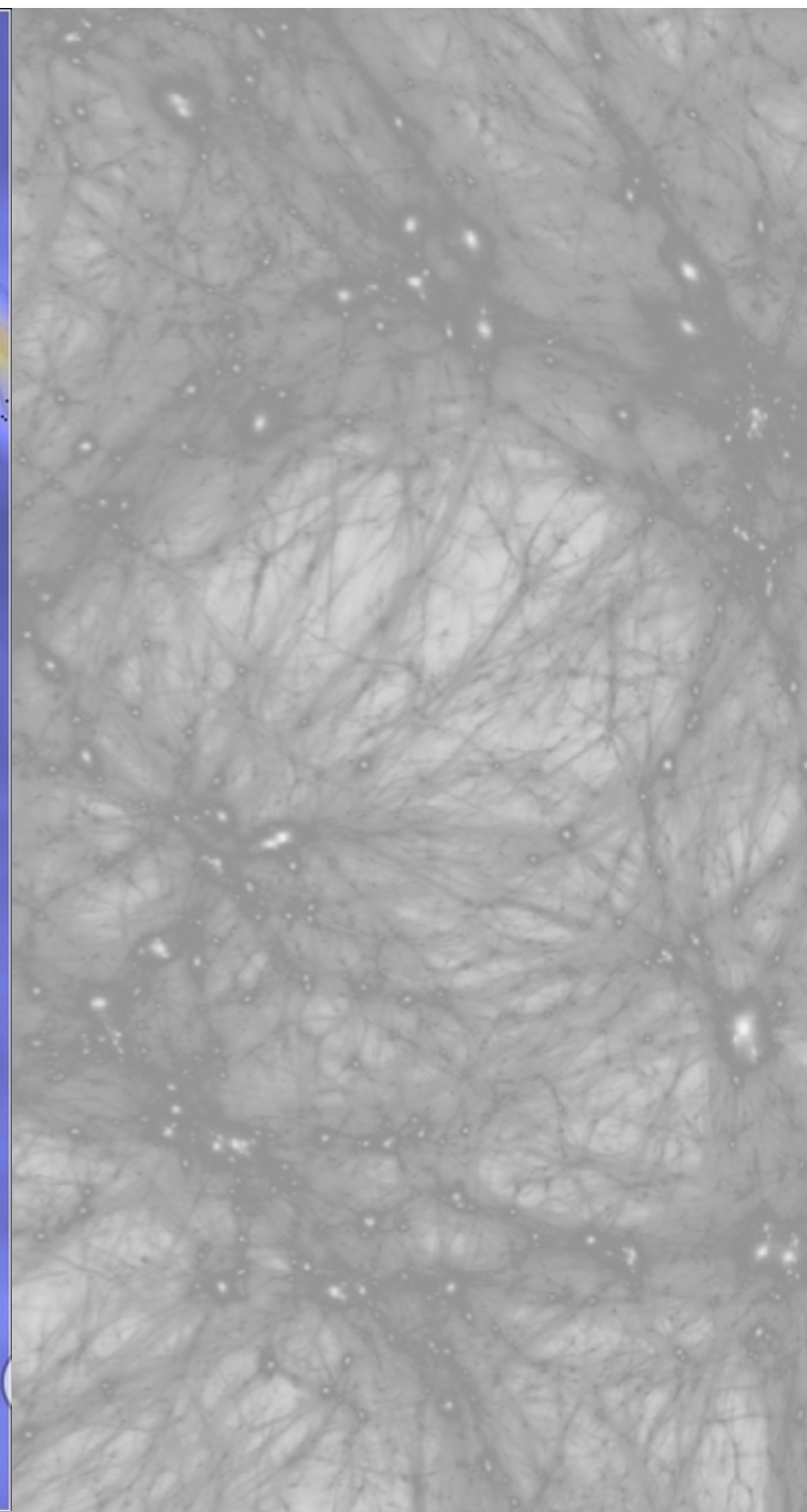
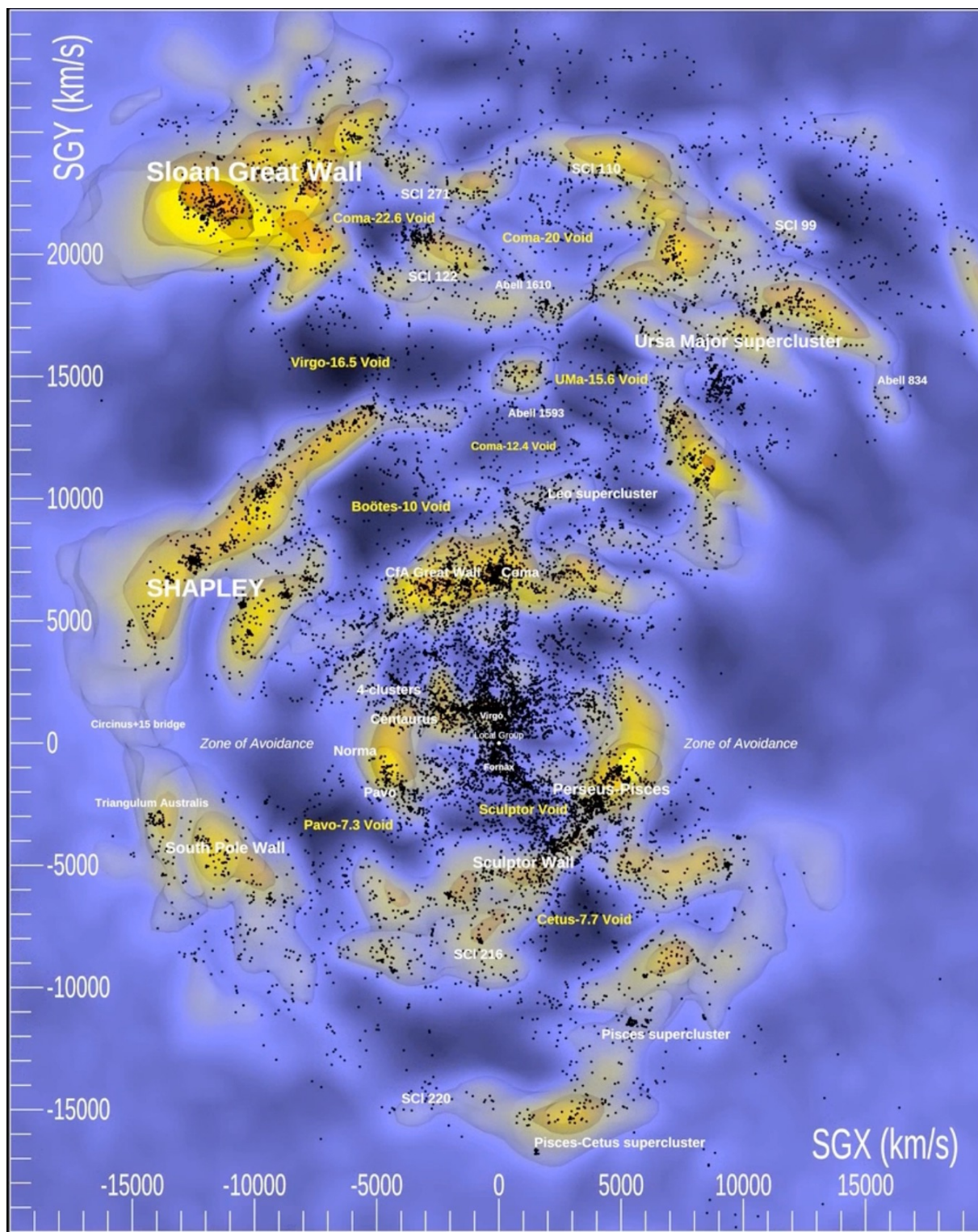
Density field and super clusters

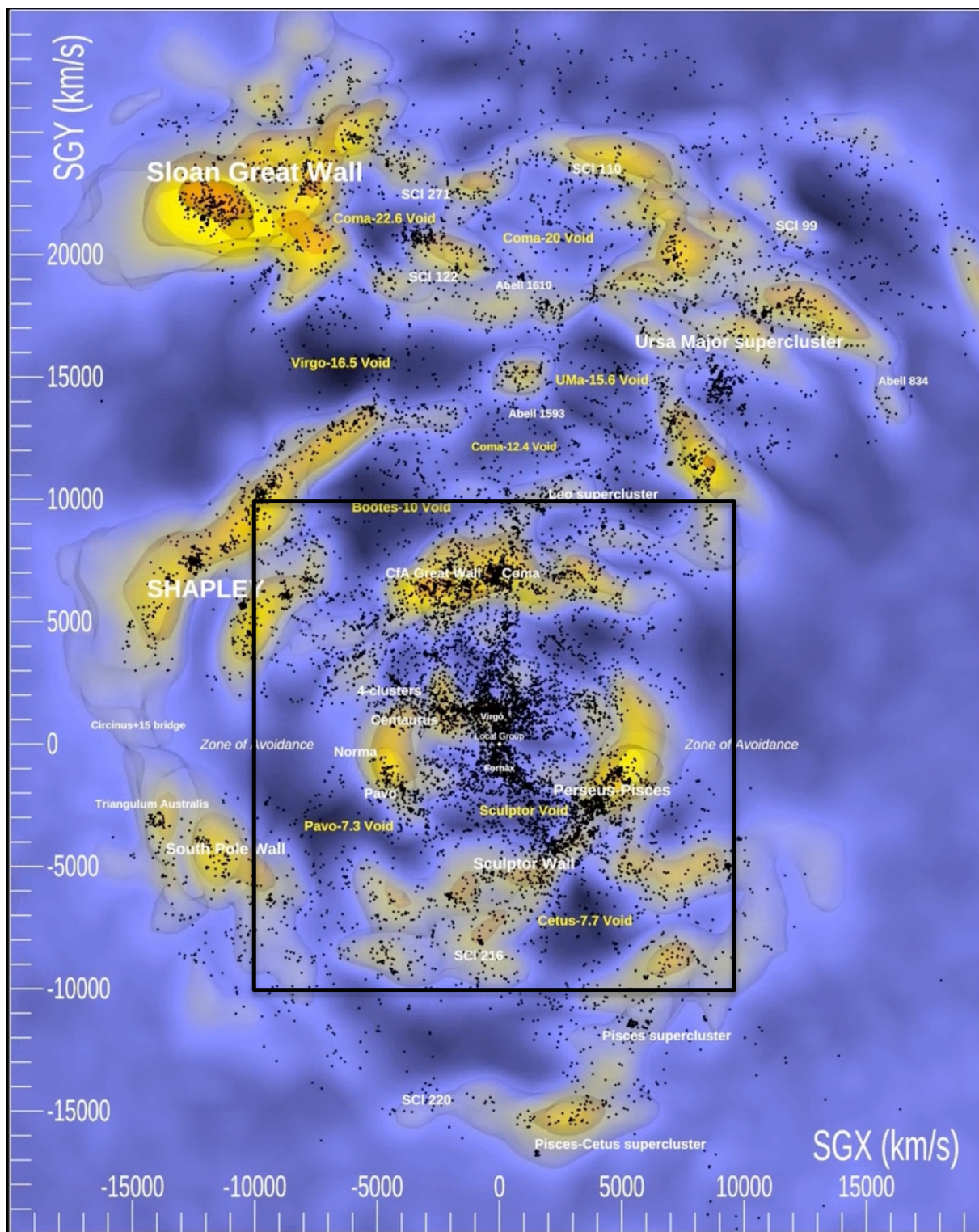
CF4 Reconstruction

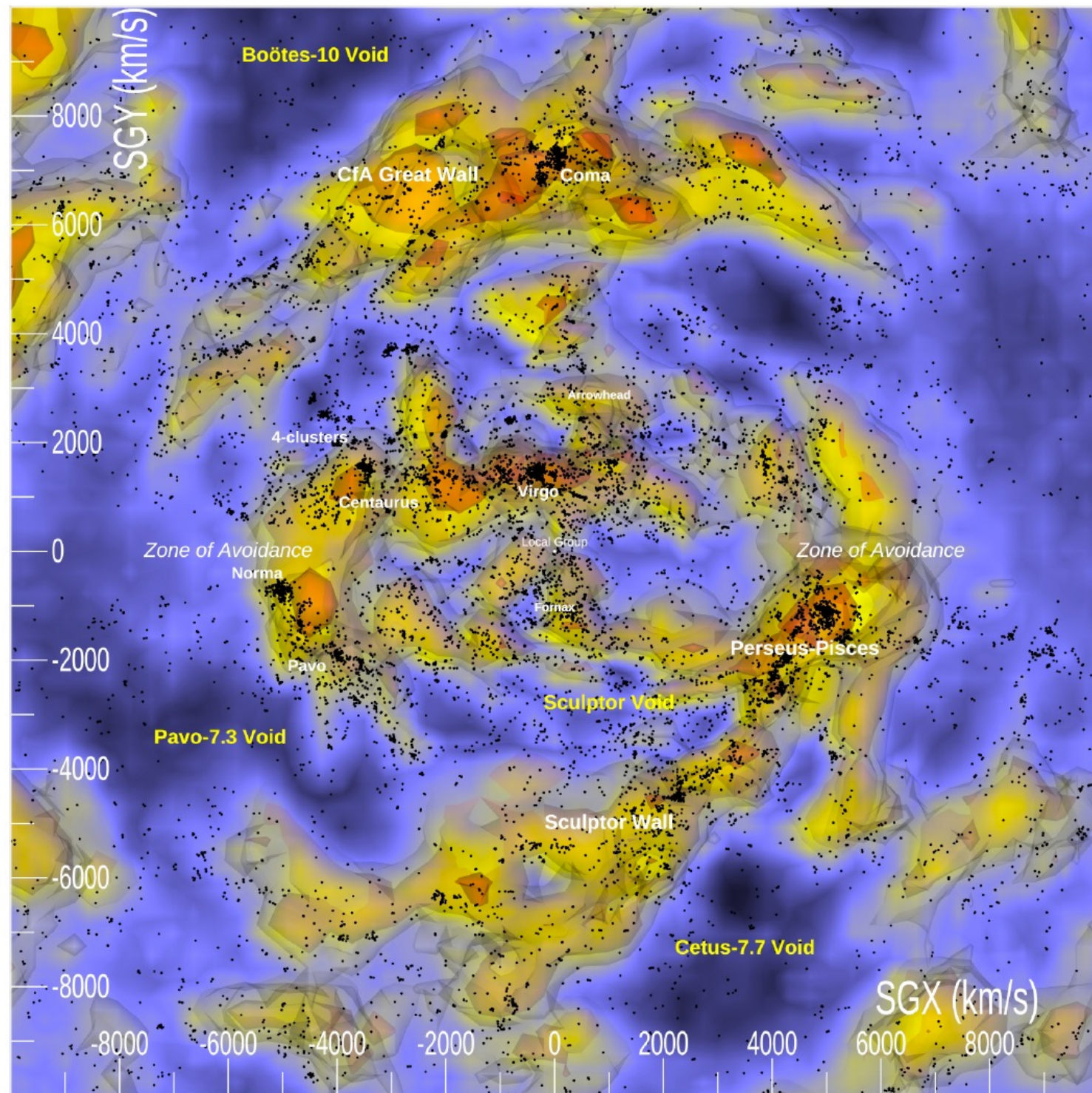
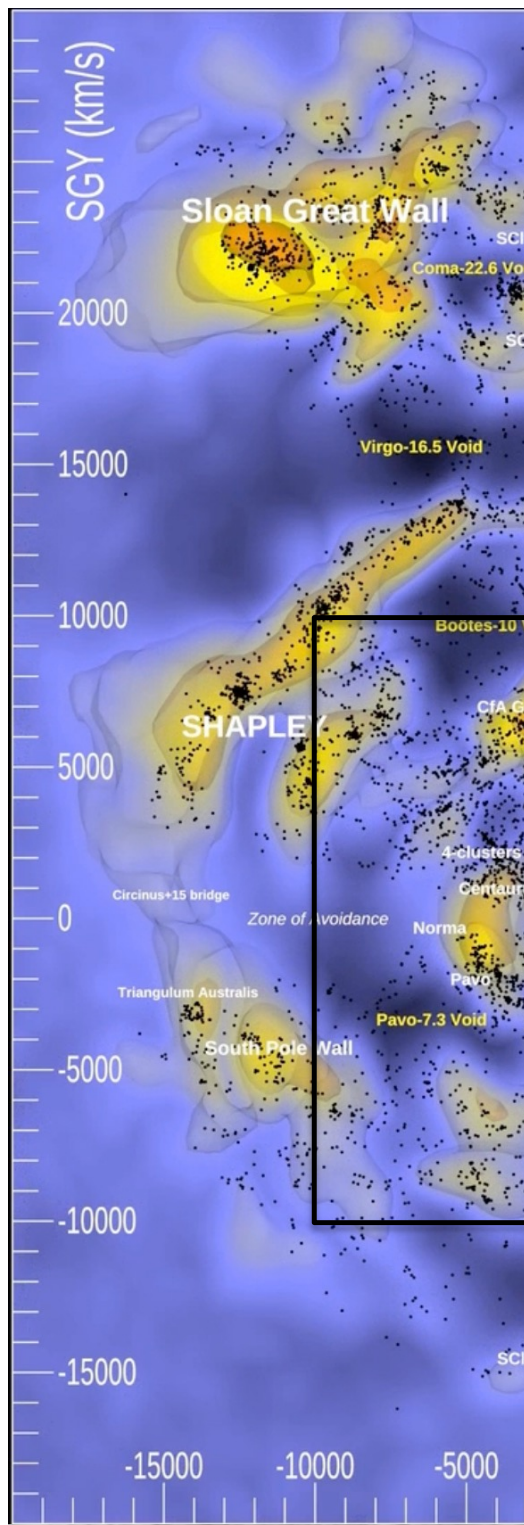


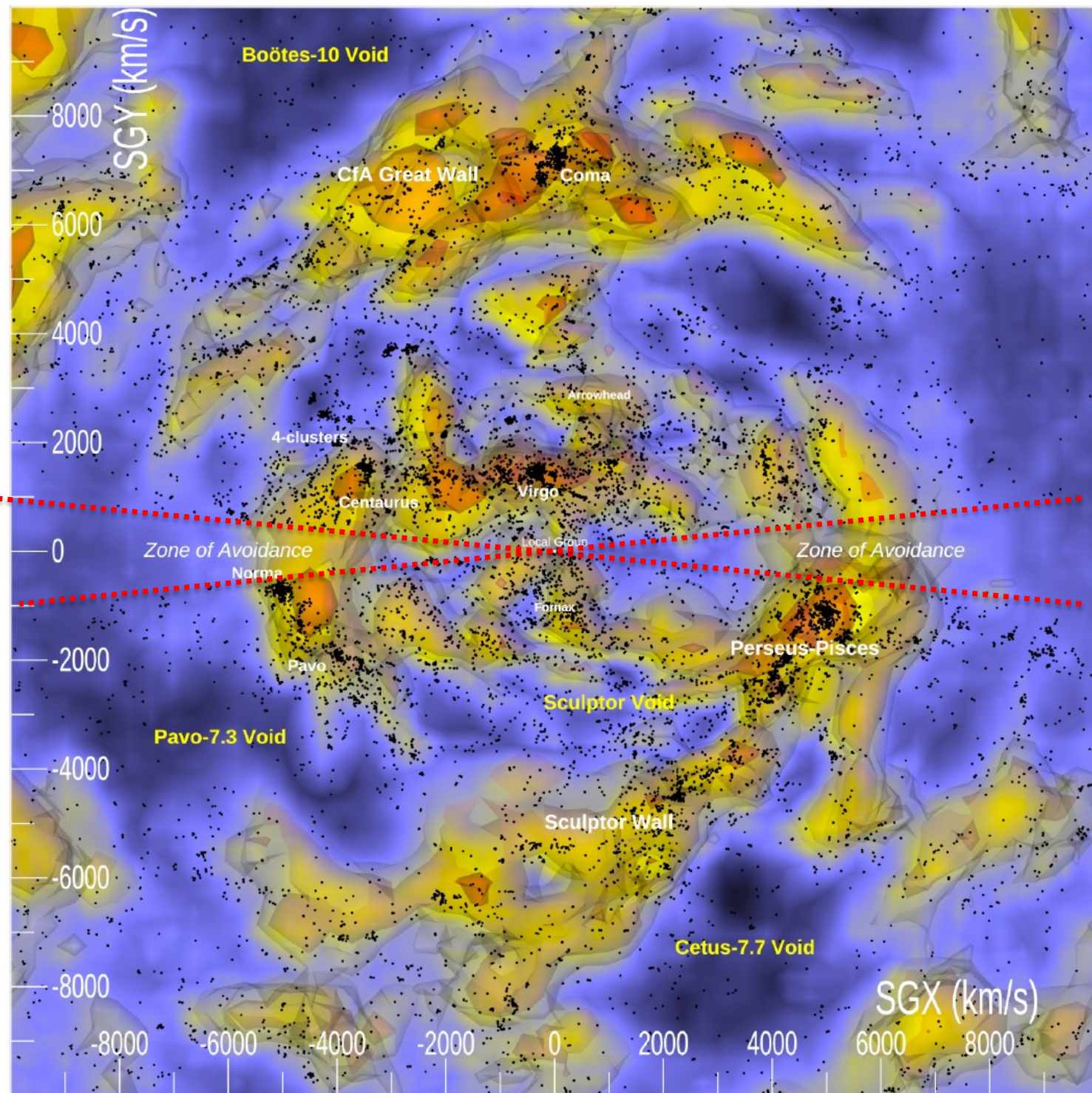
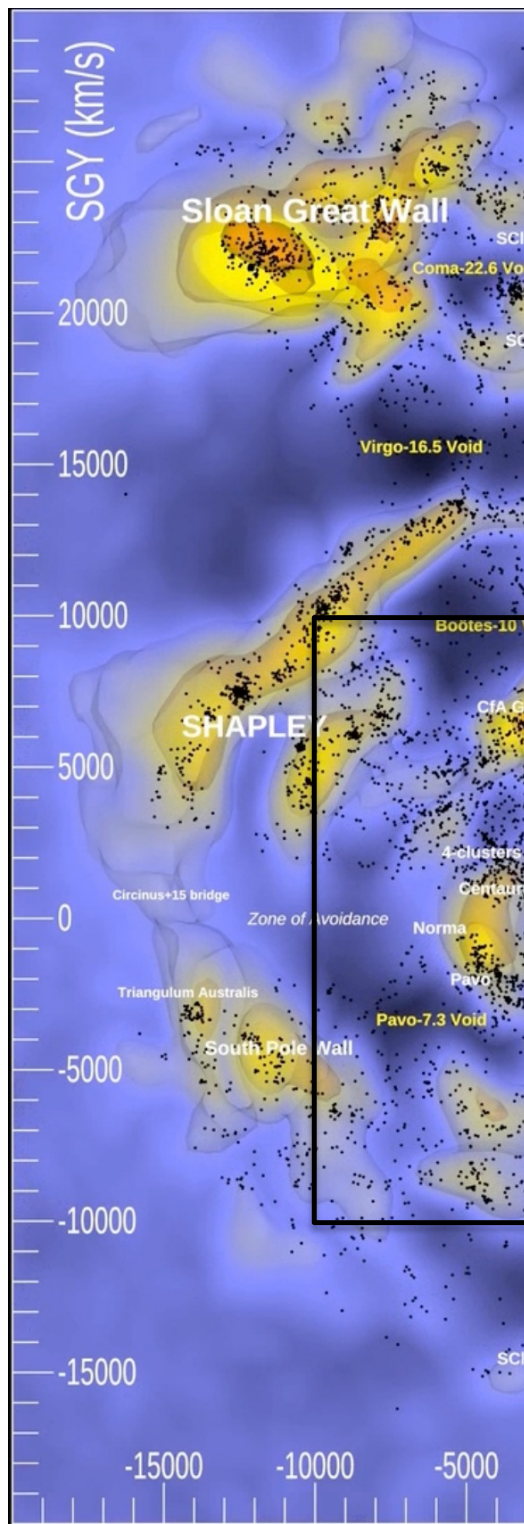
View from negative SGX





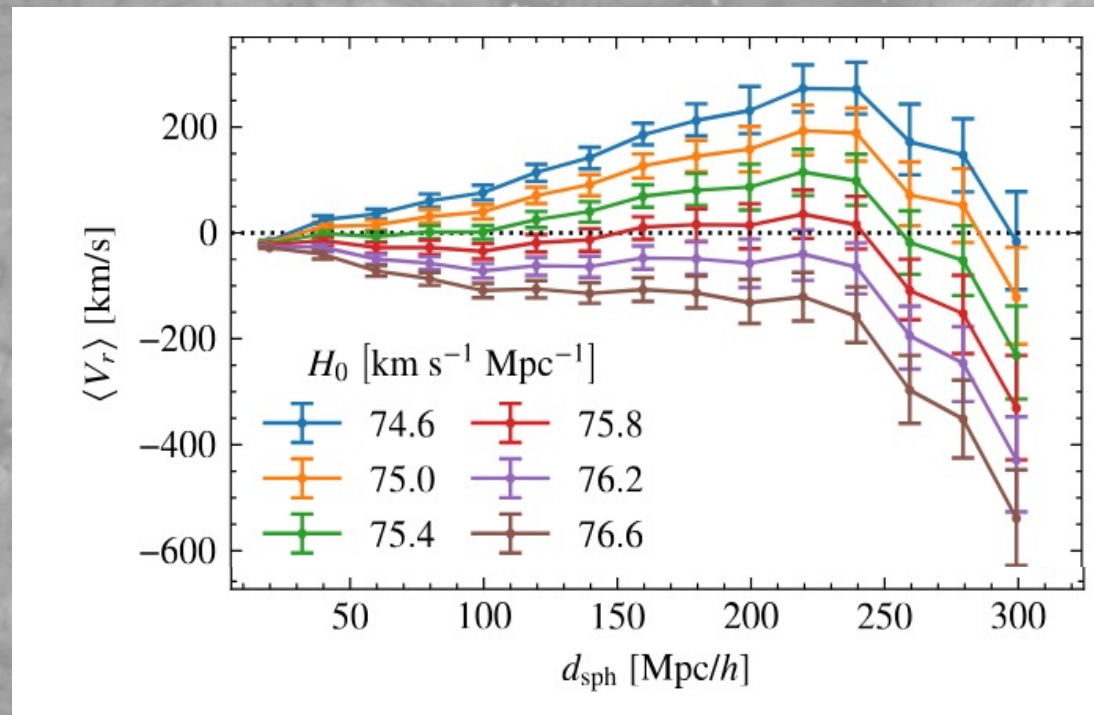




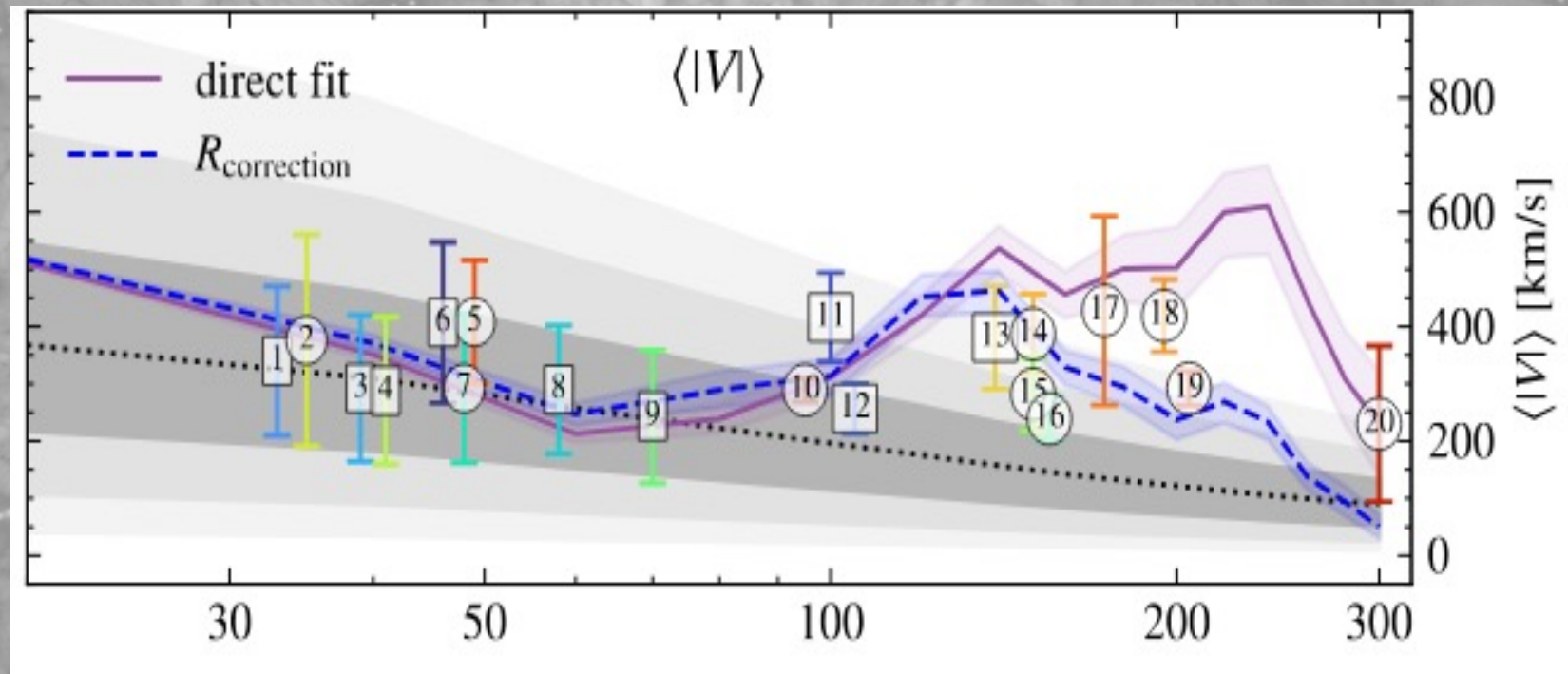


Measuring the cosmic velocity field

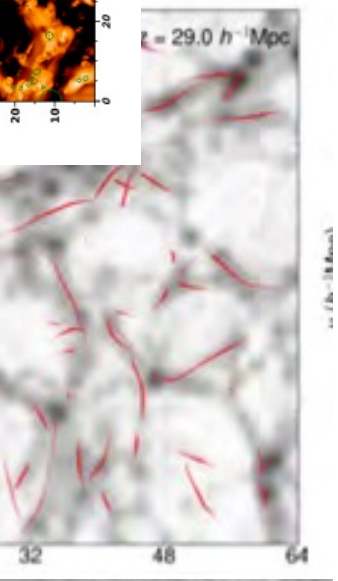
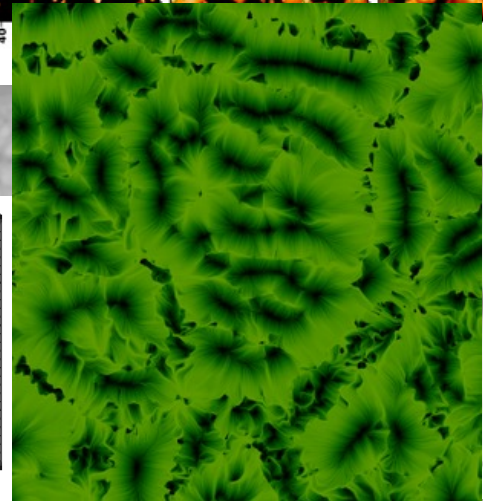
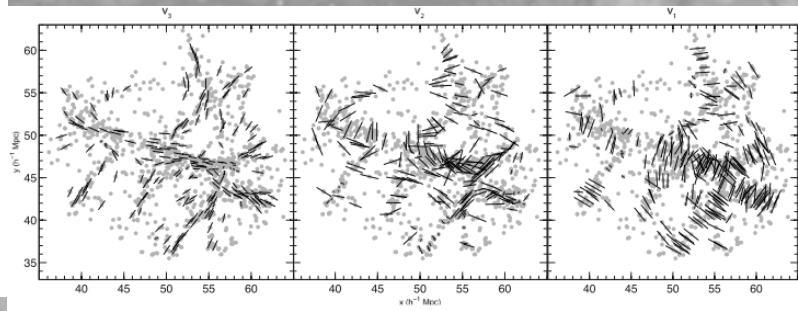
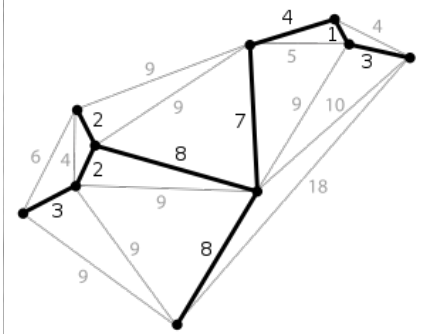
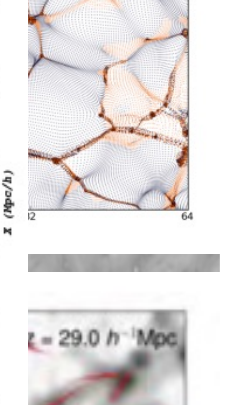
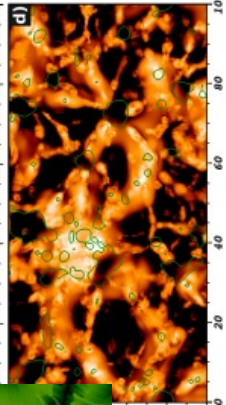
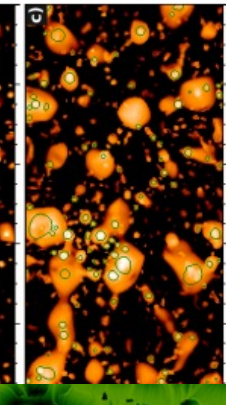
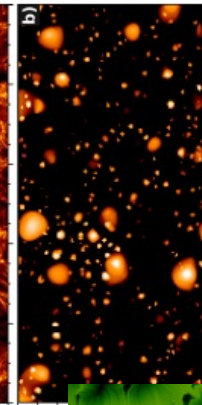
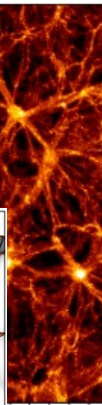
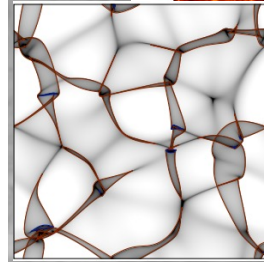
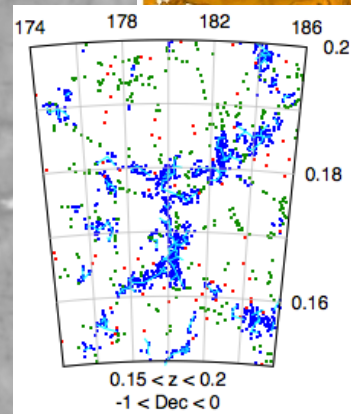
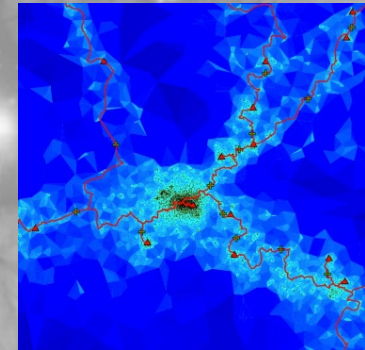
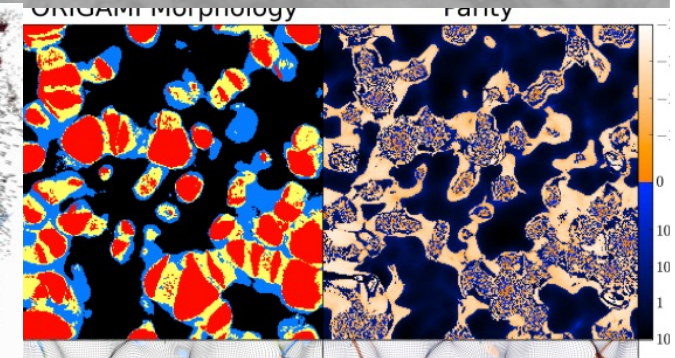
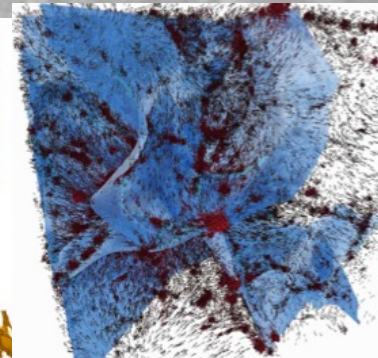
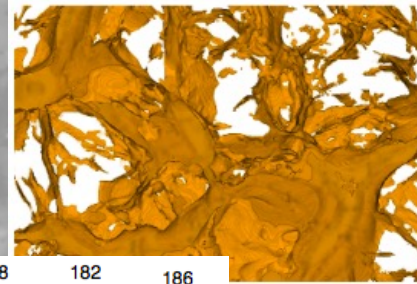
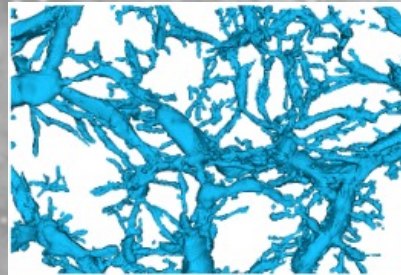
Fixing H_0 by requiring no inflow at some distance



Measuring the cosmic velocity field



The Cosmic Web

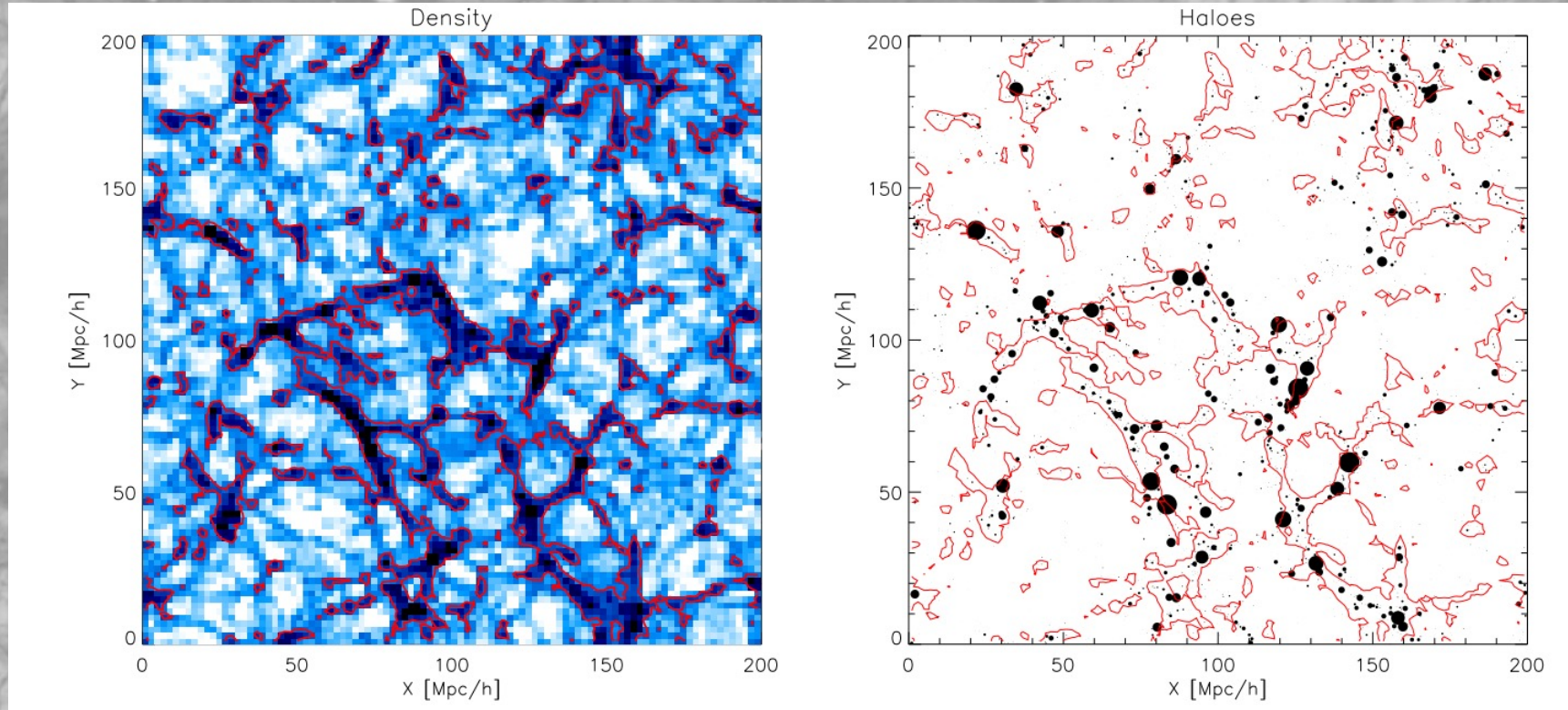




The cosmic web is a vast network of interconnected filaments, nodes, and voids, composed of dark matter, galaxies and gas.

Tracing the cosmic web

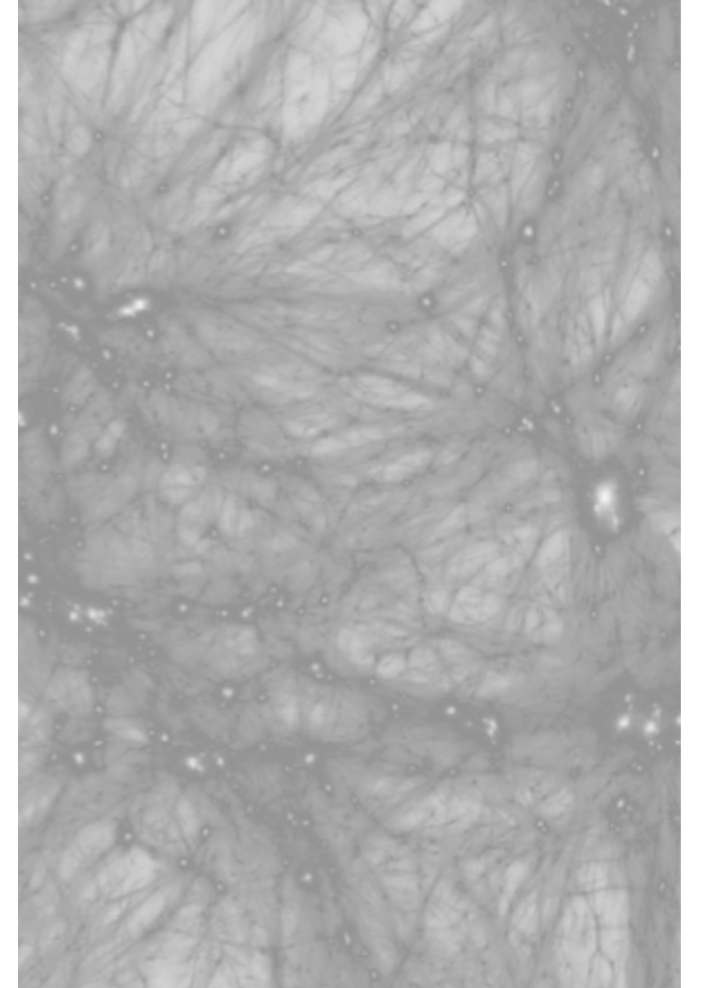
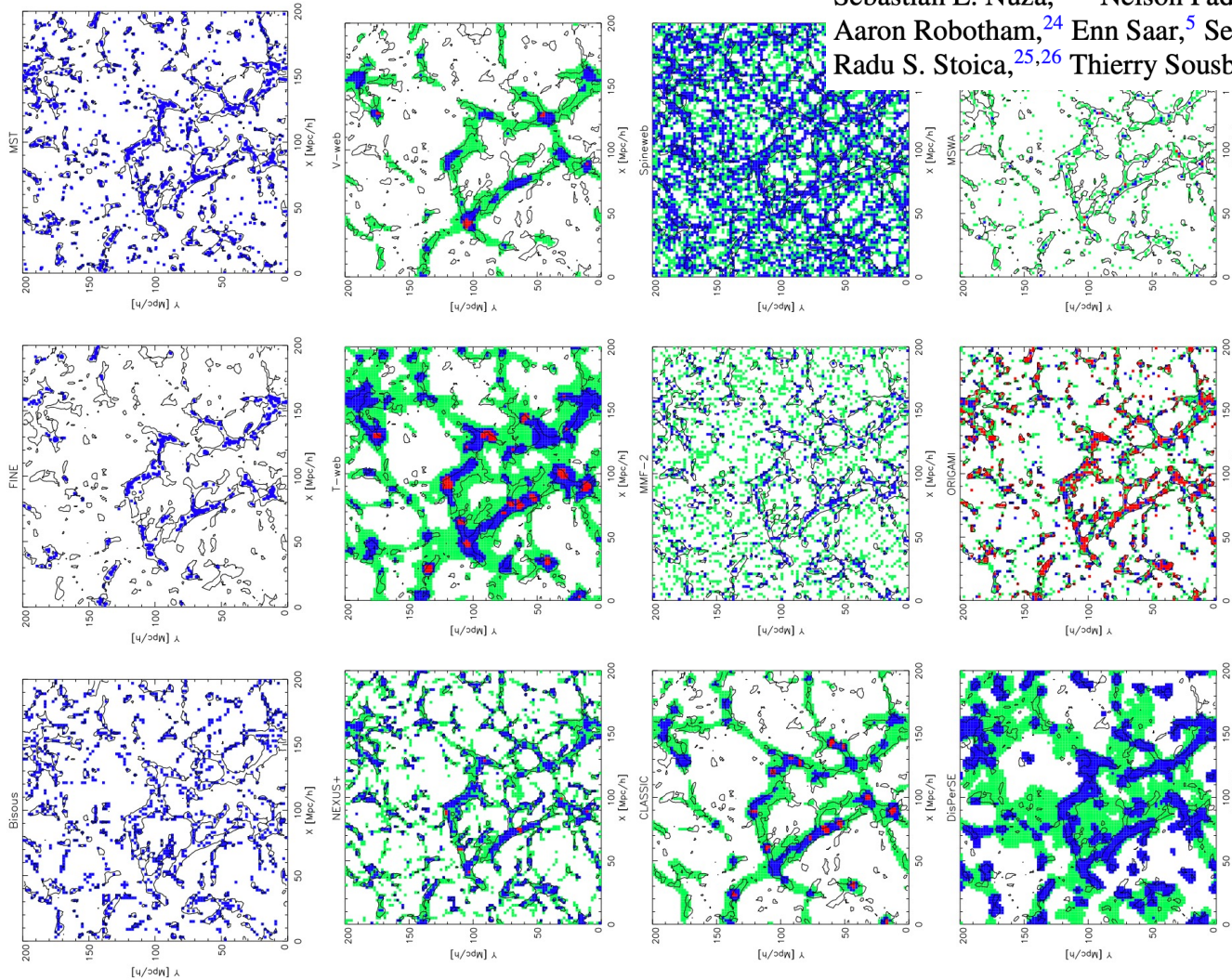
Noam I. Libeskind,^{1★} Rien van de Weygaert,² Marius Cautun,³ Bridget Falck,⁴ Elmo Tempel,^{1,5} Tom Abel,^{6,7} Mehmet Alpaslan,⁸ Miguel A. Aragón-Calvo,⁹ Jaime E. Forero-Romero,¹⁰ Roberto Gonzalez,^{11,12} Stefan Gottlöber,¹ Oliver Hahn,¹³ Wojciech A. Hellwing,^{14,15} Yehuda Hoffman,¹⁶ Bernard J. T. Jones,² Francisco Kitaura,^{17,18} Alexander Knebe,^{19,20} Serena Manti,²¹ Mark Neyrinck,³ Sebastián E. Nuza,^{1,22} Nelson Padilla,^{11,12} Erwin Platen,² Nesar Ramachandra,²³ Aaron Robotham,²⁴ Enn Saar,⁵ Sergei Shandarin,²³ Matthias Steinmetz,¹ Radu S. Stoica,^{25,26} Thierry Sousbie²⁷ and Gustavo Yepes¹⁸



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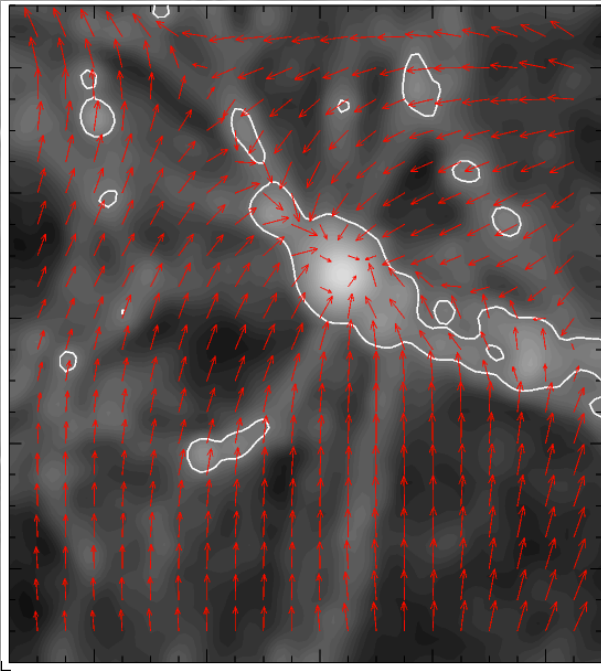
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Velocity Shear Tensor

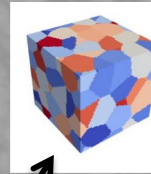
Looking at LSS from the point of view of (*peculiar*) velocity.

Specifically the deformation of the velocity field – shear, compression and rotation:

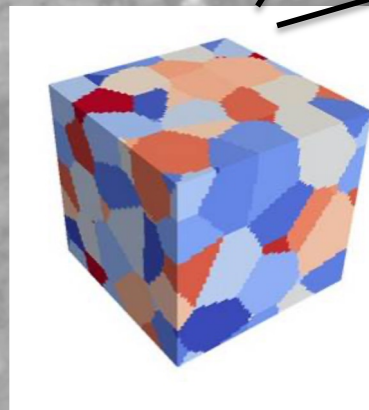
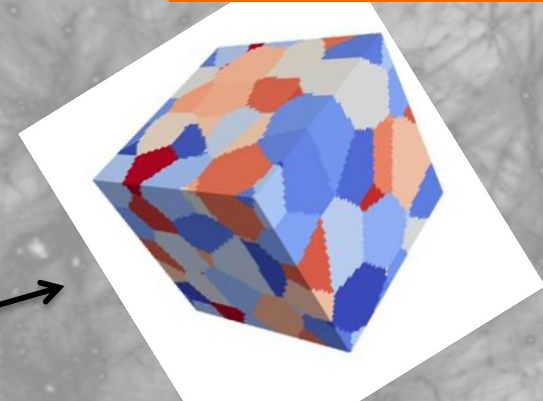


Hoffman et al 2012
Libeskind et al 2012, 2013

Compression/expansion

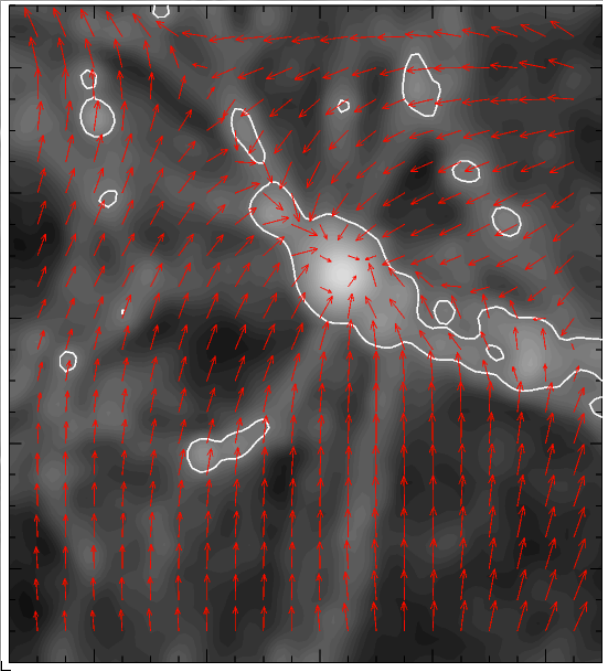


Rotation (vorticity)



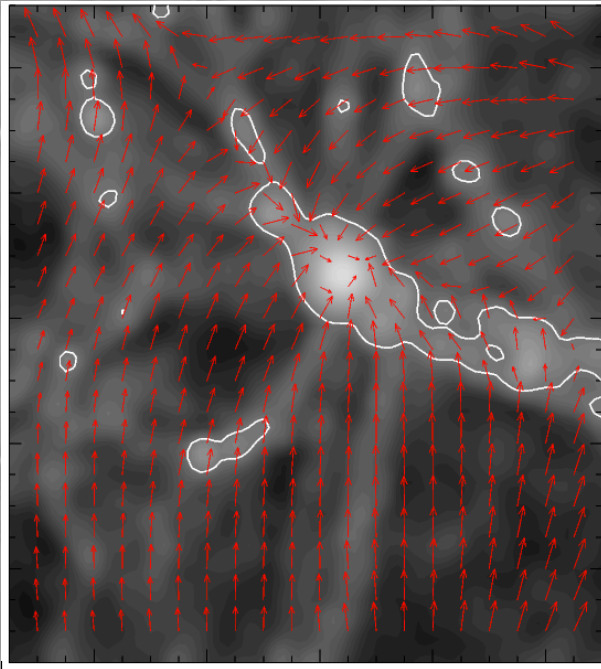
Shear





$$\mathbf{u} = H_0 \mathbf{r} \left(1 + \frac{\mathbf{v}}{H_0} \right)$$

Symmetric part is the
“**Shear**” tensor +
Divergence

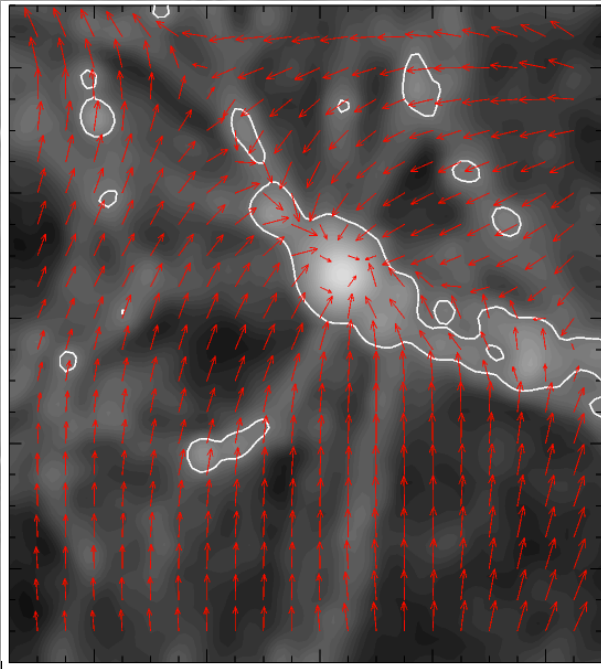


$$\mathbf{u} = H_0 \mathbf{r} \left(1 + \frac{\mathbf{v}}{H_0} \right)$$

$$\begin{aligned} \mathbf{v}(\mathbf{r}) &= \mathbf{v}(\mathbf{r}_0) + \frac{\partial \mathbf{v}(\mathbf{r})}{\partial r} d\mathbf{r} \\ &= \mathbf{v}(\mathbf{r}_0) + \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} & \frac{\partial v_x}{\partial z} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & \frac{\partial v_z}{\partial z} \end{bmatrix} \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} \\ &= \mathbf{v}(\mathbf{r}_0) + \mathbf{S}_{\alpha\beta} d\mathbf{r} \end{aligned}$$

$$\mathbf{S}_{ij} = \Sigma_{ij} + \Omega_{ij}$$

Symmetric part is the
“**Shear**” tensor +
Divergence



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“**Shear**” tensor +
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$$\mathbf{u} = H_0 \mathbf{r} \left(1 + \frac{\mathbf{v}}{H_0} \right)$$

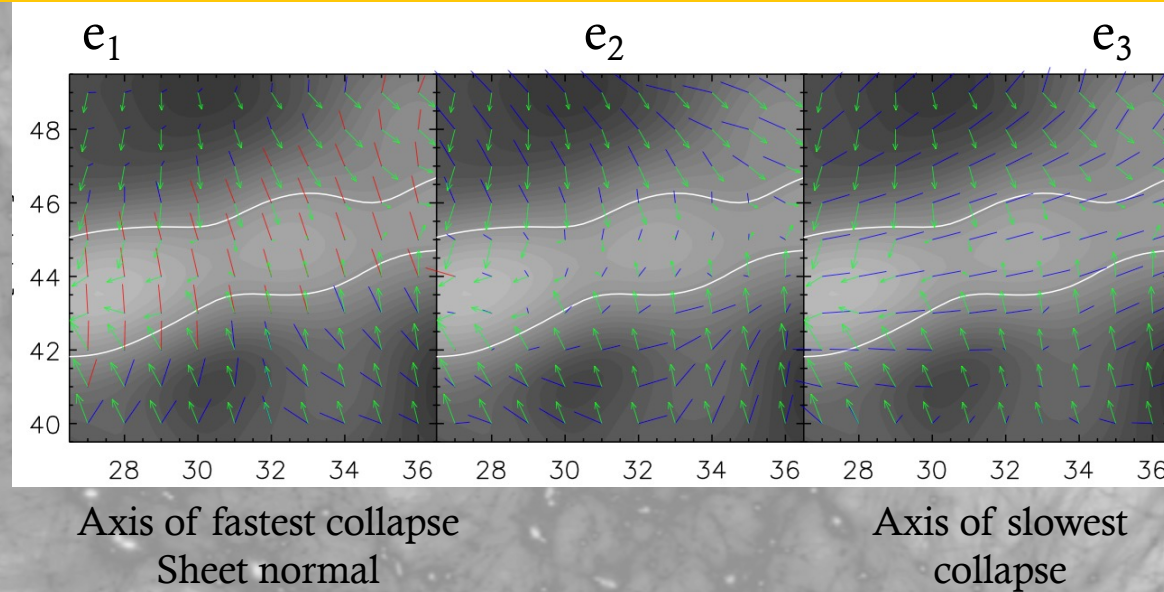
$$\begin{aligned} \mathbf{v}(\mathbf{r}) &= \mathbf{v}(\mathbf{r}_0) + \frac{\partial \mathbf{v}(\mathbf{r})}{\partial r} d\mathbf{r} \\ &= \mathbf{v}(\mathbf{r}_0) + \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} & \frac{\partial v_x}{\partial z} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & \frac{\partial v_z}{\partial z} \end{bmatrix} \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} \\ &= \mathbf{v}(\mathbf{r}_0) + \mathbf{S}_{\alpha\beta} d\mathbf{r} \end{aligned}$$

$$\mathbf{S}_{ij} = \Sigma_{ij} + \Omega_{ij}$$

$$\begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{1}{2} \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right) & \frac{1}{2} \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right) \\ \frac{1}{2} \left(\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} \right) & \frac{\partial v_y}{\partial y} & \frac{1}{2} \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right) \\ \frac{1}{2} \left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \right) & \frac{1}{2} \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right) & \frac{\partial v_z}{\partial z} \end{bmatrix}$$

$$\begin{bmatrix} 0 & \frac{1}{2} \left(\frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} \right) & \frac{1}{2} \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) \\ -\frac{1}{2} \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) & 0 & \frac{1}{2} \left(\frac{\partial v_y}{\partial z} - \frac{\partial v_z}{\partial y} \right) \\ -\frac{1}{2} \left(\frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z} \right) & -\frac{1}{2} \left(\frac{\partial v_y}{\partial z} - \frac{\partial v_z}{\partial y} \right) & 0 \end{bmatrix}$$

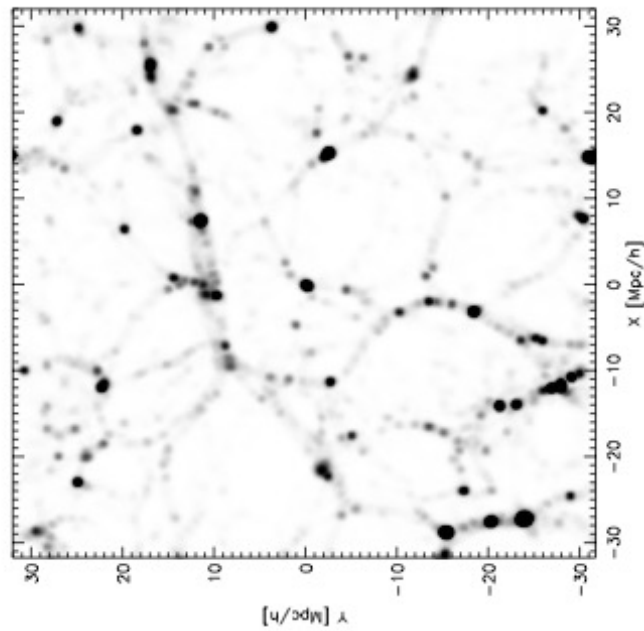
Full (3D) velocity & density field from Wiener filter reconstructions of the cosmic flows-2 survey



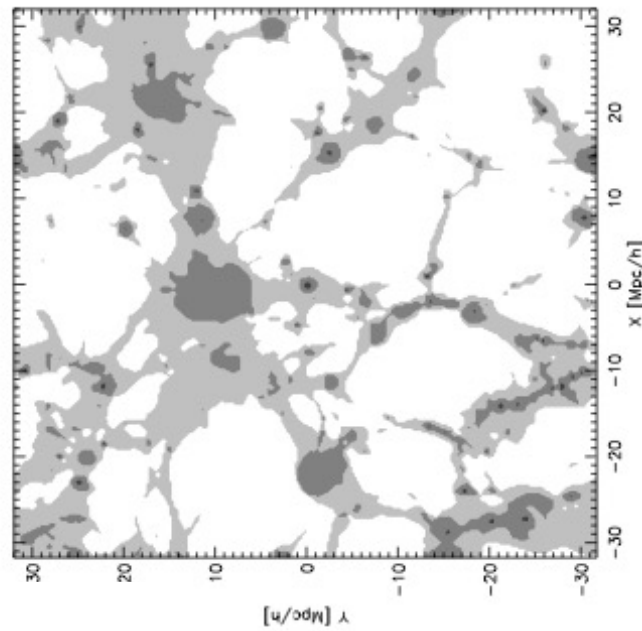
$$\Sigma_{ij} = -\frac{1}{2H(z)} \left(\frac{\partial v_i}{\partial r_j} + \frac{\partial v_j}{\partial r_i} \right),$$

$\lambda_1 > \lambda_2 > \lambda_3$ are the eigenvalues and represent the magnitude of compression (+) or collapse (-)

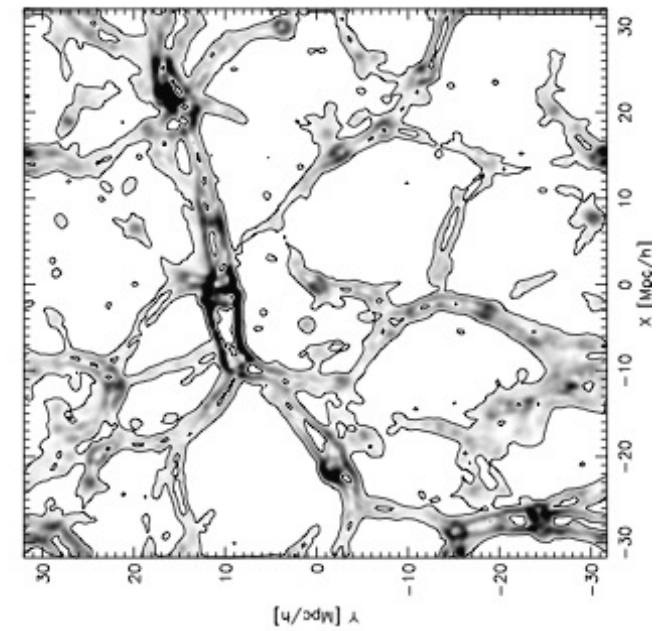
Log density



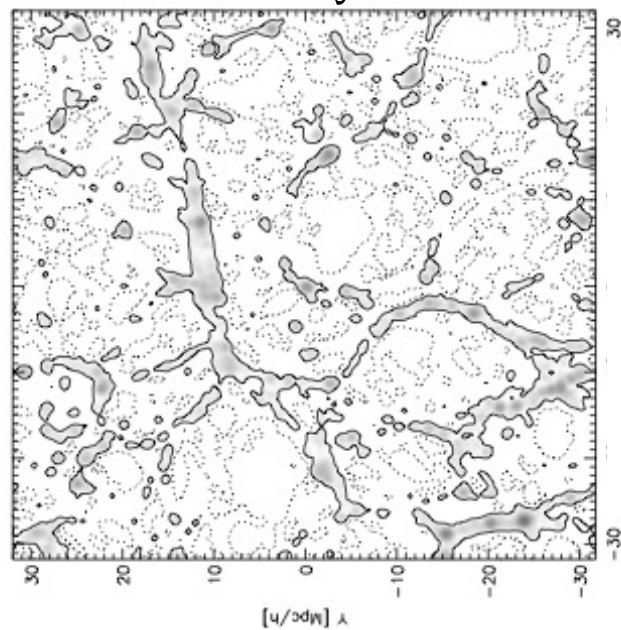
Tidal Tensor



div . v

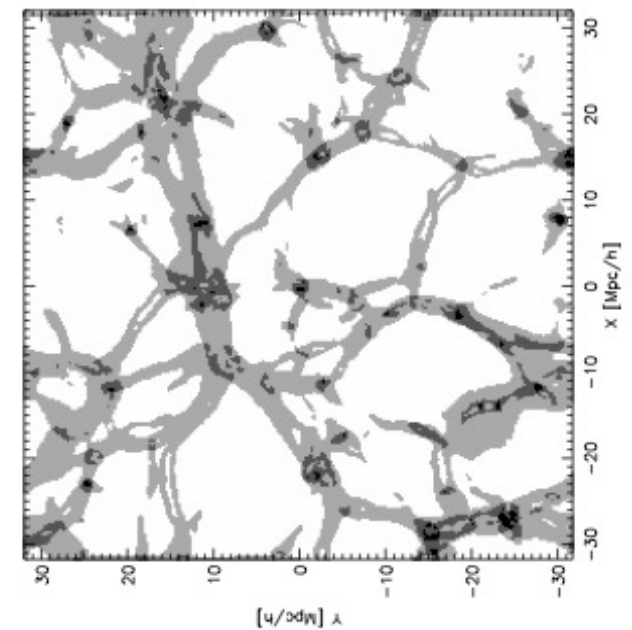


density



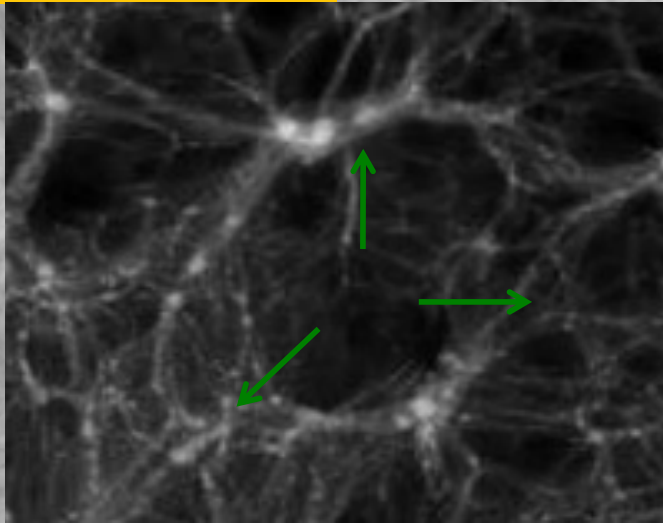
| web elements | volume | mass filling fraction |
|--------------|--------|-----------------------|
| voids | 0.68 | 0.13 |
| sheets | 0.27 | 0.36 |
| filaments | 0.046 | 0.34 |
| knots | 0.0036 | 0.17 |

Shear



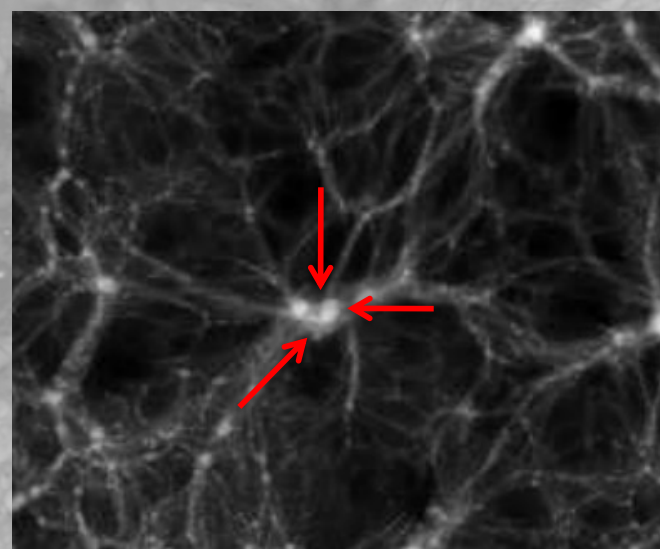
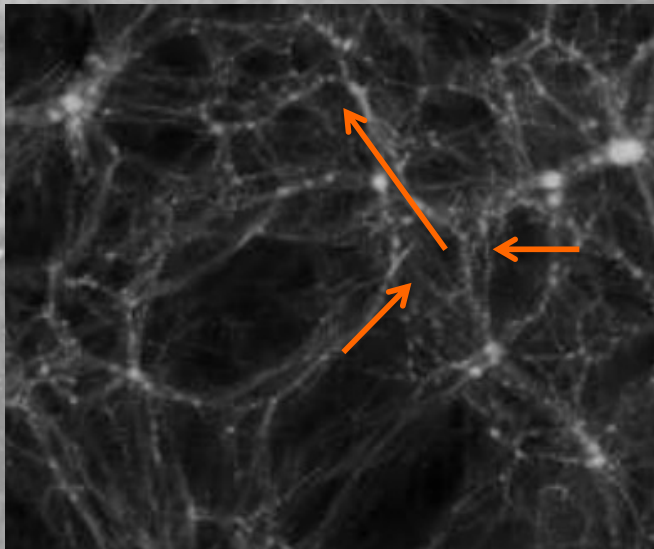
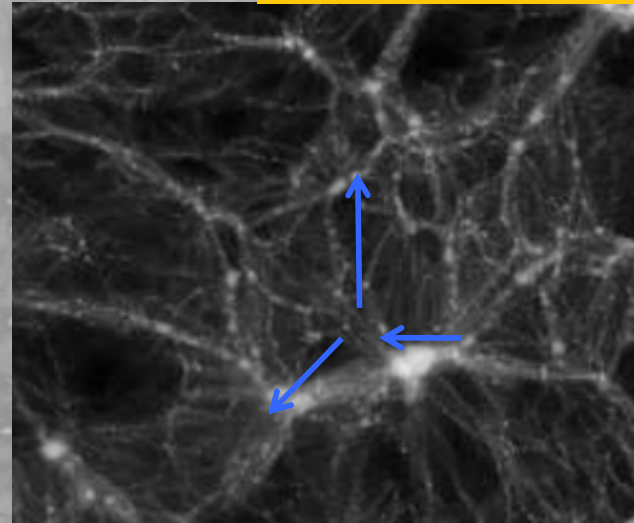
VOID $\lambda_{th} > \lambda_I > \lambda_2 > \lambda_3$

All three eigenvectors of the shear tensor are expanding



SHEET $\lambda_I > \lambda_{th} ; \lambda_2, \lambda_3 < \lambda_{th}$

Collapse along one axis (\hat{e}_1), expansion along the two (\hat{e}_2, \hat{e}_3)



FILAMENT $\lambda_I, \lambda_2 > \lambda_{th} ; \lambda_3 < \lambda_{th}$

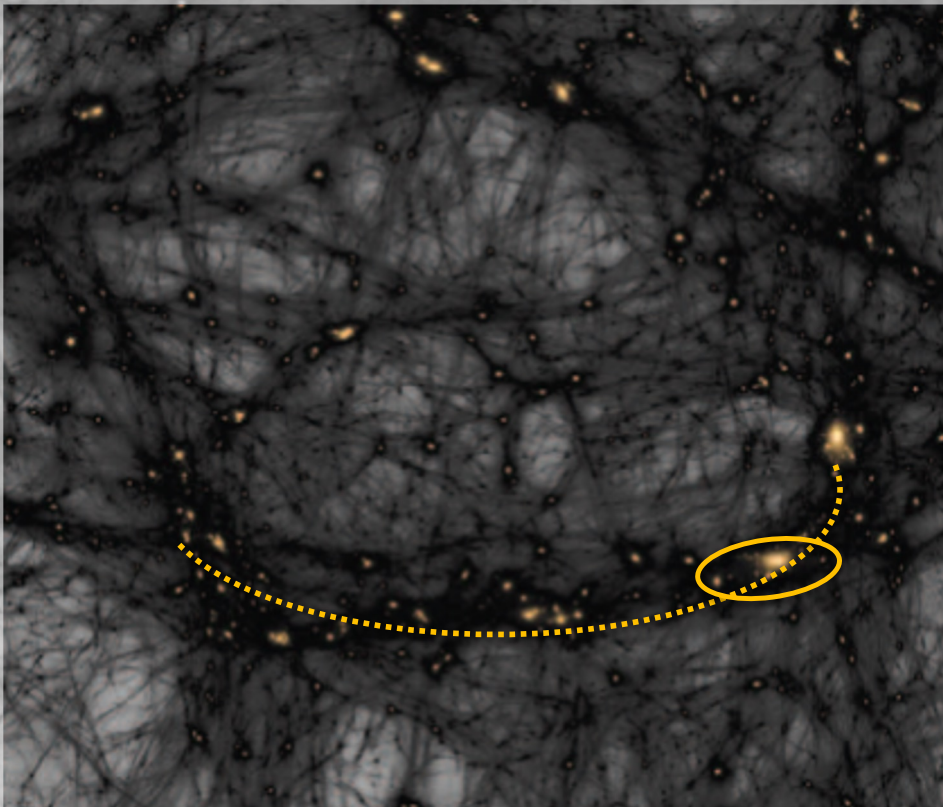
Collapse along two axes (\hat{e}_1, \hat{e}_2), expansion along the other (\hat{e}_3)

KNOT $\lambda_I > \lambda_2 > \lambda_3 > \lambda_{th}$

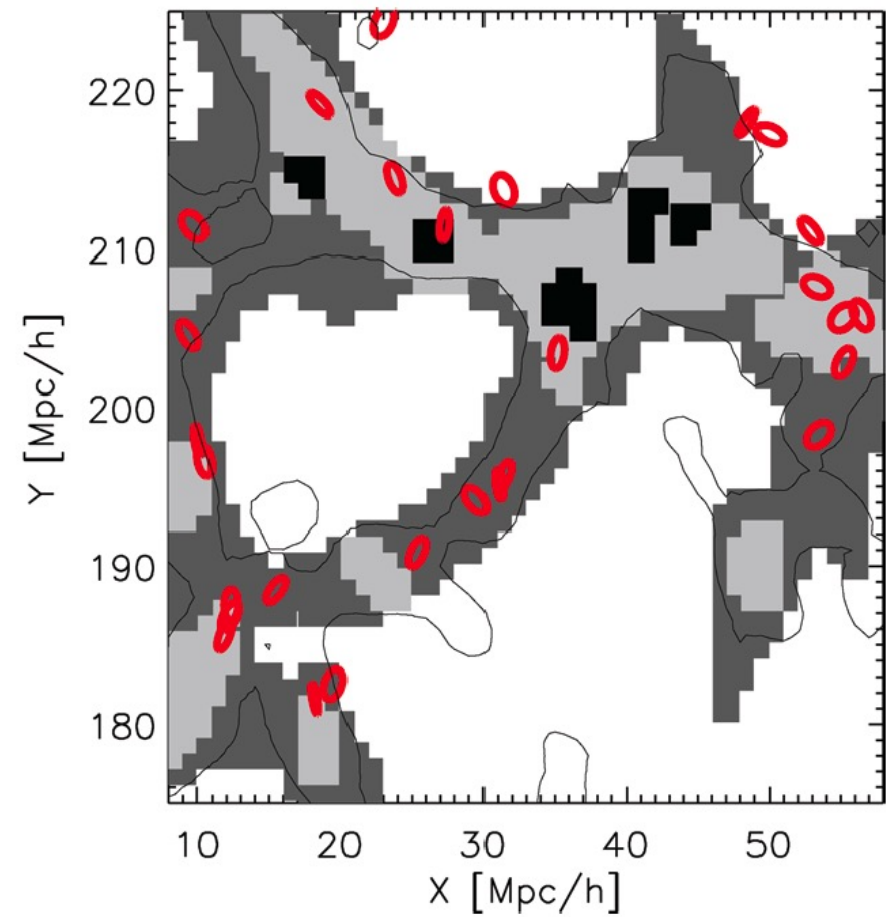
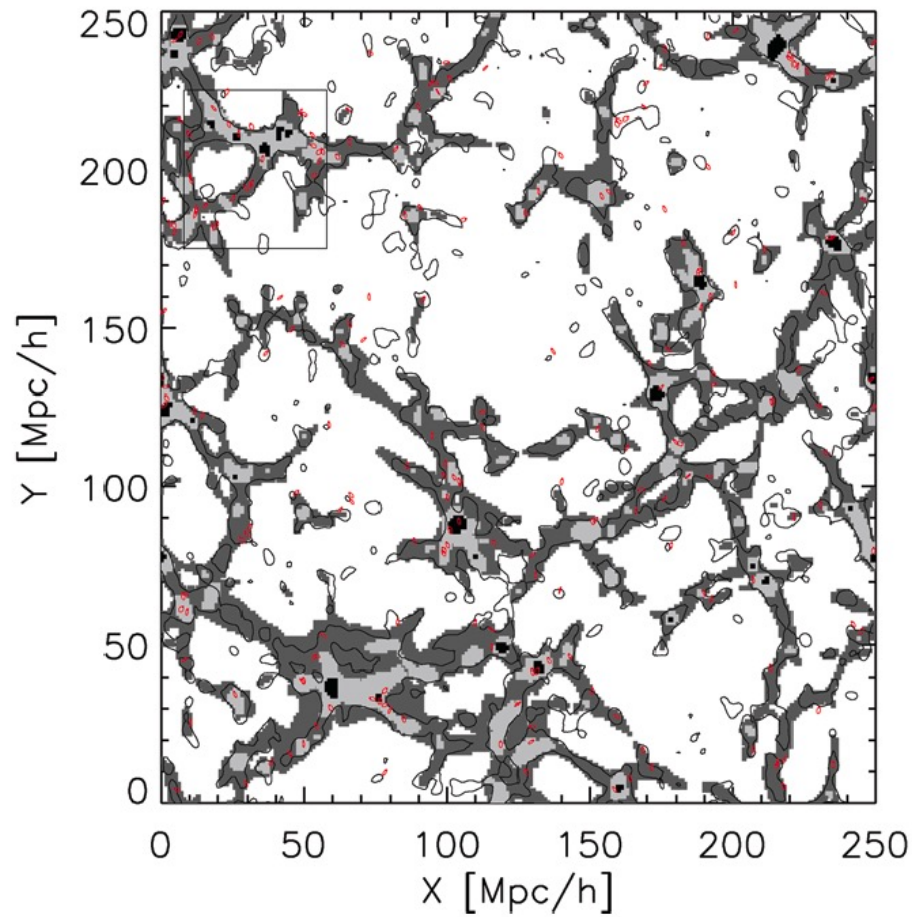
All three eigenvectors of the shear tensor are collapsing

The Shear Tensor: Alignment of halo shape

Examine short axis of a DM halo with the shear field



The Shear Tensor: Alignment of halo shape

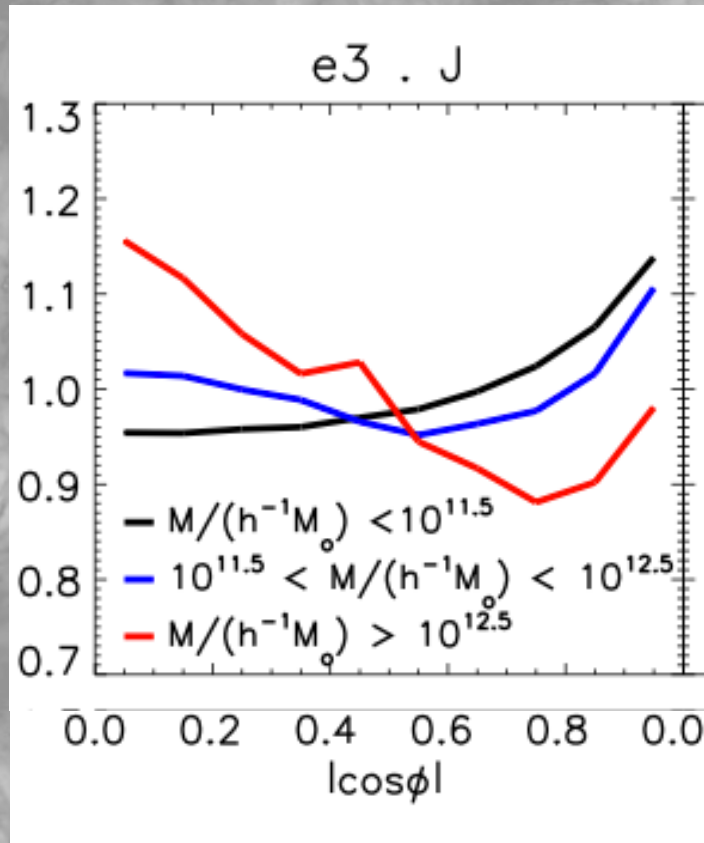


Libeskind et al 2012

Haloes are aligned with the large scale structure

How do spins align with the cosmic web defined by the shear?

Libeskind et al 2013



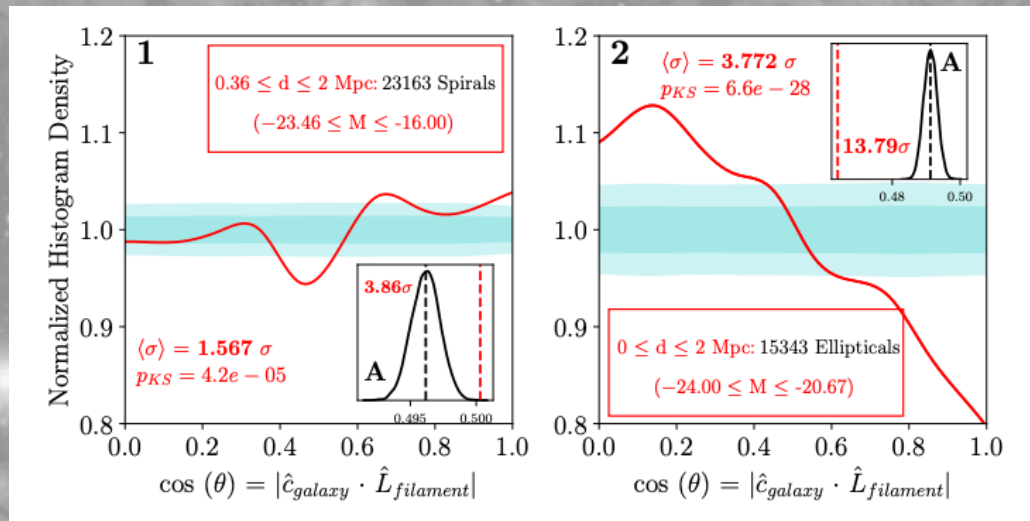
Low mass haloes – **spin aligned** with filament axis: spins wind up with filament

High mass haloes have a **spin flip** due to accretion

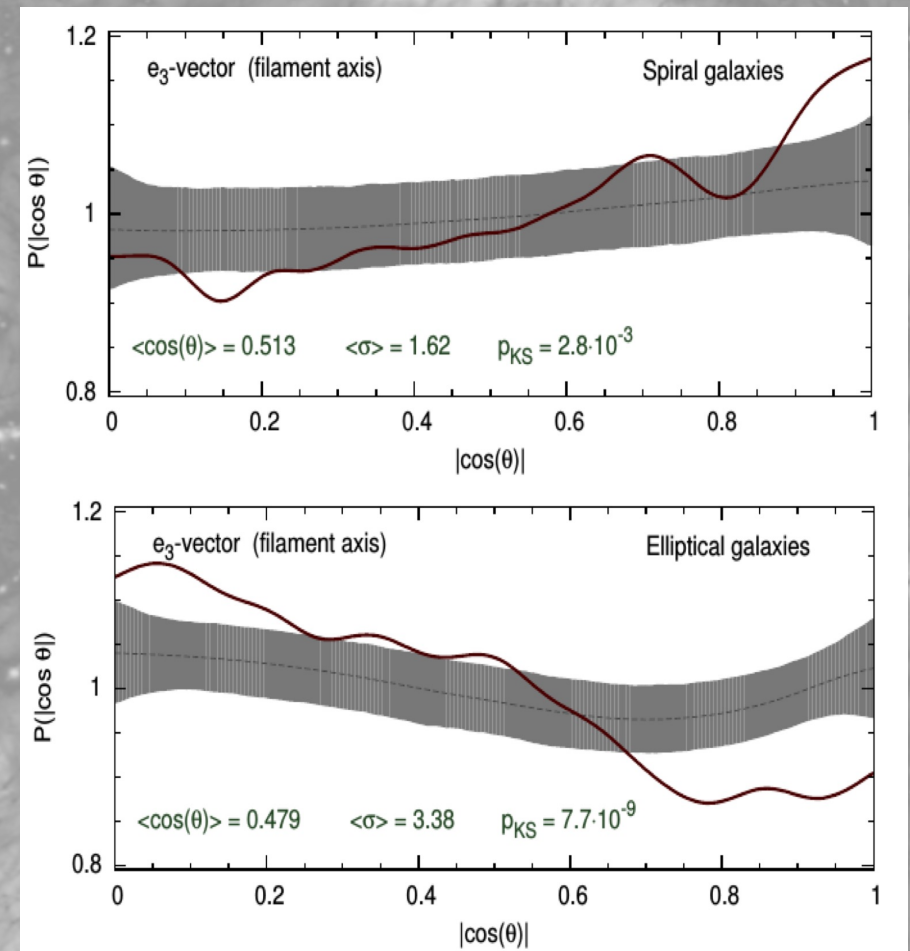
Aragon-Calvo et al 2007, 2013

Can we observe spin flip in data? Do observations confirm theoretical picture?

Tempel & NL 2013



Muralichandran, NL *in press*



A grayscale visualization of the cosmic web, showing a complex network of dark, filamentary structures (galaxy clusters and filaments) against a lighter background. Numerous bright, point-like sources representing galaxies are scattered throughout the network.

Conclusions

The universe can be simulated with great success

The universe can be mapped – even if there are enormous biases and errors

The cosmic web is beautiful