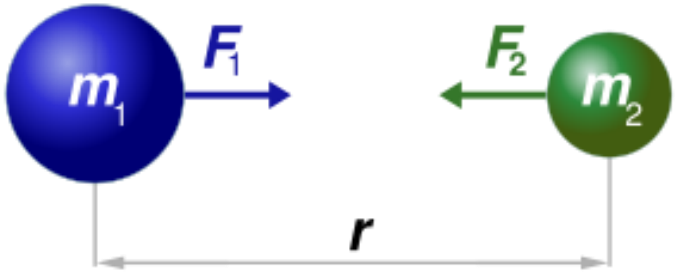
KEPLER LAW

And application to exoplanet

All is about gravitational force

$$F = \frac{GM_1M_2}{r^2}$$



$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

F – Attraction force between masses

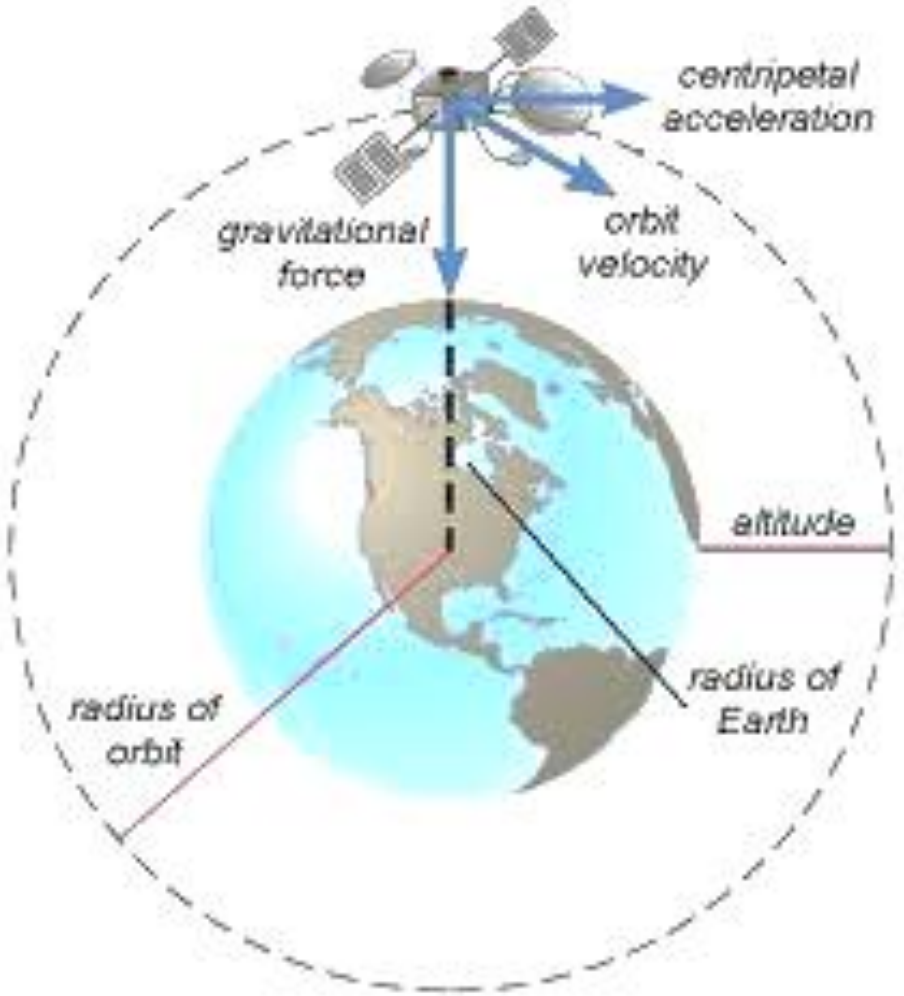
M_1 – Mass of object 1

M_2 – Mass of object 2

r – Distance (between CM of the two masses)

G - Gravitational constant = 6.67 x 10^{-11} N m^2 per kg^2

SIMPLE CASE: CIRCULAR MOTION



$$v = \frac{2\pi r}{T}$$

v velocity of an object in n circular motion

r Distance between the two masses (CM)

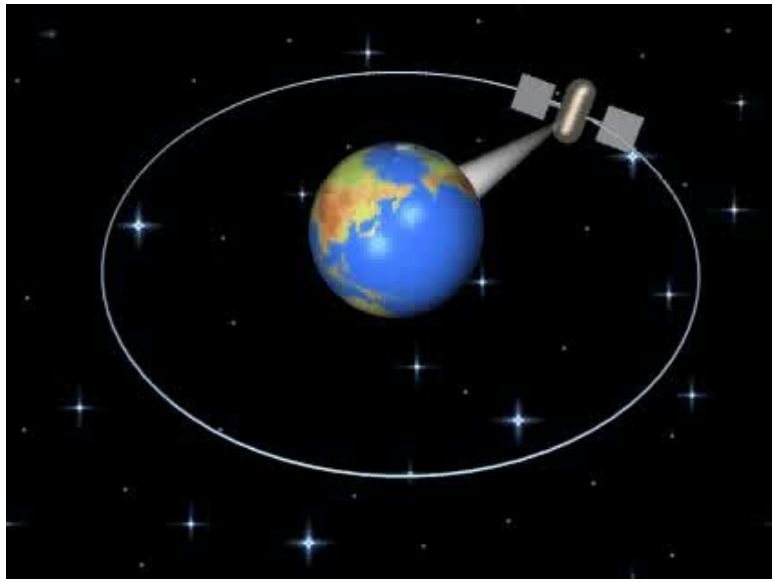
T Time to complete one orbit: a period.

Circular Motion

$$F_c = F_g$$

Under an influence of Earth Gravity

$$\frac{mv^2}{r} = G\frac{Mm}{r^2}$$



$$v = \sqrt{\frac{GM}{r}}$$

To calculate orbital velocity (constant for circular orbit).

Constant v in a circular orbit $\quad v = \frac{2\pi r}{T}$

Re-arranging, we have $\quad T^2 = \frac{4\pi^2}{GM} r^3$

However….

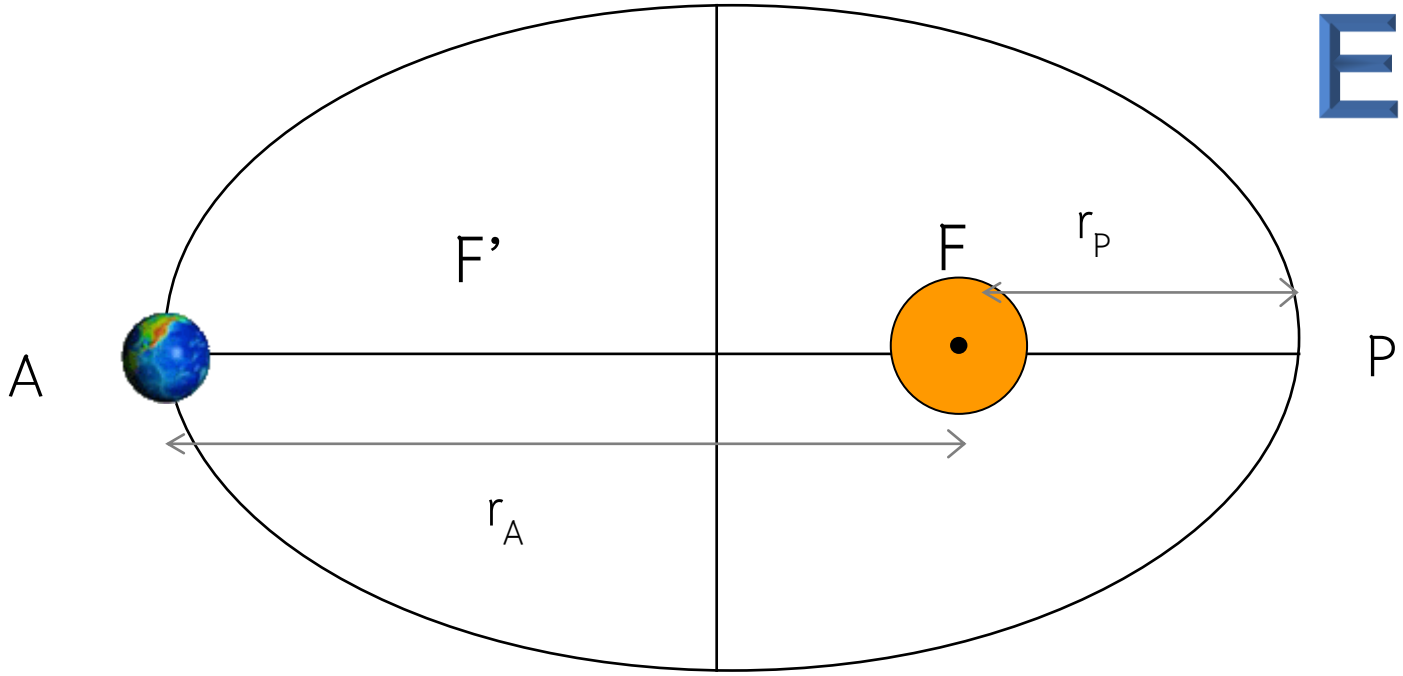
The motions of objects in our (and any) solar system are "Elliptical Orbit"

Astronomer
Johannes Kepler
(1571-1630)

Kepler's law

https://www.universetoday.com/

Elliptical Orbit

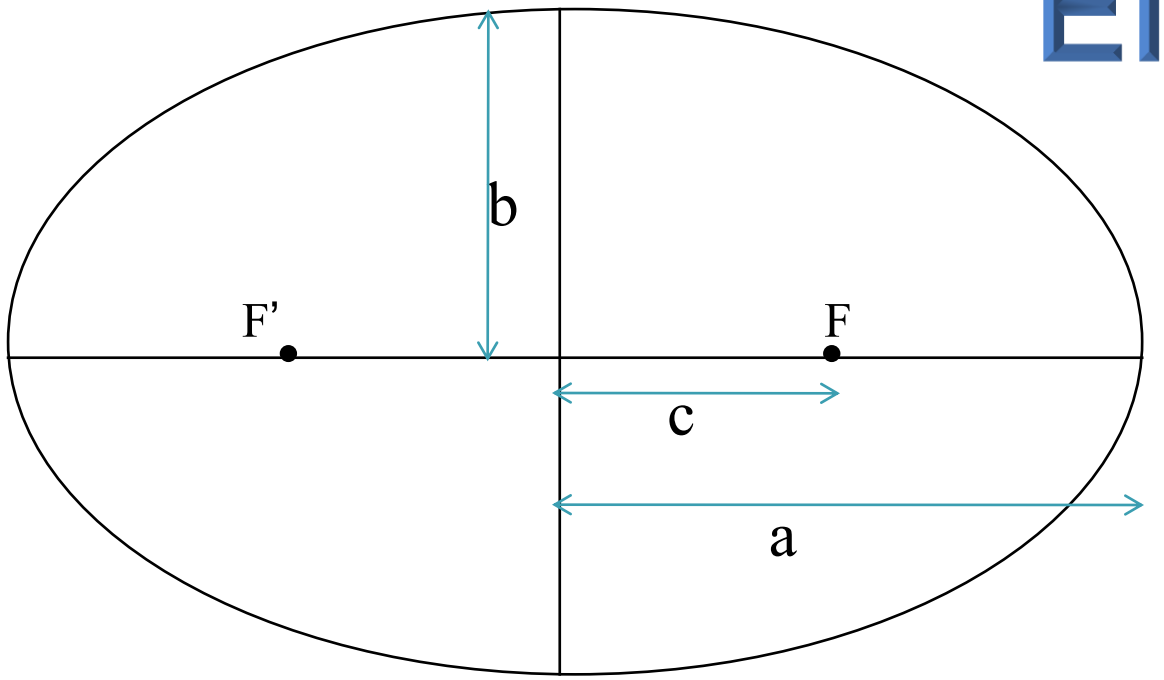


Earth and other planets orbit around the Sun with elliptical orbits, while the Sun stays at one of the focal points of the ellipse (Kepler's FIRST law).

P – **A position nearest to the Sun (perihelion)**

A – **A position furthest from the Sun (aphelion)**

Ellipse Geometry



a – **Semi-major axis**

b – **Semi-minor axis**

c – **Focal length**

They have a relationship as

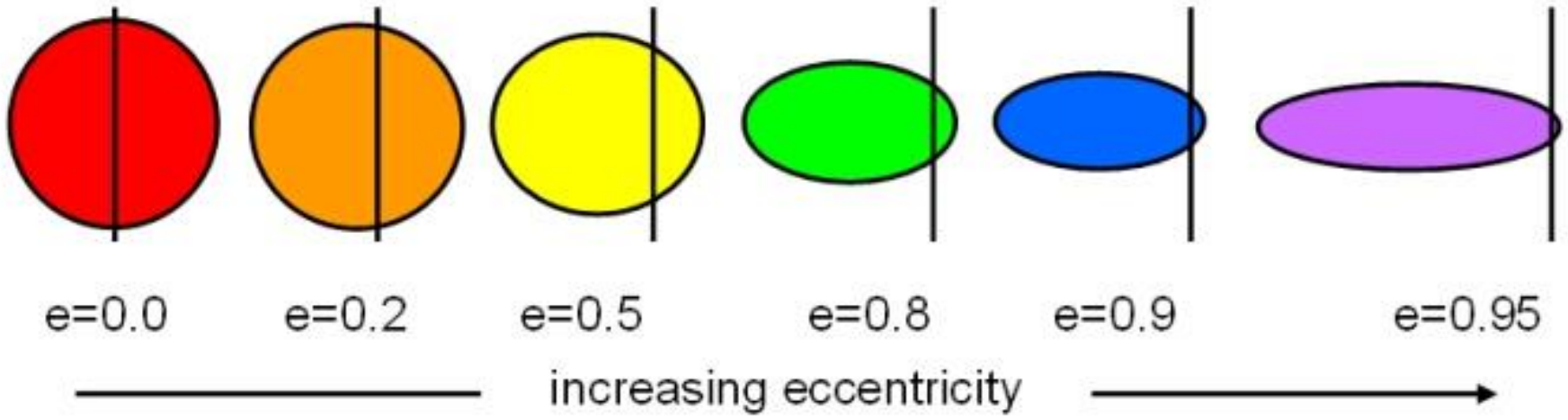
$$a^2 = b^2 + c^2$$

ELLIPTICITY (ECCENTRICITY), e

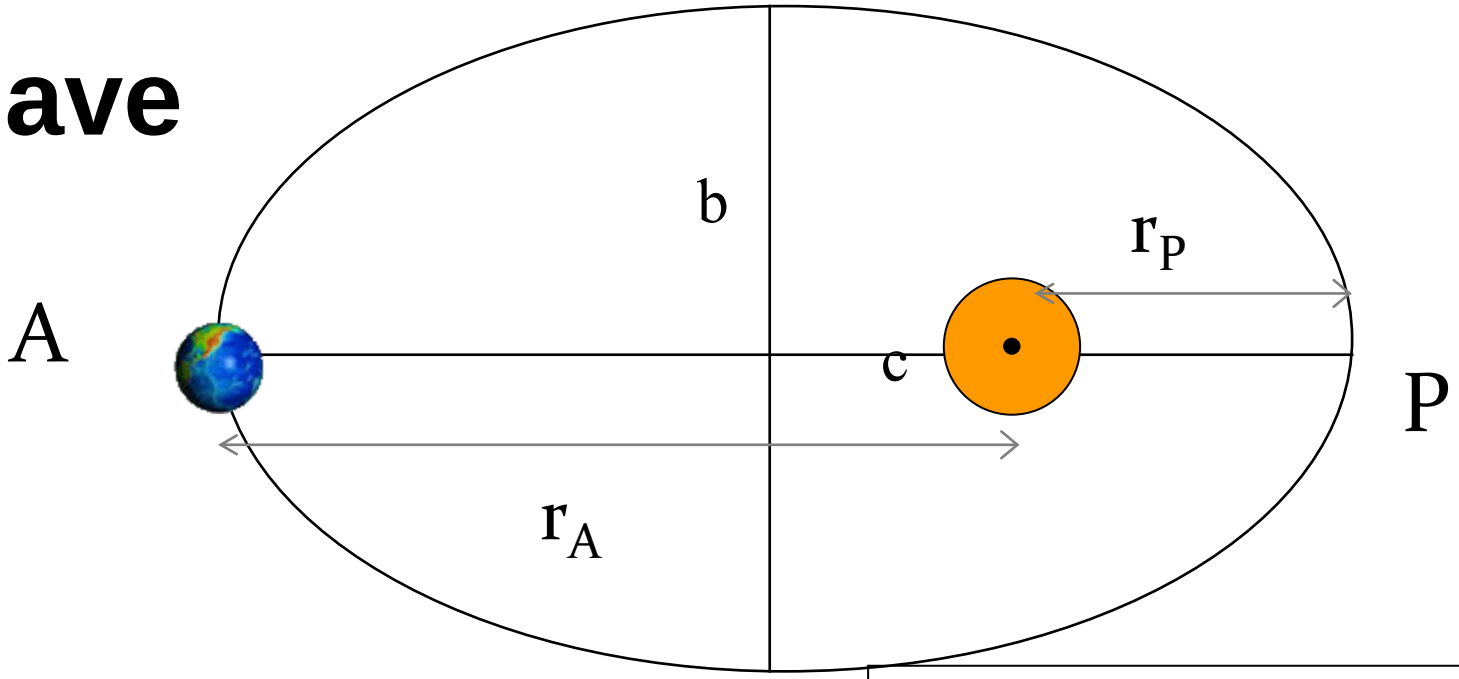
Definition

$$e = \frac{c}{a}$$

$$\therefore \quad c = a \times e$$



Now we have



$$r_P = a - c$$

$$r_P = a - ae$$

$$\therefore \quad r_P = a(1-e)$$

$$r_A = a + c$$

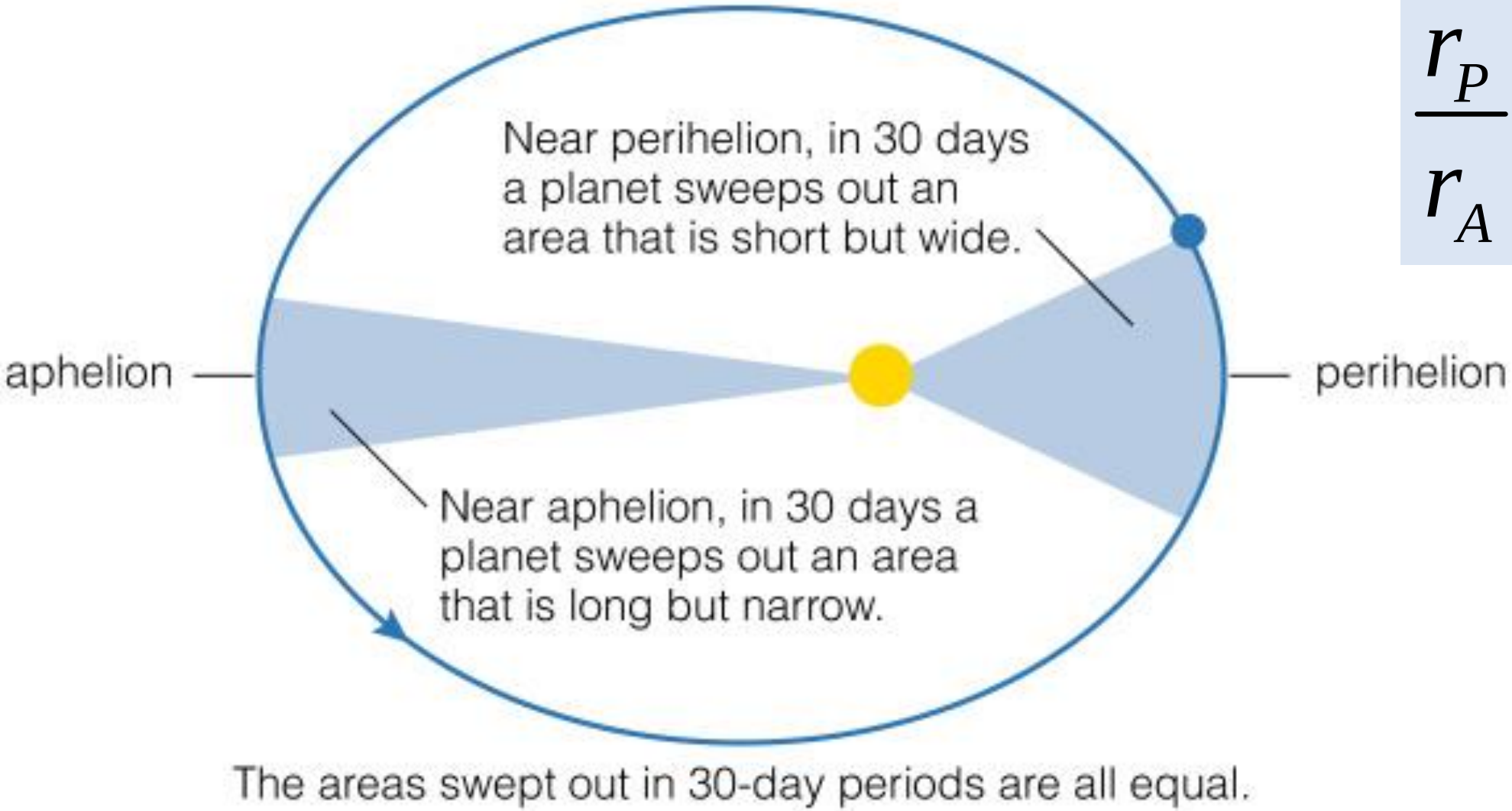
$$r_A = a + ae$$

$$\therefore \quad r_A = a(1+e)$$

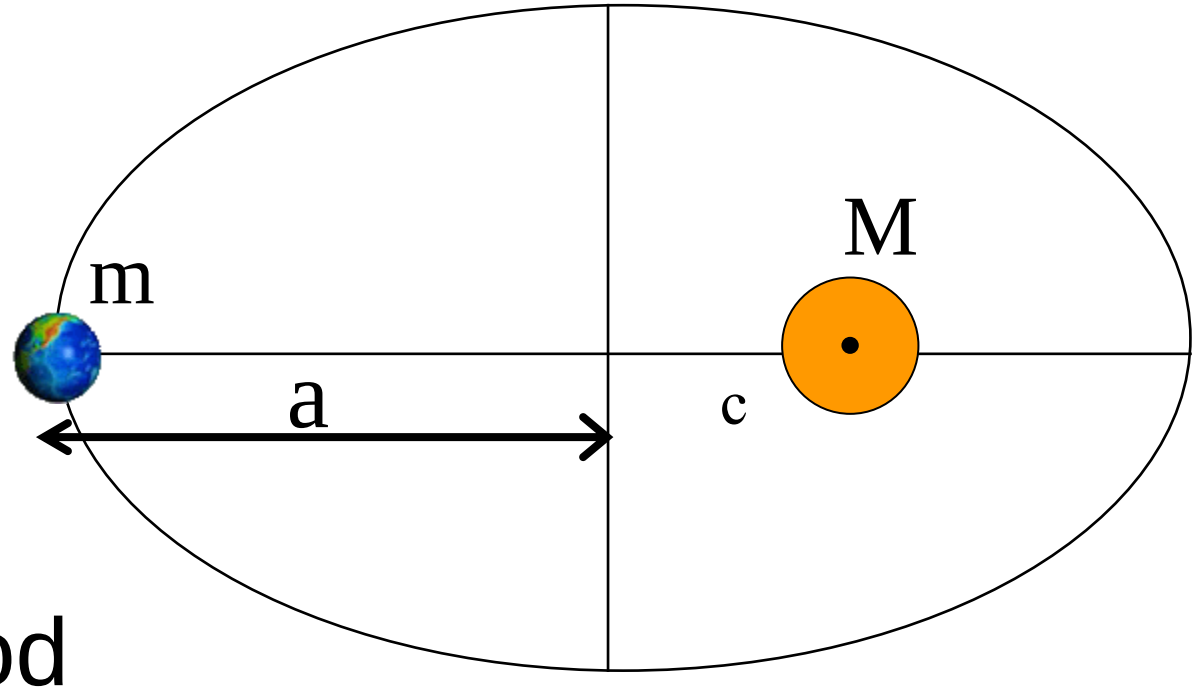
นั่นคือ $$\frac{r_A}{r_P} = \frac{1+e}{1-e}$$

Kepler's Second law

$$\frac{r_P}{r_A} = \frac{v_A}{v_P}$$

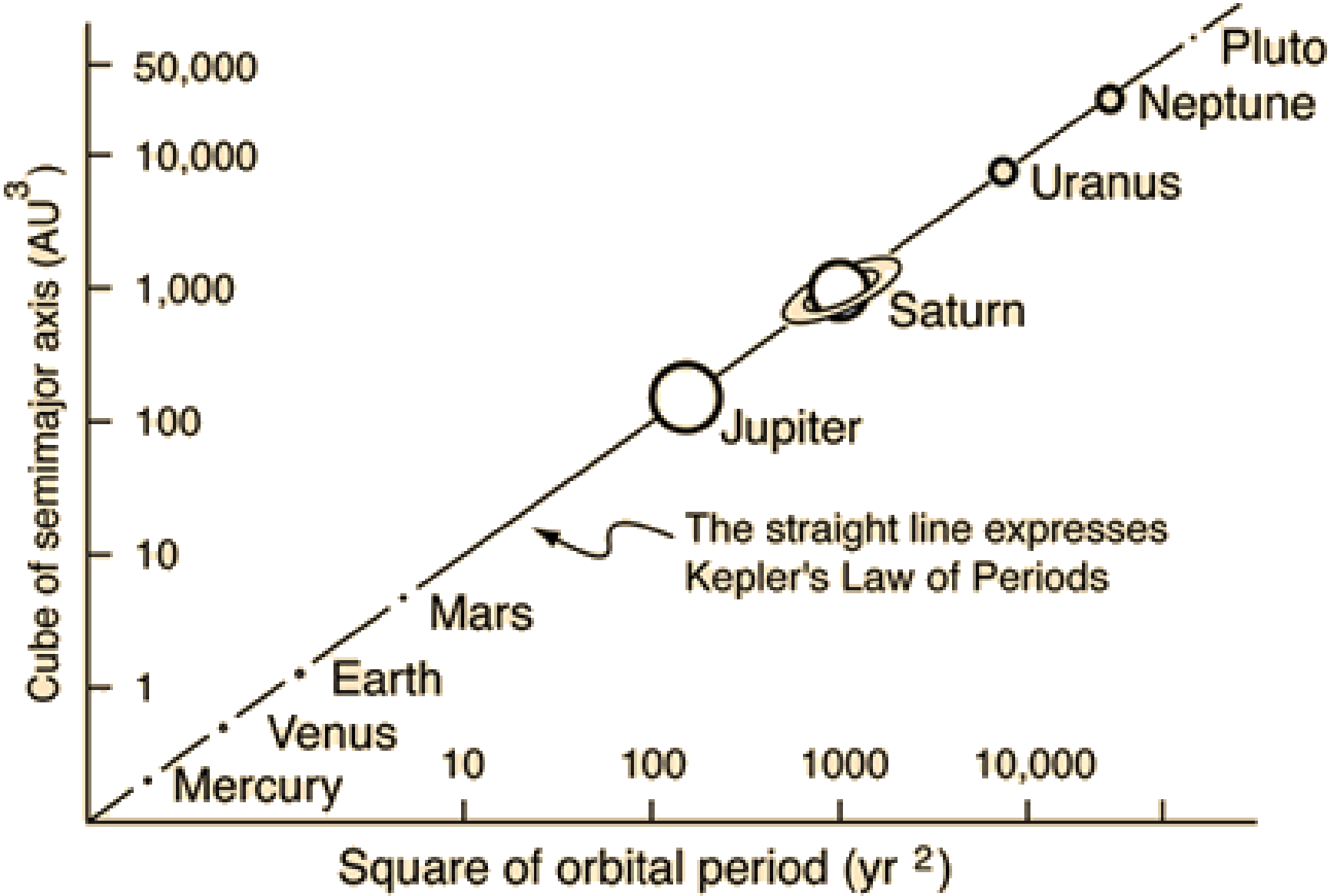


Kepler's Third Law



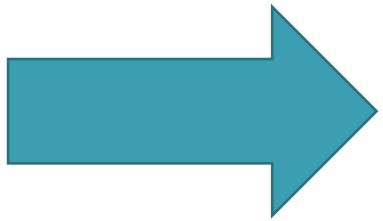
T – Orbital Period

$$T^2 = \frac{4\pi^2}{G(M+m)} a^3$$


Cube of semimajor axis (AU^3)
50,000
10,000
1,000
100
10
1
Square of orbital period (yr ^{2})
10
100
1000
10,000
Mercury
Venus
Earth
Mars
Jupiter
Saturn
Uranus
Neptune
Pluto
The straight line expresses
Kepler's Law of Periods

Simplified Kepler's third law

$$T^2 = \frac{4\pi^2}{G(M+m)} a^3$$



$$T^2 = a^3$$

SI unit

T has a unit of 'seconds'

a has a unit of 'meter'

BEWARE!!! The equation of the right is only for our solar system

T has a unit of 'years'

a has a unit of A.U. (Astronomical Unit)

Astronomical Unit (AU)
1.496×10^{11} m, or about 150 million kilometers