2 points statistics for weak lensing and intensity maps



Work with **David Bacon**,





- Introduction to ICG
- Weak Gravitational Lensing
- 21cm cosmology
- HI intensity map
- Spherical Harmonics and Angular Power Spectrum
- Lensing-HI 2-point functions

Introduction to ICG





Research Groups In ICG



- Very early Universe
- Dark energy
- Testing gravity on cosmological scales
- Large scale structure
- Gravitational lensing
- Supernovae
- Galaxy evolution
- Stellar population modelling
- Gravitational Wave

Basic of Gravitational Lensing



- Small perturbation in flat FLRW $ds^{2} = a^{2}(\tau) \left[-(1+2\Psi)d\tau^{2} + (1-2\Phi)d\vec{x}^{2} \right]$
 - Deflection angle

$$\alpha(\hat{n}) = \int_0^{\chi_s} d\chi' q_L(\chi', \chi_s) \nabla_\perp \phi_W(\chi', \chi'\hat{n})$$

- Weyl potential (Growth structure)

$$\phi_W = \frac{1}{2}(\Psi + \Phi)$$

Lensing weight function (Geometry)

$$q_L(\chi',\chi_s) = \frac{\chi'(\chi_s - \chi')}{\chi_s}$$



 ${
m Cr}:{
m ESA}$ In LCD $abla \phi_W \propto rac{1}{a} \delta_m(\chi,\chi \hat{n})$

Basic of Gravitation Lensing : continue



 $\vec{\alpha}(\chi \hat{n}) = \nabla_{\theta} \psi(\chi \hat{n})$

Convergence

$$\kappa(\chi \hat{n}) = \frac{1}{2} \nabla^2_{\theta}(\chi \hat{n})$$

Shear

 $\gamma(\chi,\chi\hat{n}) = \frac{1}{2}\eth \eth \psi(\chi,\chi\hat{n})$

	< 0	> 0
к	0	
Re[γ]		
lm[γ]		\bigcirc

Why is lensing a good cosmological probe ?

- 2 in one : measure both geometry (lensing weight function) and growth structure (Weyl / Gravitational Potential)
- Sensitive to modified gravity (MG) : (Weyl Potential)
- Deflection angle is small > Flat sky approximation > 2D > very powerful when considering joint statistic with other tracers
- 3D weak lensing > growth index > constraint M
- Muti-wavelength phenomena

21 CM Cosmology



The most abundant element inter galactic medium!!



$$\nu_{10} = \frac{8}{3} g_{\rm I} \left(\frac{m_{\rm e}}{m_{\rm p}} \right) \alpha^2 (R_{\rm M} c) \approx 1420.405751 \text{ MHz} => \lambda \approx 21 \text{ cm}$$

HI sources







Intensity Map



Lower Resolution but deeper

$$\Delta T_{\rm HI}(z,\hat{n}) = \bar{T}_{\rm HI}(z)b_{\rm HI}(z)\delta(z,\hat{n})$$

$$\bar{T}_{\rm HI}(z) = 180\Omega_{\rm HI}(z)h\frac{(1+z)^2}{H(z)/H_0}$$
[mK]

Battye et al. (2013)

 $\Omega_{\rm HI}(z) = 0.00048 + 0.00039z - 000065z^2$

SKA cosmology SWG et. al 2018

2-point functions



- Cosmological Principle = Isotropic + Homogenous
- Imply, any process is randomly in all directions and everywhere
- Central limit theorem > Gaussian process
- 2 point functions contain all statistical properties of Gaussian random fields.

2-point functions (con)



In 2D projection on the spherical surface, 2-point functions can easily measure by the amplitude square of spherical harmonics modes, in which we call "angular power spectrum"

$$f(\theta,\phi) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta,\phi)$$

Angular power spectra

$$C_{\ell}^{XY} = \langle a_{\ell m}^X a_{\ell m}^{Y*} \rangle_m$$

Limber Approximation

 Most of lensing power spectra (and its cross correlations) can be approximated by Limber approximation as if expect small deflection angle => flat sky approximation => Limber approximation

$$C_{\ell}^{XY} = \int d\chi \ q^X(\chi) q^Y(\chi) P_{\delta}\left(\frac{\ell + 1/2}{\chi}, z(\chi)\right)$$

$$q^{\rm HI}(\chi) = \bar{T}_{\rm HI}(\chi) b_{\rm HI}(\chi) \frac{n_{\rm HI}^i(z(\chi))}{\bar{n}_{\rm HI}^i} \frac{dz}{d\chi}, \quad q^{\kappa}(\chi) = \frac{3\Omega_m H_0^2}{2c^2} \frac{\chi}{a(\chi)} \int_{\chi}^{\infty} d\chi' \frac{n_s^i(z(\chi')) \frac{dz}{d\chi'}}{\bar{n}_s^i} \frac{\chi' - \chi}{\chi'}$$





108 realisations of full sky ray tracing code: 0.05 < z < 5.3 + CMB lensing Takashi et. al 2017 http://cosmo.phys.hirosakiu.ac.jp/takahasi/allsky_raytracing/

Result from Mock Catalogues (con)





Result from Mock Catalogues (con)





What we would expect (or what will we do) ?



 Include, EOR + CMB lensing Portsmout

 Make fisher forcast

DES Year I : joint analysis of 2points function : cosmological constrain