Computational Fluid Dynamics:

A first approach to Aerodynamics

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GCRF Big Data and Digital Technology Workshop

September 20th 2019

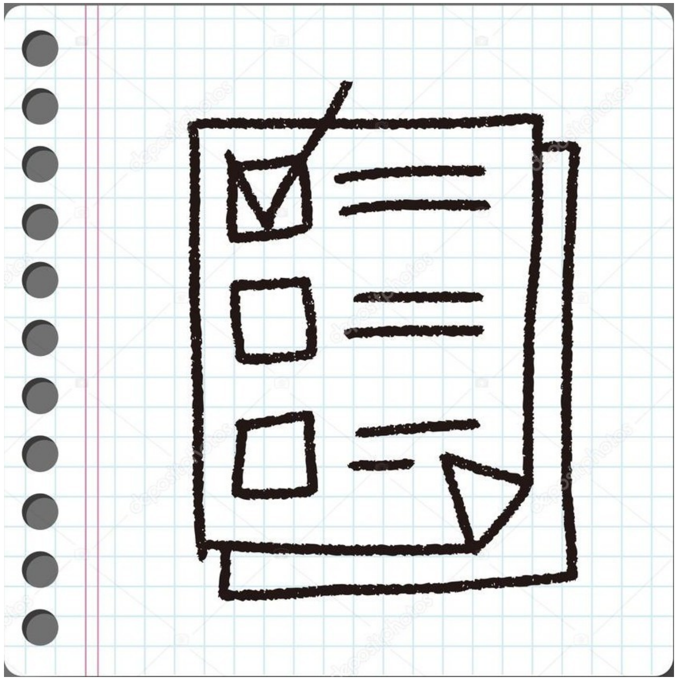
Chiang Mai - Thailand

Observatorio Astronómico Nacional

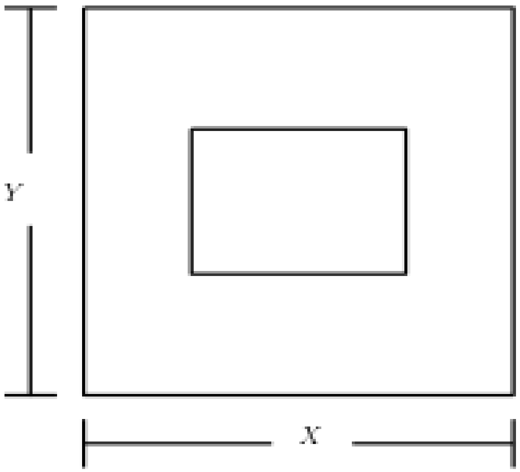
Making a Simulation

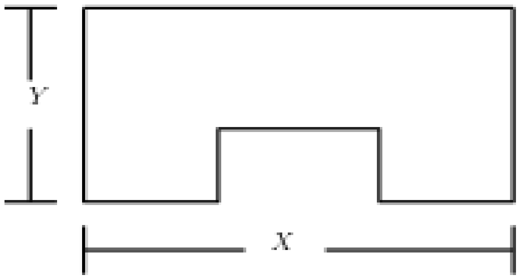
Steps

- Understanding physical problem:
  - Governing Equations and Boundary Conditions
- Gridding Strategy
- Numerical Methods and optimizing
  - Efficiency of the solution
- Visualization
- Validation
  - Experiments or analytical solution



Problem Specification – Type of flow





External Fluid

- Time independent
- Incompressible
- Viscous
- 2-Dimensional

$$R = \frac{V_0 h}{\nu} = 1,2$$

Problem Specification – Governing equations

Conservation Equations

$$\frac{\partial}{\partial t}\rho + \nabla \cdot (\rho \boldsymbol{V}) = 0$$

$$\rho \left[\frac{\partial}{\partial t} \boldsymbol{V} + (\boldsymbol{V} \cdot \nabla) \boldsymbol{V} \right] = -\nabla P + \mu \nabla^2 \boldsymbol{V}$$

$$\boldsymbol{V} = (u, v)$$

Making a Simulation

Problem Specification – Governing equations

Complications

$$P = P\left(\rho, T\left(\boldsymbol{x}, t\right)\right)$$

Simplified Equations

$$\nabla \cdot \boldsymbol{V} = 0$$

$$\rho\left(\boldsymbol{V} \cdot \nabla\right)\boldsymbol{V} = -\nabla P + \mu \nabla^2 \boldsymbol{V}$$

Problem Specification – Governing equations

Stream – Vorticity formulation

$$(\mathbf{V}, P, \rho) \rightarrow (\psi(x,y), \zeta(x,y))$$

$$u = \frac{\partial}{\partial y}\psi; \; v = -\frac{\partial}{\partial x}\psi$$

- Decoupling of pressure
- The velocity field is the rotational of other field!

Making a Simulation

Problem Specification – Governing equations

Stream – Vorticity formulation

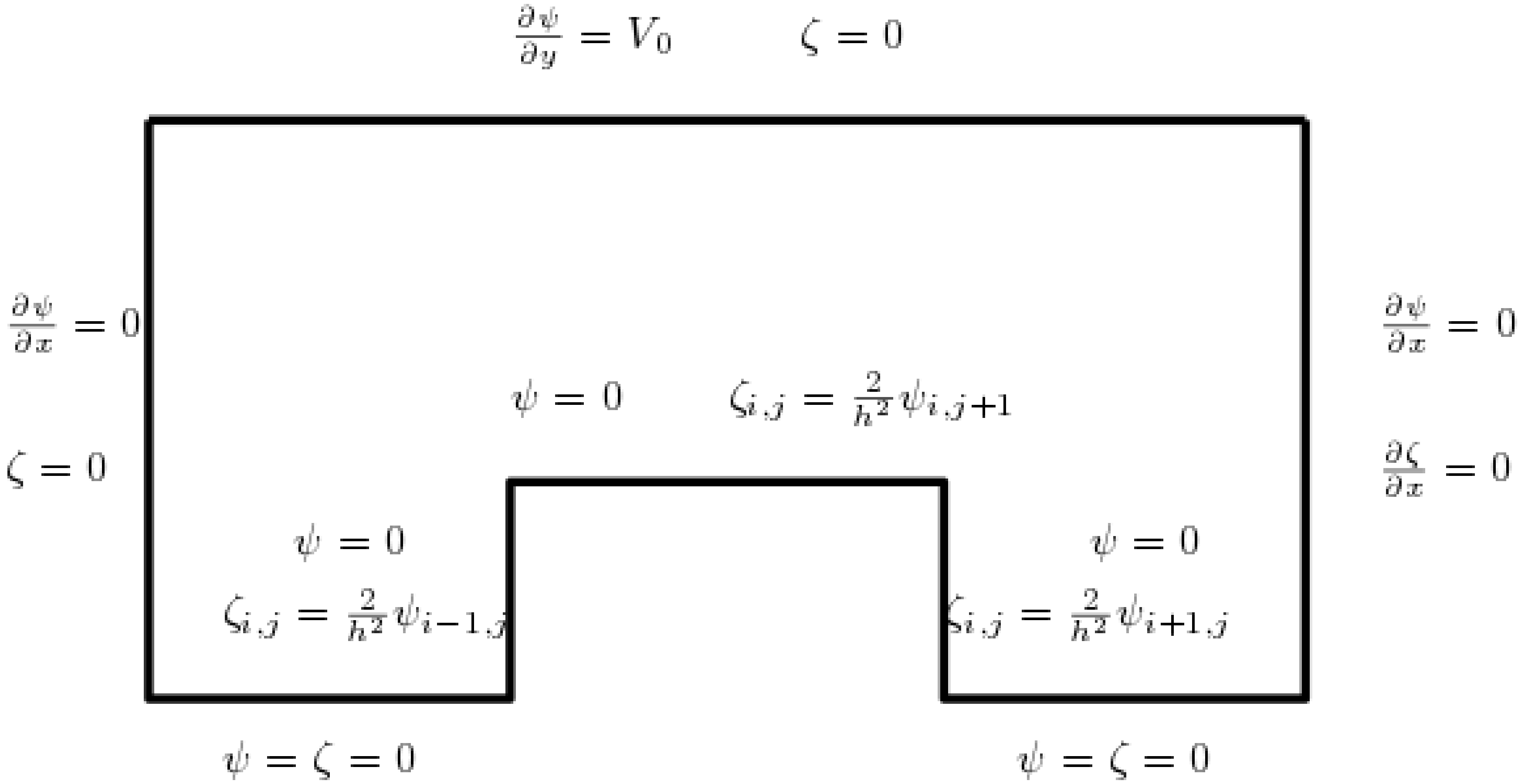
Equations to be solved...What did we win?

$$\nabla^2 \psi = \zeta$$

$$\nabla^2 \zeta = \frac{1}{\nu} \left(\frac{\partial \psi}{\partial y} \frac{\partial \zeta}{\partial x} - \frac{\partial \zeta}{\partial y} \frac{\partial \psi}{\partial x} \right)$$

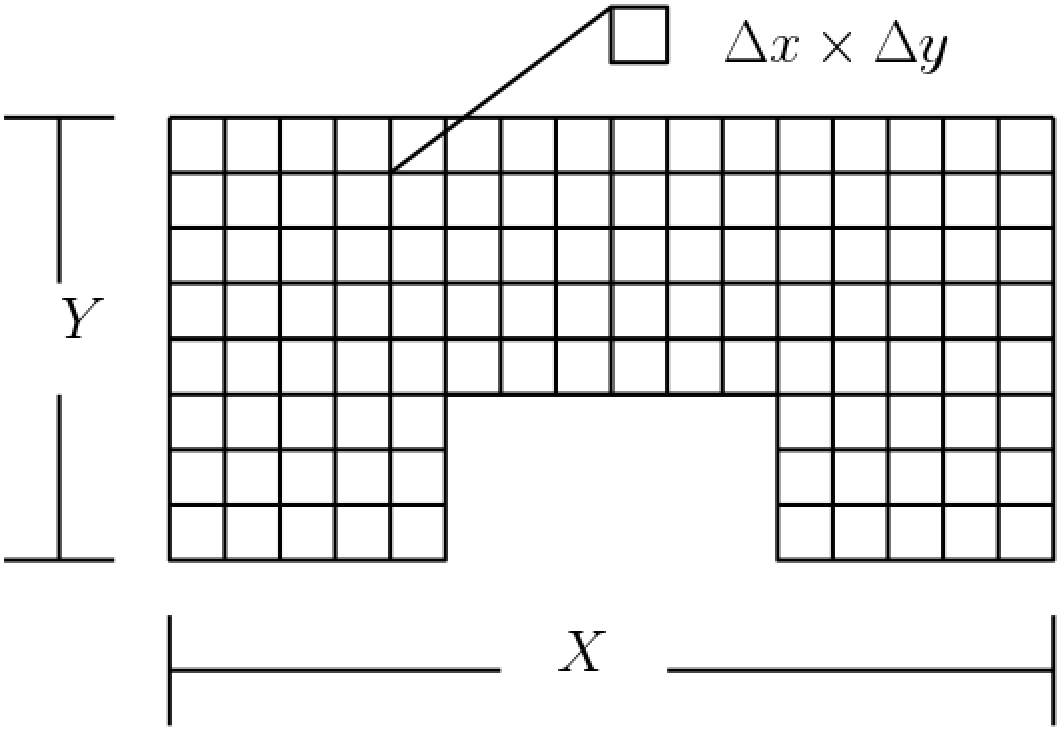
Making a Simulation

Problem Specification – Boundary Conditions



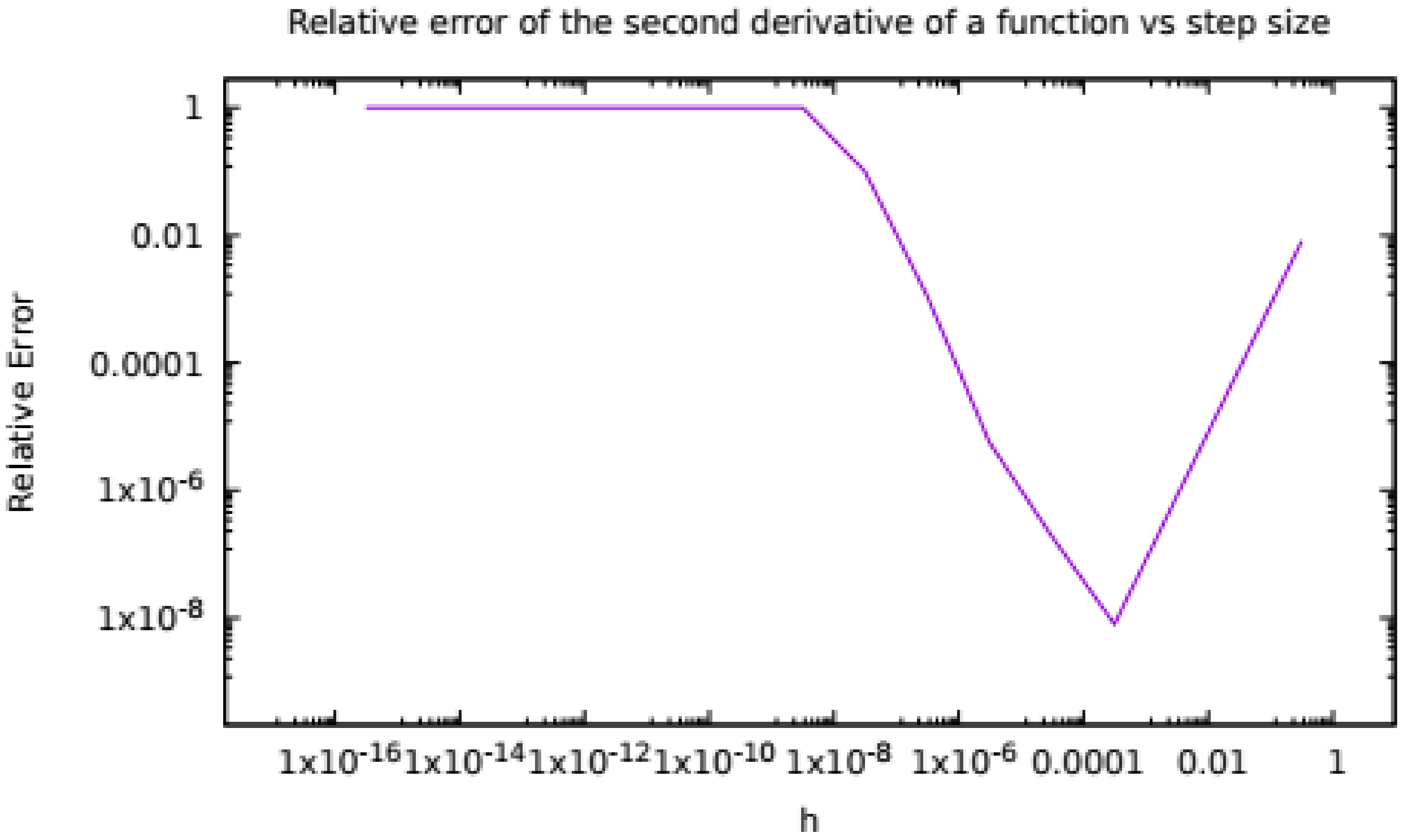
Making a Simulation

Gridding: Structured grid in rectangular coordinates



$$\Delta x = \Delta y$$
$$= h$$
$$= 0.1$$

Gridding: is it the appropriate size to h?



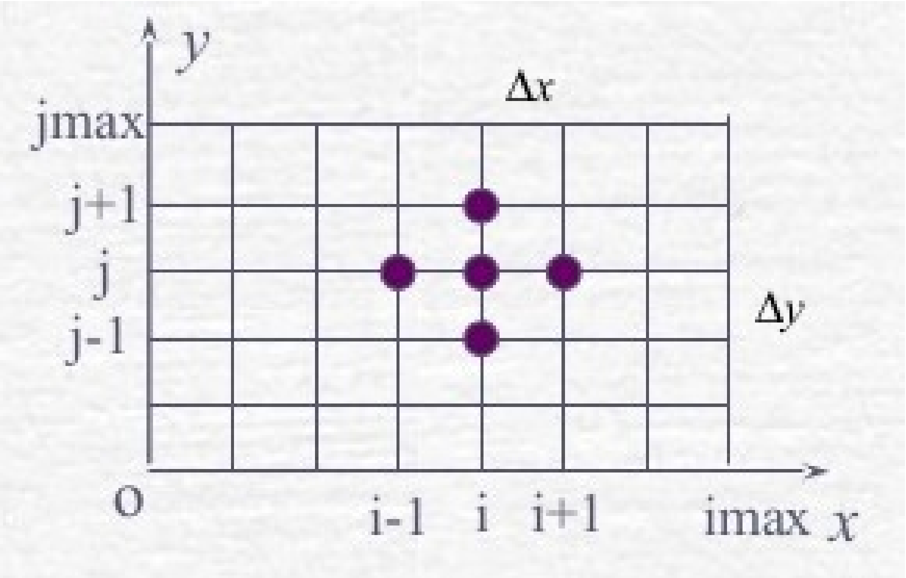
$$f(x) = cos(x)$$

$$f''(x = \pi) = -1$$

Numerical Methods: Finite Differences

$$\frac{\partial\phi}{\partial\lambda} \approx \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta\lambda}$$

$$\frac{\partial^2\phi}{\partial\lambda^2} \approx \frac{\phi_{i+1} - 2\phi_i + \phi_{i-1}}{\Delta\lambda^2}$$

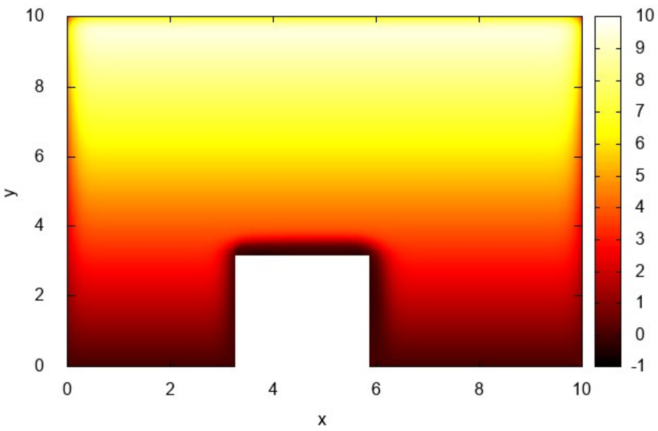


$$\Gamma = (1-\omega)\,\Gamma_{old} + \omega\Gamma_{new}$$ → Relaxation method

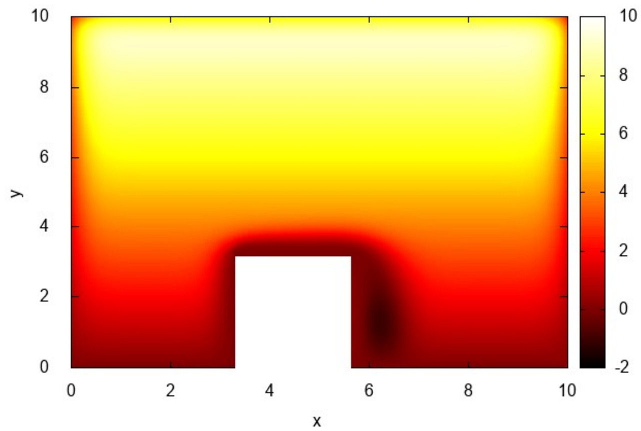
Making a Simulation

Visualization : Results of the digital experiment

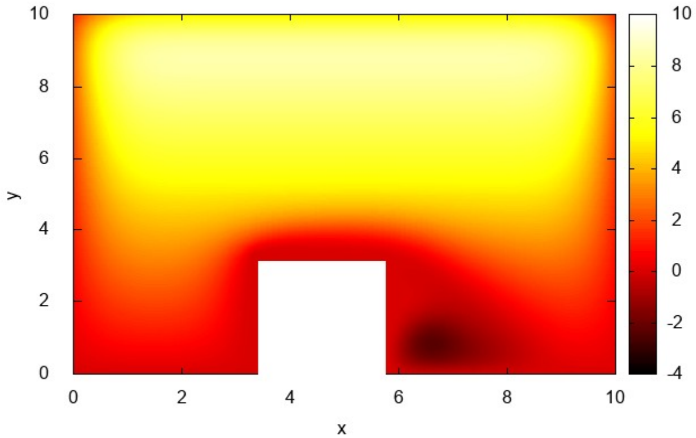
Maps of stream $R = 1$



$t = 1000$



$t = 3000$

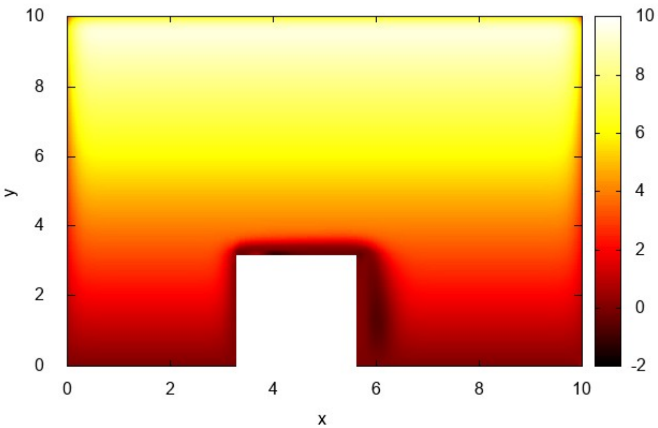


$t = 10000$

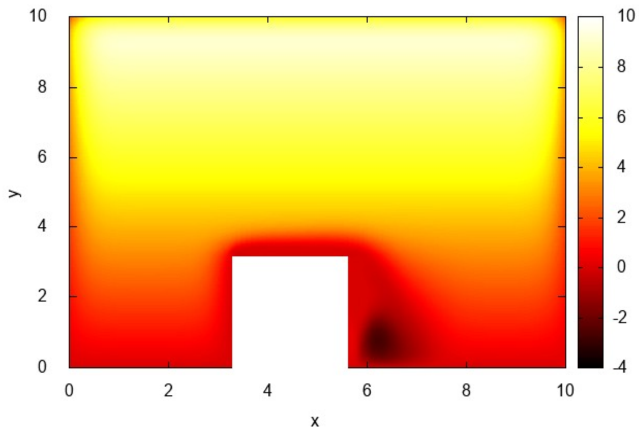
Making a Simulation

Visualization : Results of the digital experiment

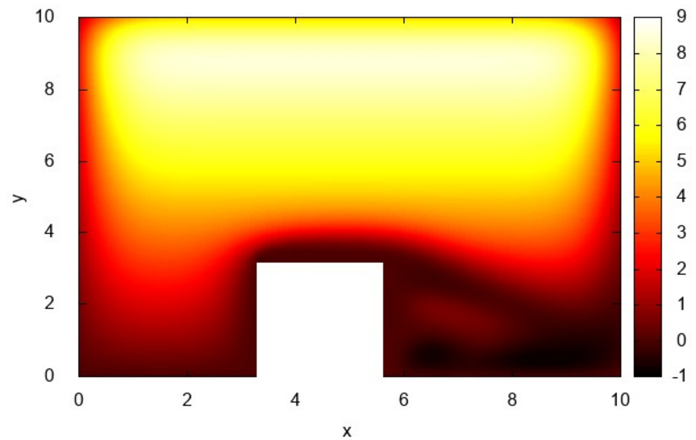
Maps of stream $R = 2$



$t = 1000$



$t = 3000$

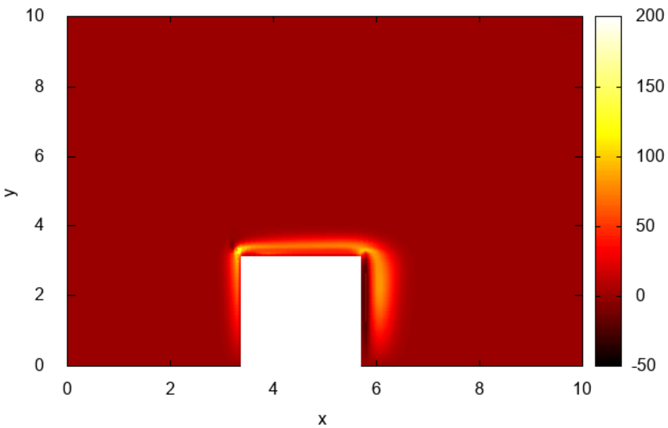


$t = 10000$

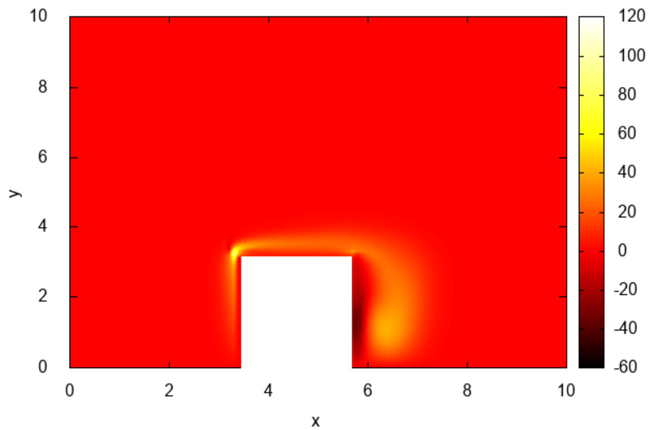
Visualization : Results of the digital experiment

Maps of vorticity

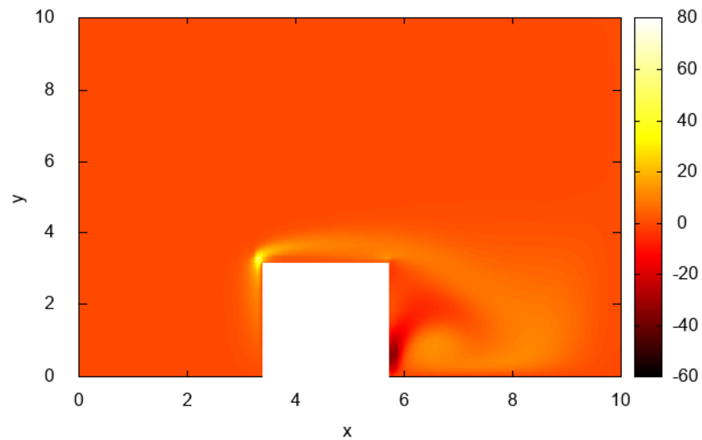
$$R = 1$$



$$t = 1000$$



$$t = 3000$$

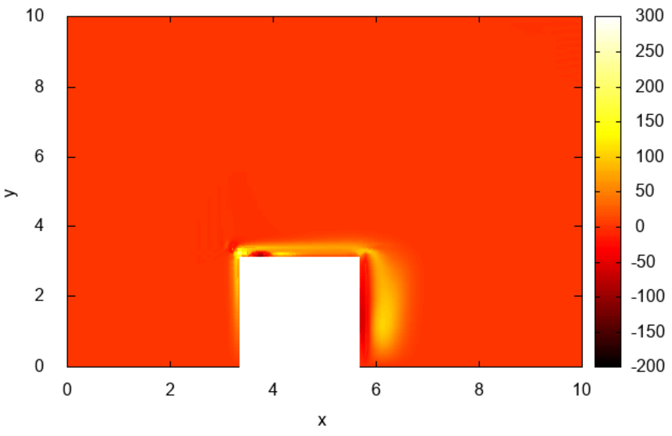


$$t = 10000$$

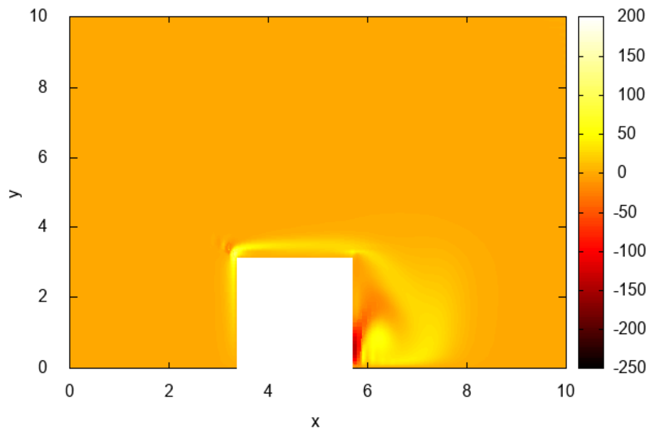
Making a Simulation

Visualization : Results of the digital experiment

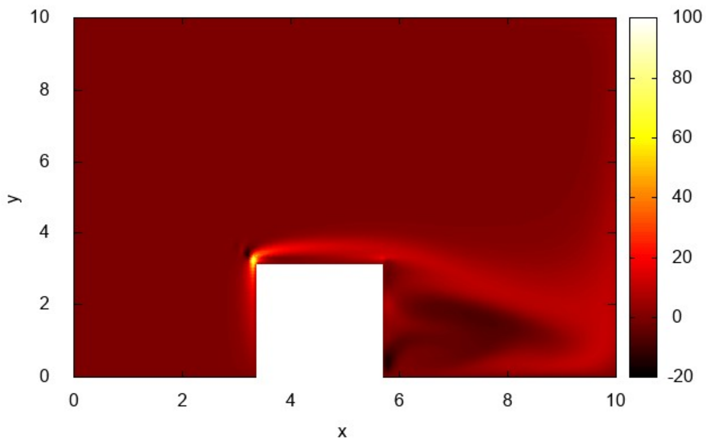
Maps of vorticity $R = 2$



$t = 1000$



$t = 3000$

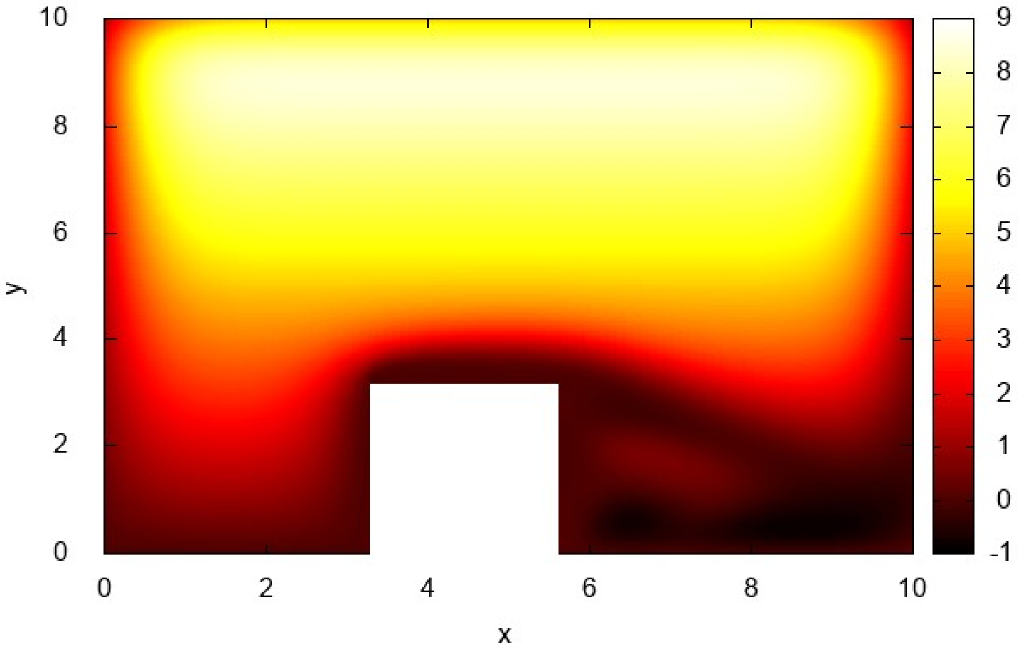


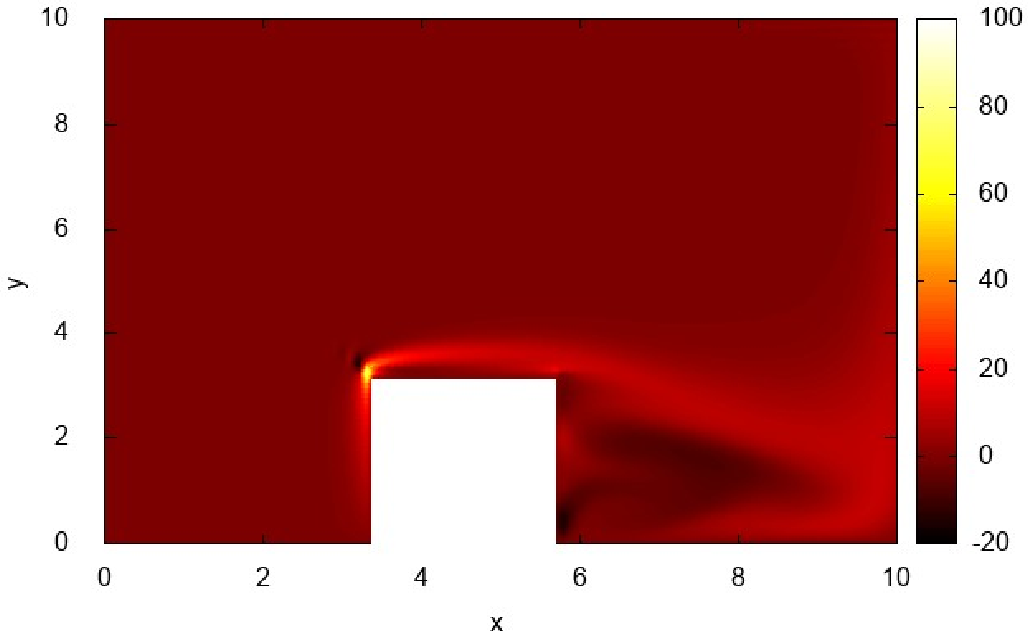
$t = 10000$

Visualization : Results of the digital experiment

Maps of stream and vorticity

$$R = 2$$
$$t = 10000$$



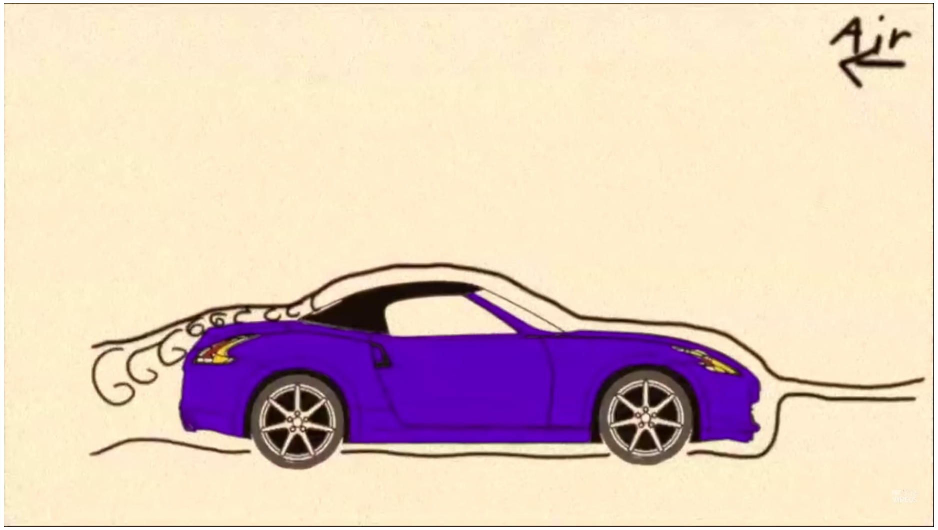


Making a Simulation

Visualization : Experiment

Application

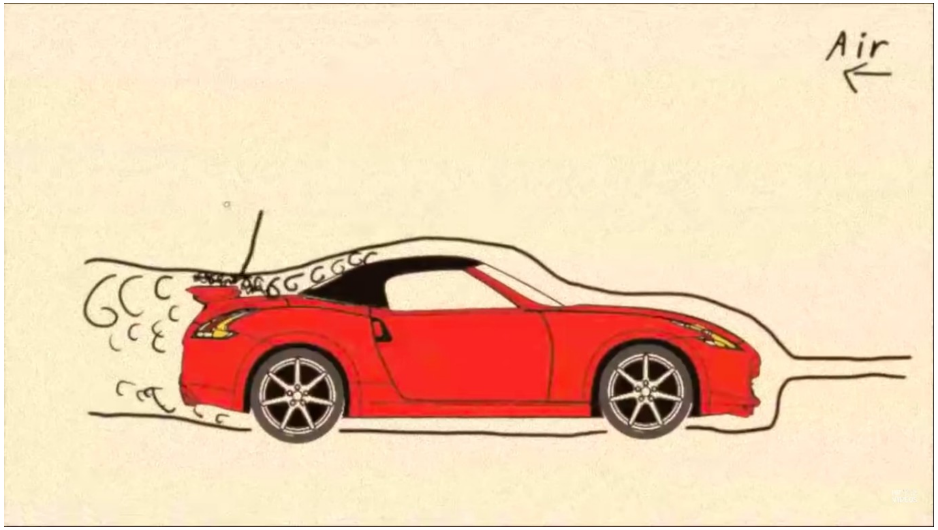
Why the spoilers of the cars?



Drag forces

$$F \propto \rho v^2 A$$

$$P = \frac{F}{A}$$



Conclusions

- Computational complex processes.
- No pointed corners are useful in Aerodynamics.
- Computational Fluid Dynamics is an important tool with many applications in Science and Industry.
- It is not about reinvent the wheel, it is about to understand the principles involved in the problem.
- Use specialized software. Do not understand it as a 'black box'.

thank you!