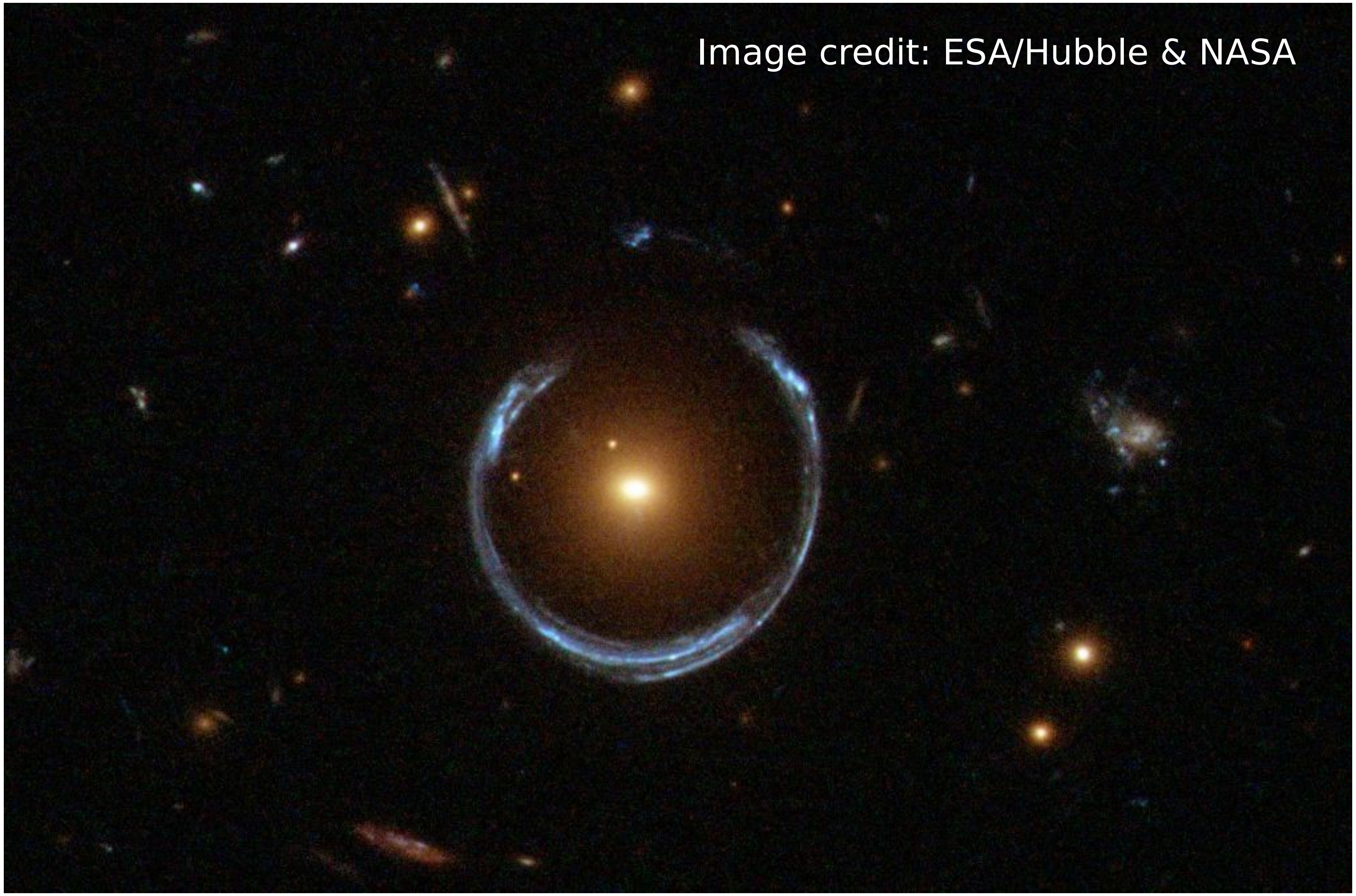
# Gravitational microlensing

Eamonn Kerins
Jodrell Bank Centre for Astrophysics
University of Manchester

# Gravitational lensing

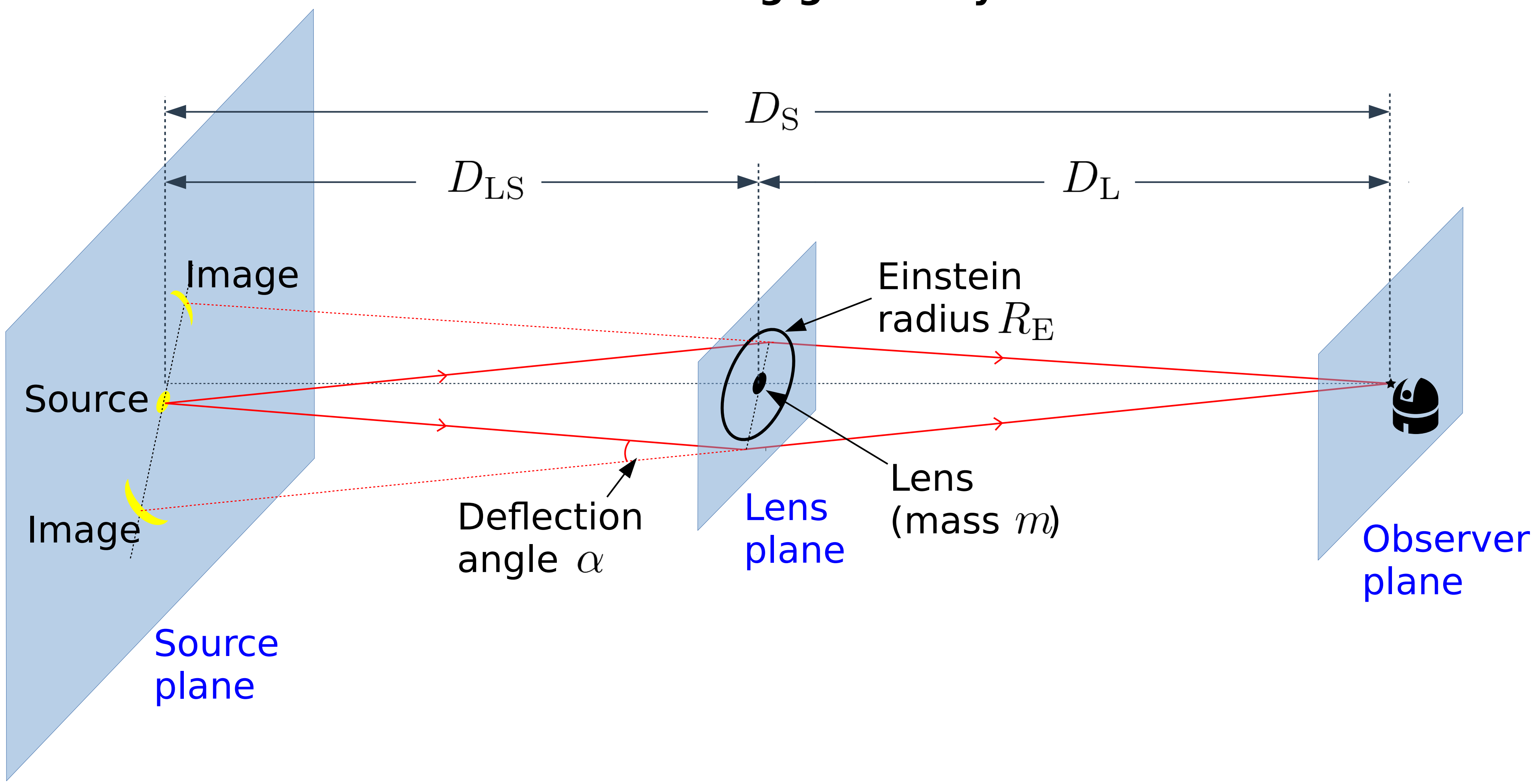
- Gravitational lensing and microlensing
- What does a microlens look like?
- How likely is it to see one?
- Where to look for them?
- The planetary signal
- Extracting physics from microlens exoplanets
- Some microlensing results to date

# Gravitational lensing




A gravitationally-lensed galaxy: a “familiar” view of gravitational lensing in action

## Lensing geometry



# Gravitational microlensing

Probably the least intuitive detection method

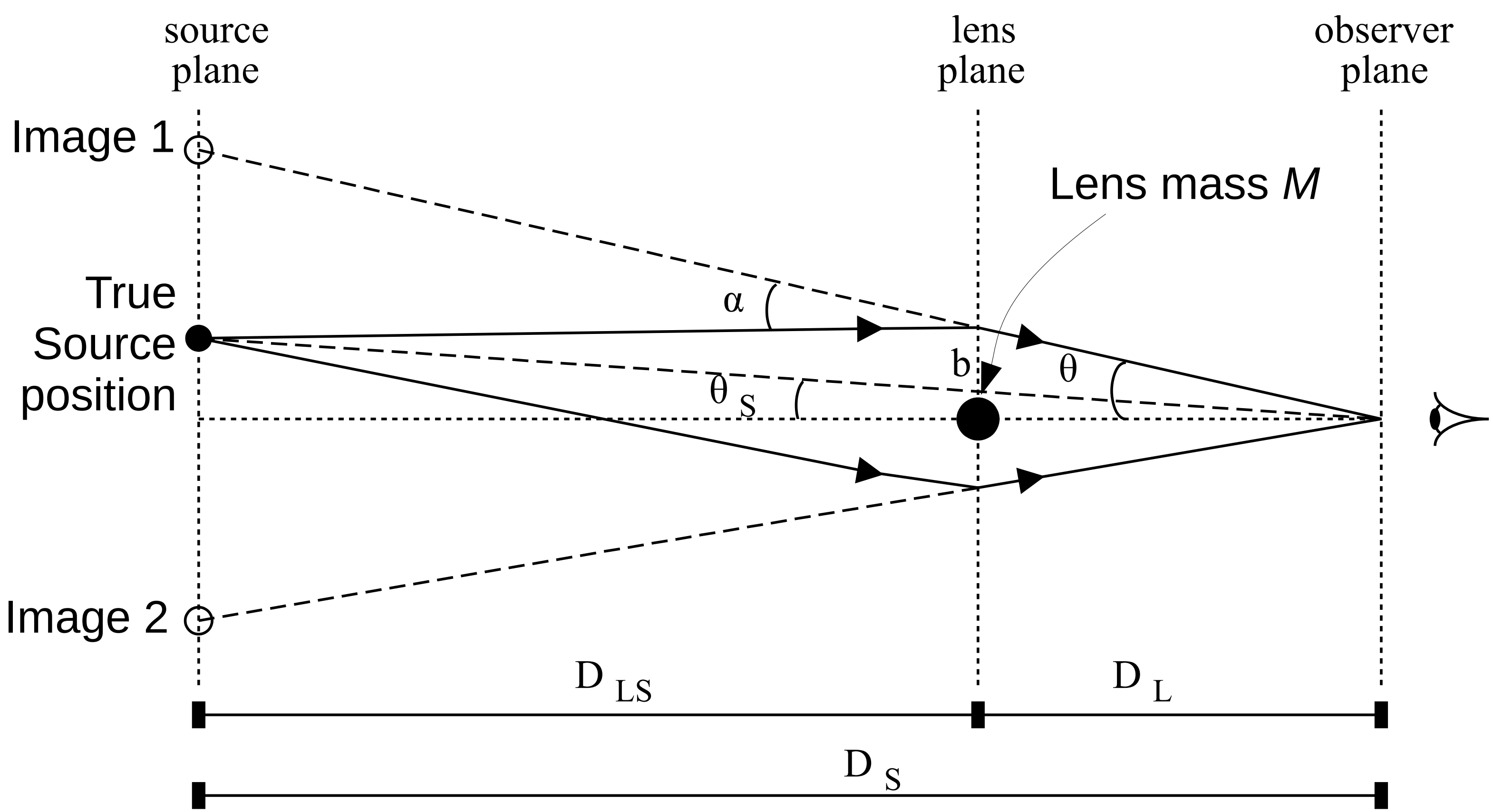
- First verification of Einstein's theory of General Relativity came in 1919 by the observation of the deflection of star light by the Sun. In the simplest case a foreground compact mass will deflect the light from a background by an angle (in radians):

$$\boxed{\alpha = \frac{4GM}{c^2 b}} \qquad (1)$$

where *M* is the mass of the deflector body (known as the “lens”) and *b* is the closest approach of the light path to the lens. The gravitationally lensed background star is known as the “source”

# Gravitational microlensing

Point source – point lens (PSPL) system




$$\theta D_S = \theta_S D_S + \alpha D_{LS} \qquad (2)$$

# Gravitational microlensing

Expanding eq (2) by substituting for $\alpha$ in eq (1), and noting that $b = \theta D_L$, gives

$$\theta D_S = \theta_S D_S + \frac{4GMD_{LS}}{c^2 b} = \theta_S D_S + \frac{4GMD_{LS}}{c^2 \theta D_L} \qquad (3)$$

For the special case $\theta_s = 0$ (source is directly behind the lens) we have

$$\theta D_S = \frac{4GMD_{LS}}{c^2 \theta D_L} \qquad (4)$$

The case where $\theta_s = 0$ defines the **angular Einstein radius**:

$$\boxed{\theta_E \equiv \theta(\theta_S = 0) = \sqrt{\frac{4GMD_{LS}}{c^2 D_S D_L}}} \qquad (5)$$

**Physical interpretation:**
A source located exactly behind the lens will produce images which merge to form a ring with angular radius $\theta_E$

The corresponding physical Einstein radius (measured in the lens plane) is

$$\boxed{R_E = \theta_E D_L = \sqrt{\frac{4GMD_L D_{LS}}{c^2 D_S}}} \qquad (6)$$

# The lens equation: image locations

For a given deflection angle we can compute how many images of the source are produced and where they are located. We note from eq (1) and (5) that

$$\alpha = \frac{4GM}{c^2 b} = \frac{4GM}{c^2 \theta D_L} = \frac{\theta_E^2}{\theta} \frac{D_S}{D_{LS}} \qquad (7)$$

So we can substitute $\theta_E$ for $\alpha$ in eq (2) to get:

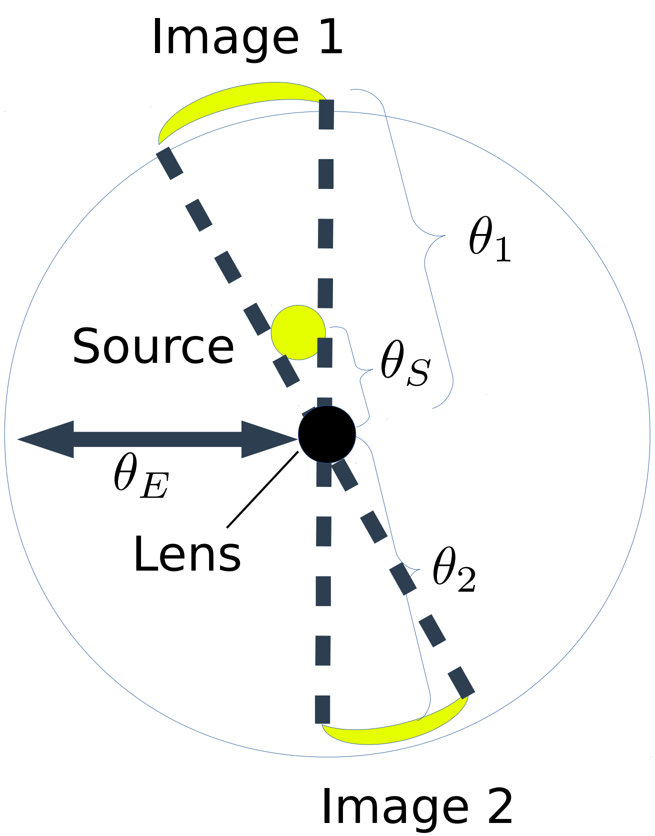
$$\boxed{\theta^2 = \theta_S \theta + \theta_E^2} \qquad (8)$$

This is the **lens equation**. This simple quadratic equation has two solutions for the image positions $(\theta_1, \theta_2)$:

$$\theta = \theta_{1,2} = \frac{\theta_S}{2} \pm \frac{1}{2}\sqrt{\theta_S^2 + 4\theta_E^2} \qquad (9)$$

A negative value for $\theta_2$ indicates that this image is formed on the opposite side of the the lens to the true source position, and is inverted

# The lens equation: image locations




**The observer's "view"**

**Points to note:**

- The observer cannot observe the true source position, only the image positions
- The images are always co-aligned with the lens and source
- One image forms outside of the Einstein radius $(\theta_1 > \theta_E)$ and one inside $(\theta_2 < \theta_E)$
- From eq (9), as the source moves away from the lens $\theta_1 \to \theta_S,\ \theta_2 \to 0$. So the first image becomes coincident with the true source location and the second image becomes coincident with the lens position ("hides" behind it)

# Microlensing

Consider the size of the image separation for typical stars within our Galaxy

- Typical stellar distances: a few kpc
- Typical stellar masses: around a solar mass
- Image separation is $\Delta\theta = \theta_1 + \theta_2 \simeq 2\theta_E$ where, from Eq (5)

$$\theta_E = 2.85 \text{ mas} \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{D'}{\text{kpc}}\right)^{-1/2}, \qquad D' = \frac{D_S D_L}{D_{LS}} \qquad (10)$$

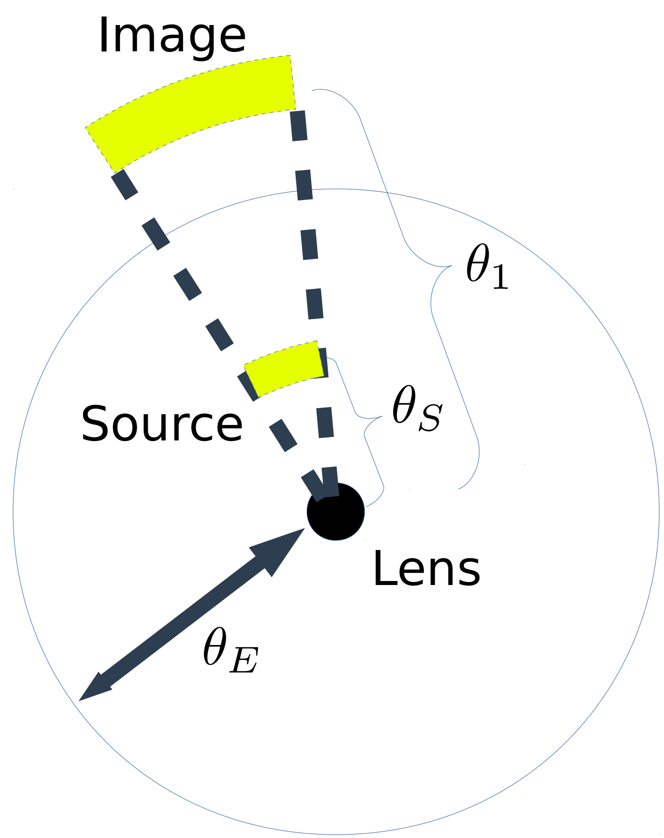
**Images cannot be resolved from one another!**

All we can observe is the combined brightness of the images and how this changes with time due to the relative motion of the lens and source.

# Image magnification

- **How magnified are the images?**
  - The surface brightness of the source is conserved whilst gravitationally lensed
  - Due to surface brightness conservation the magnification is due entirely to changes in the angular size of the source images
  - The magnification factor is given by the ratio of the image angular size to the unlensed source angular size
  - The magnification can be derived analytically under the point-source point-lens (PSPL) approximation

# Image magnification




Magnification $A$ = (area of image) / (area of source) :

$$A_1 = \left| \frac{\theta_1}{\theta_S} \frac{\mathrm{d}\theta_1}{\mathrm{d}\theta_S} \right| \qquad (11)$$

Convenient to normalise angles to the Einstein radius $\theta_E$. Therefore we define:

$$\begin{aligned} u &= \theta_S / \theta_E \\ u_1 &= |\theta_1| / \theta_E \end{aligned} \qquad (12)$$

$u$ is referred to as the **impact parameter**

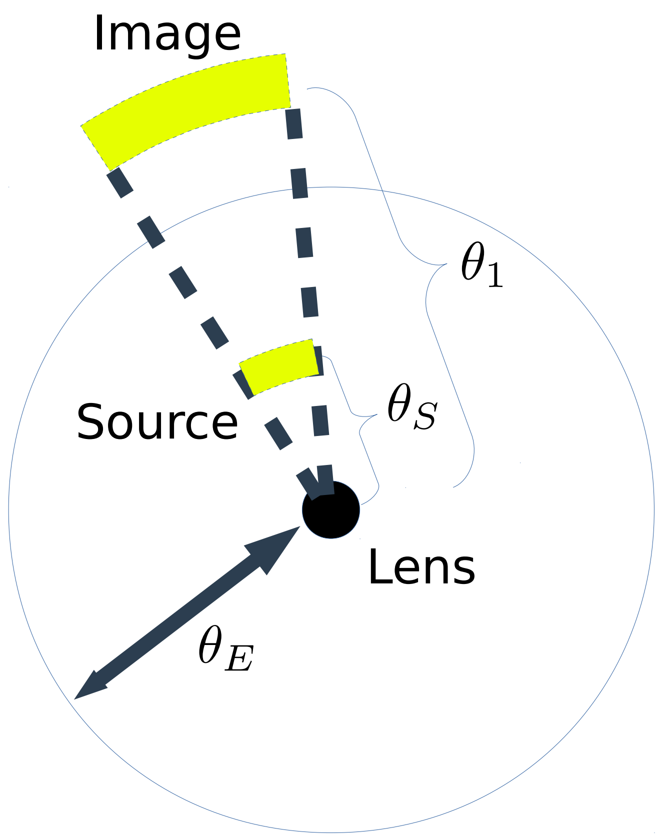
Eq (9) for image 1 can therefore be written as:

$$u_1 = \frac{1}{2}(u^2 + 4)^{1/2} + \frac{1}{2}u \qquad (13)$$

and the magnification factor becomes:

$$A_1 = \frac{u_1}{u} \frac{\mathrm{d}u_1}{\mathrm{d}u} \qquad (14)$$

# Image magnification




From eq (13) we have

$$\frac{\mathrm{d}u_1}{\mathrm{d}u} = \frac{u}{2(u^2+4)^{1/2}} + \frac{1}{2}$$

$$\therefore A_1 = \frac{u^2+2}{2u(u^2+4)^{1/2}} + \frac{1}{2} \qquad (15)$$

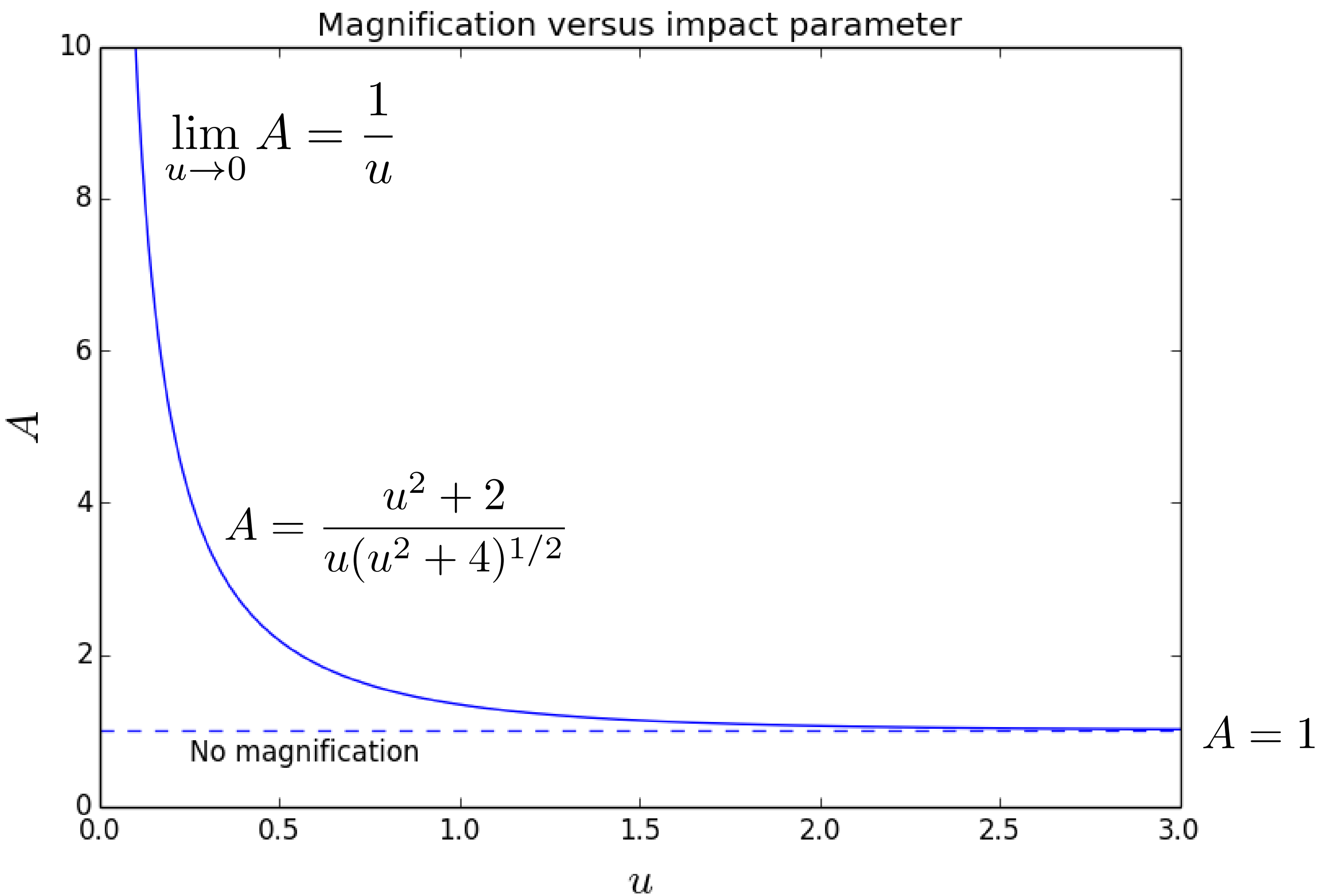
Following a similar method for the second image at $\theta_2$ we get:

$$A_2 = \frac{u^2+2}{2u(u^2+4)^{1/2}} - \frac{1}{2} \qquad (16)$$

Since we cannot resolve the individual images we observe an overall magnification

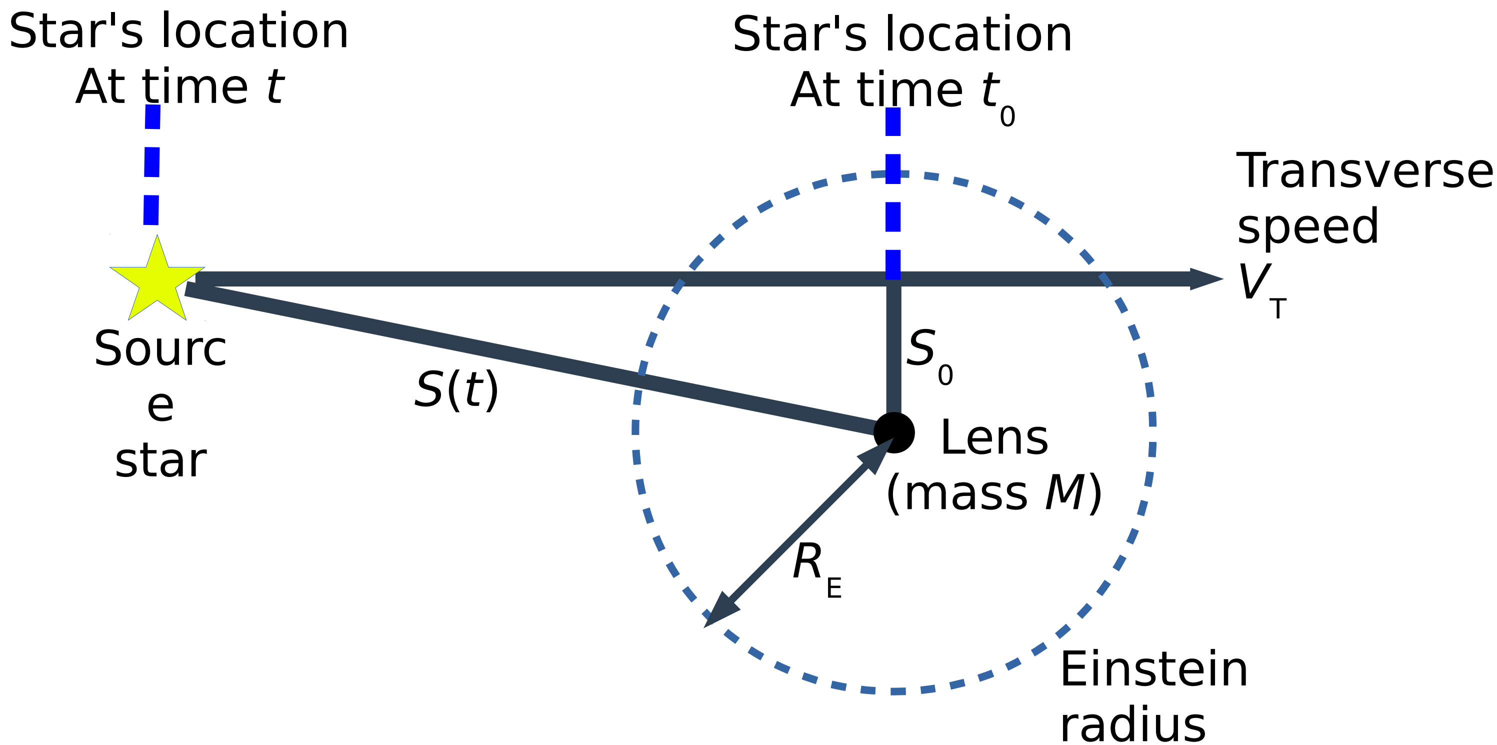
$$\boxed{A = A_1 + A_2 = \frac{u^2+2}{u(u^2+4)^{1/2}}} \qquad (17)$$

# Image magnification




PSPL magnification function diverges as $u \to 0$

# Microlensing lightcurve




For uniform relative motion between the lens and source we have

$$S(t)^2 = S_0^2 + V_T^2 (t - t_0)^2 \qquad (18)$$

# Microlensing lightcurve

- Stars in our Galaxy have motions with respect to each other.
- This means that microlensing is a transient effect lasting only whilst the lens is closely aligned to the source on the sky.
- The microlensing magnification changes with time and the plot of magnification vs time is known as a lightcurve.

# Microlensing lightcurve

From eq (12) we note that $S/R_E = \theta_S/\theta_E = u$ We can therefore divide eq (18) by $R_E$ to re-cast it as

$$u(t)^2 = u_0^2 + \left(\frac{t - t_0}{t_E}\right)^2 \qquad (19)$$

where $u_0 = S_0/R_E$ is the dimensionless **minimum impact parameter** and

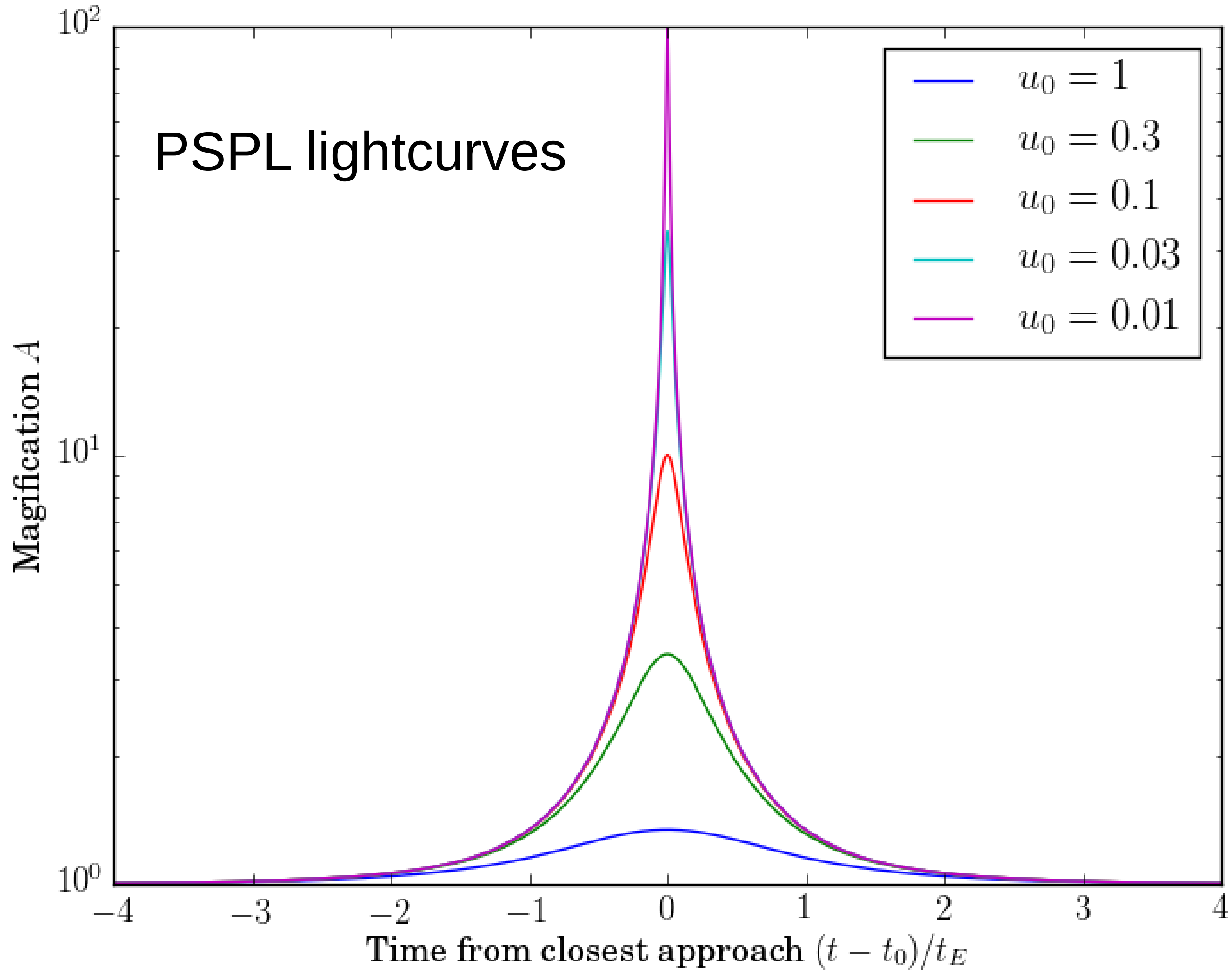
$$\boxed{t_E = \frac{R_E}{V_T} = \frac{\theta_E}{\mu_{\rm rel}}} \qquad (20)$$

is the **Einstein radius crossing time**.

In eq (20) $\mu_{rel}$ is the **relative proper motion** (angular speed).

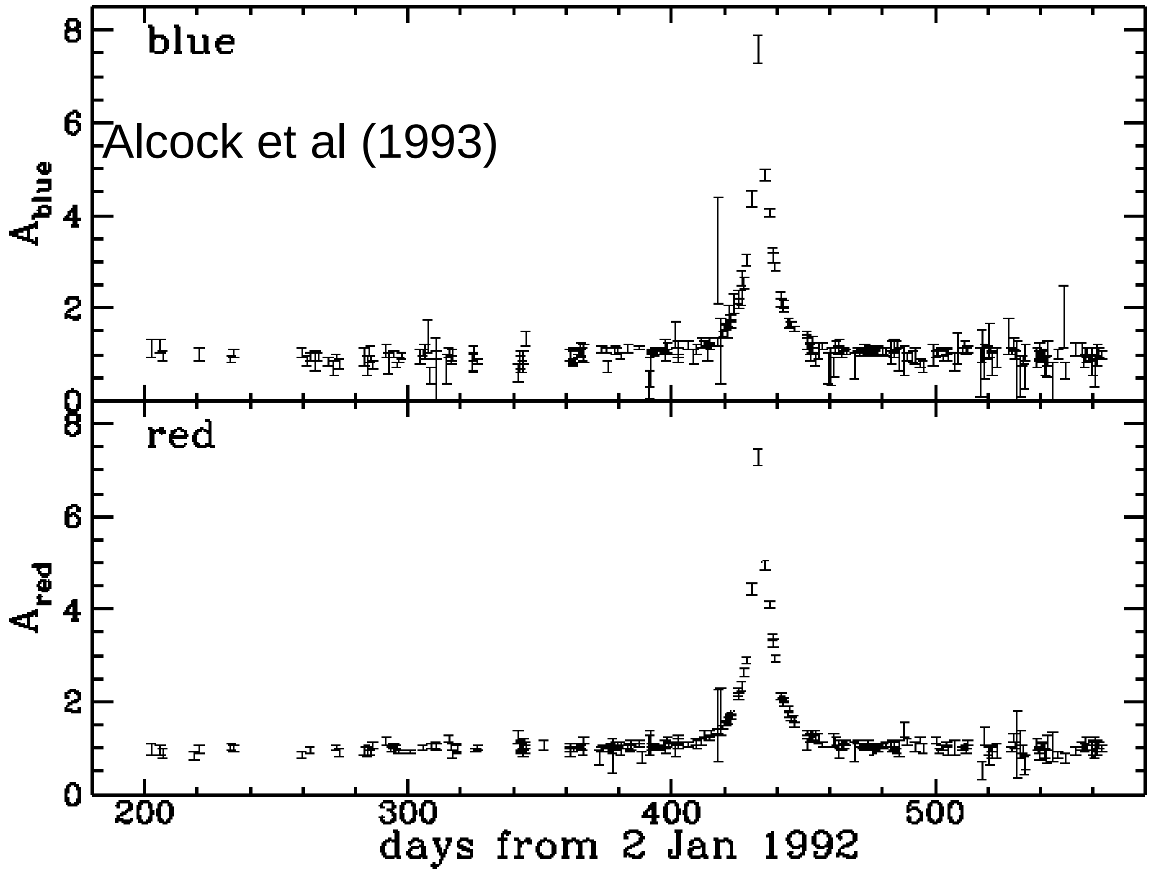
$t_E$ contains all the interesting physics (lens mass, motion and distance). But it is only one parameter. If we only have $t_E$ then we have a **three-way mass-motion-distance degeneracy**.

# Microlensing lightcurve



# A real lightcurve

First ever published microlensing event by the MACHO team, who were hunting for astrophysical dark matter towards the Large Magellanic Cloud. Subsequent studies showed an insufficient number of events to explain most of the dark matter in our Galaxy



# And another

From the OGLE Early Warning System (Udalski et al 2003)

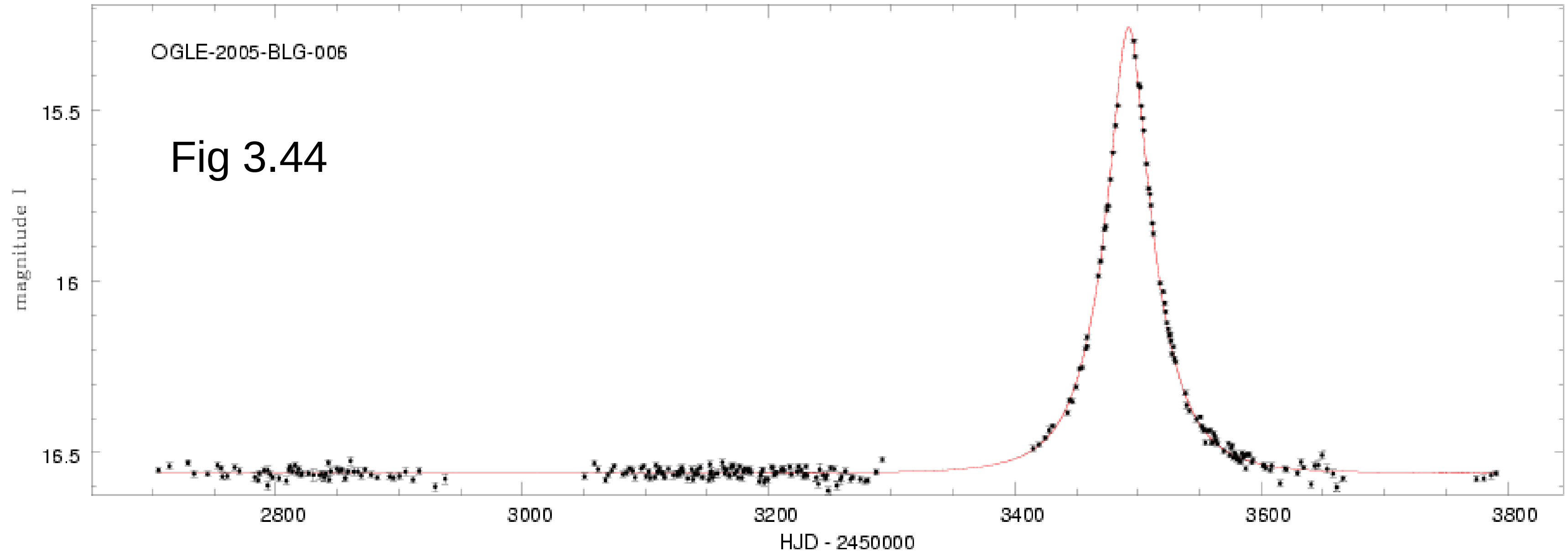
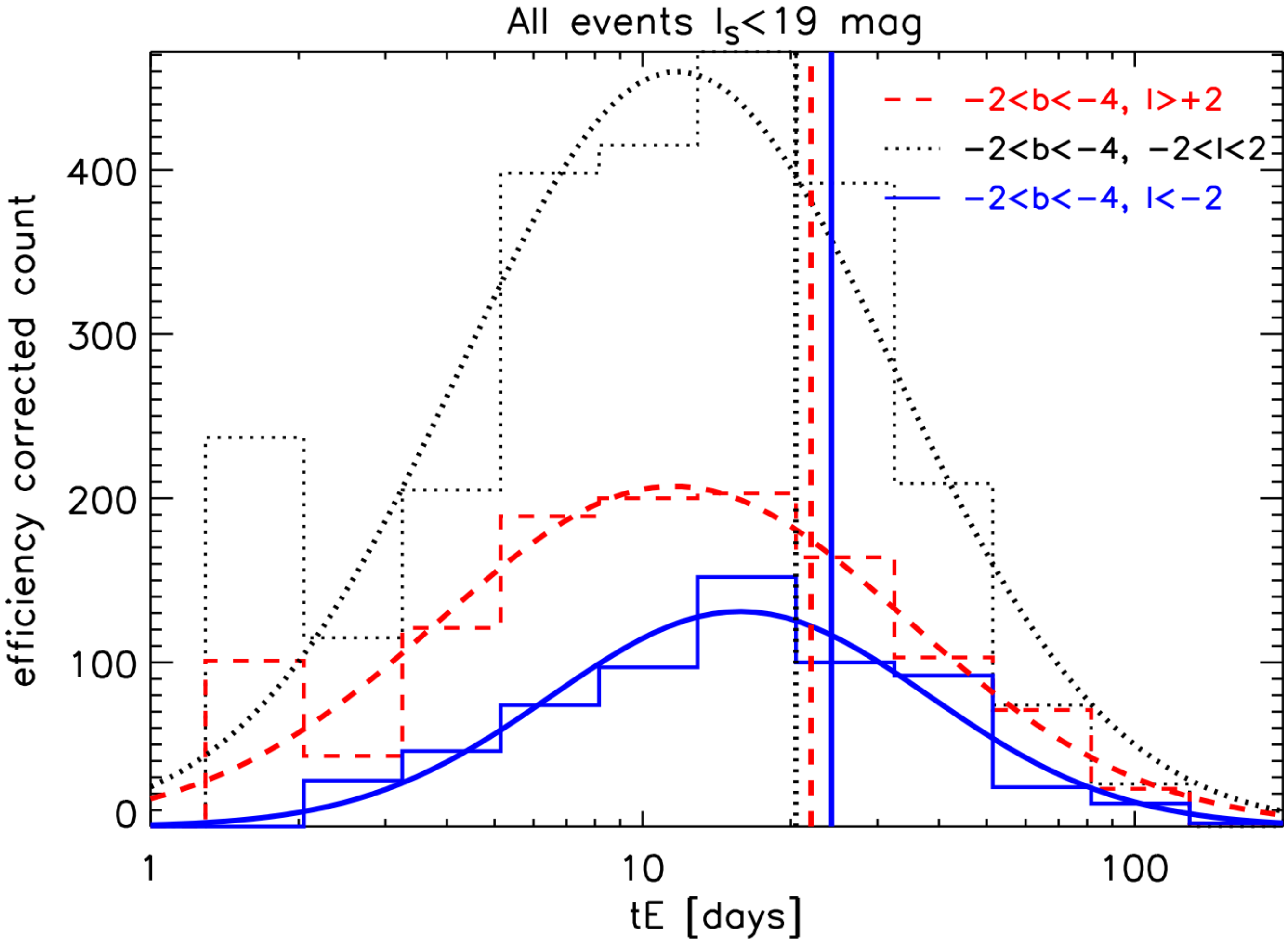



Fig 3.44

Today, microlensing teams such as OGLE, MOA and KMTNet look towards the Galactic Centre, where they observe thousands of microlensing events each year. These are not due to dark matter but instead are due to foreground stars microlensing background stars.

# Observed timescales




The microlensing event timescale distribution of stars observed by the OGLE-III survey for three regions near the Galactic Centre (Wyrzykowski et al 2014)

# Typical timescales

From Eq (20) the microlensing event timescale is set by the lens—source relative transverse motion $V_T$ and the size of $R_E$. Putting in typical values gives:

$$t_E = 50 \text{ days } \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{D}{\text{kpc}}\right)^{1/2} \left(\frac{V_T}{100 \text{ km/s}}\right)^{-1} \qquad (21)$$

where

$$D = \frac{D_L D_{LS}}{D_S} \qquad (22)$$

Since stars span a range of masses, distances and speeds, we observe events with a broad distribution of timescales

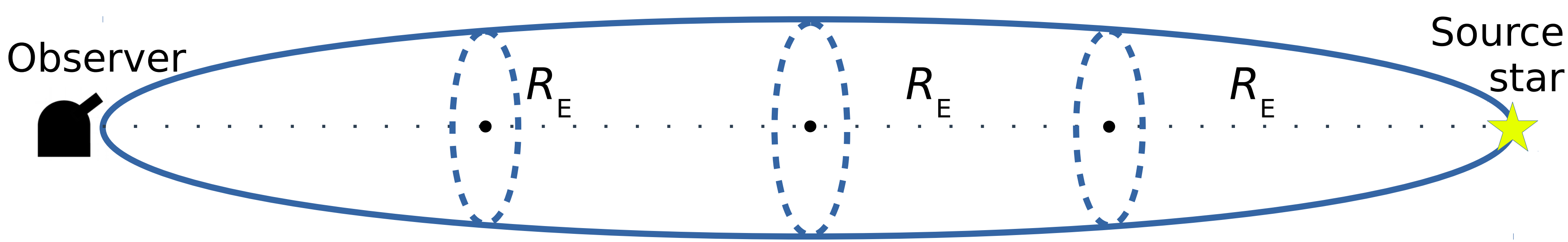
This means that we cannot use $t_E$ to solve directly for $M$ unless we have independent information about $V_T$ and $D$ (**timescale degeneracy**).

# Microlensing optical depth

Eq (6) defines the physical Einstein radius $R_E$ of a lensing star at distance $D_L$.

$R_E$ defines the cross section of a microlensing “tube” between the observer and background source star. Any foreground star lying within this tube will significantly gravitationally lens the source.

Since $R_E$ is a function of $D_L$ the cross section of the microlensing tube varies with distance, producing a “cigar”-shaped tube.




Microlensing “tube” of variable cross-section radius $R_E$

# Microlensing optical depth

The number of foreground stars within the microlensing tube is determined by the number density of stars along the line of sight.

Consider lenses with mass $M$ and a number density distribution $n(D_L)$. The surface mass density of the lenses within the distance interval $(D_L, D_L + dD_L)$ is

$$d\Sigma(D_L) = Mn(D_L)\,dD_L = \rho(D_L)\,dD_L \qquad (21)$$

where $\rho$ is the volume mass density and is generally a function of $D_L$.
The surface mass density contained within an Einstein radius is

$$\Sigma_{\rm lens} = \frac{M}{\pi R_E^2} \qquad (22)$$

The total microlensing **optical depth** for a star at distance $D_S$ is given by the ratio of (21) and (22) integrated along the sight line:

$$\tau = \int_0^{D_S} \frac{d\Sigma}{\Sigma_{\rm lens}} = \int_0^{D_S} \frac{\pi R_E^2}{M}\rho(D_L)\,dD_L = \frac{4\pi G}{c^2 D_S}\int_0^{D_S} D_L D_{LS}\rho(D_L)\,dD_L$$

# Microlensing optical depth

$$\tau = \frac{4\pi G}{c^2 D_S} \int_0^{D_S} D_L D_{LS} \rho(D_L)\, dD_L \qquad (23)$$

- Note:
  - Physically, the optical depth gives the number of ongoing microlensing events at a given time for a random source star at distance $D_s$.
  - Optical depth scales with the lens **mass density** (not with number density!)
  - The optical depth is **independent of lens mass *M***
  - Even towards the Galactic Centre, where the density of stars is high, the optical depth is still tiny: $\tau \sim \mathcal{O}(10^{-6})$
  - Microlensing is an intrinsically rare phenomenon. But there are billions of stars in the Galaxy so many events are ongoing (somewhere) all of the time

# Microlensing rate

The microlensing rate Γ is the number of lenses per unit time which cross within their own Einstein radius from a background source star. It is given by the ratio of the optical depth (eqn 23) to the mean Einstein radius crossing time:

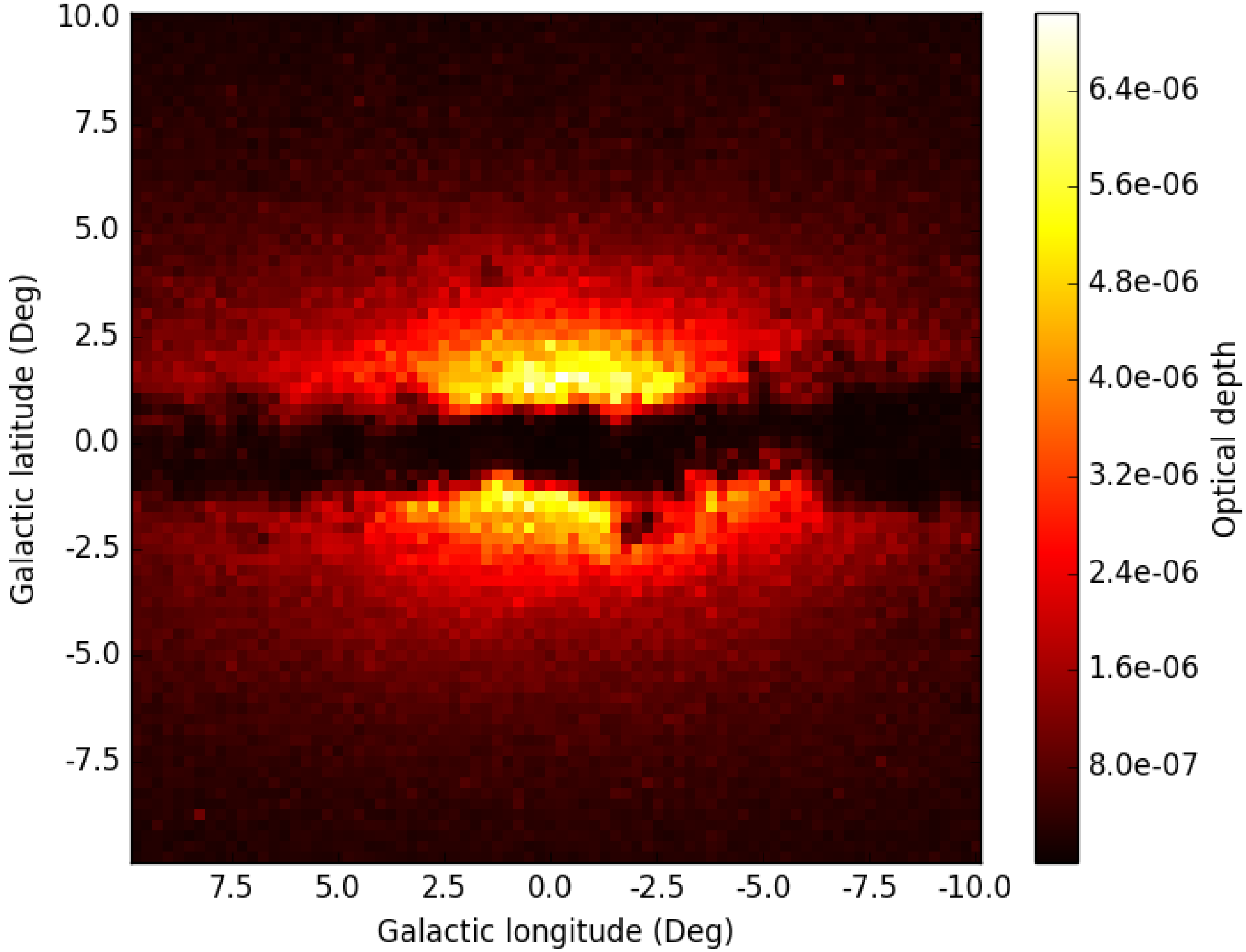
$$\Gamma = \frac{2}{\pi} \frac{\tau}{\langle t_E \rangle} \qquad (24)$$

The factor 2/π results from averaging over minimum impact distances $u_0$.

For ordinary stars towards the Galactic Centre the microlensing rate is $\Gamma \sim 10^{-5}$ events per year per background source star. Ground-based optical surveys such as OGLE and MOA find around 2000 new events each year towards the Galactic Centre by continuously monitoring around 100 million stars. Around 0.1-1% of these show evidence of one or more **planetary companions** around the foreground lensing star.

# Simulating the microlensing distribution

Awiphan, Kerins & Robin (2016)
www.mabuls.net



# Planetary microlensing

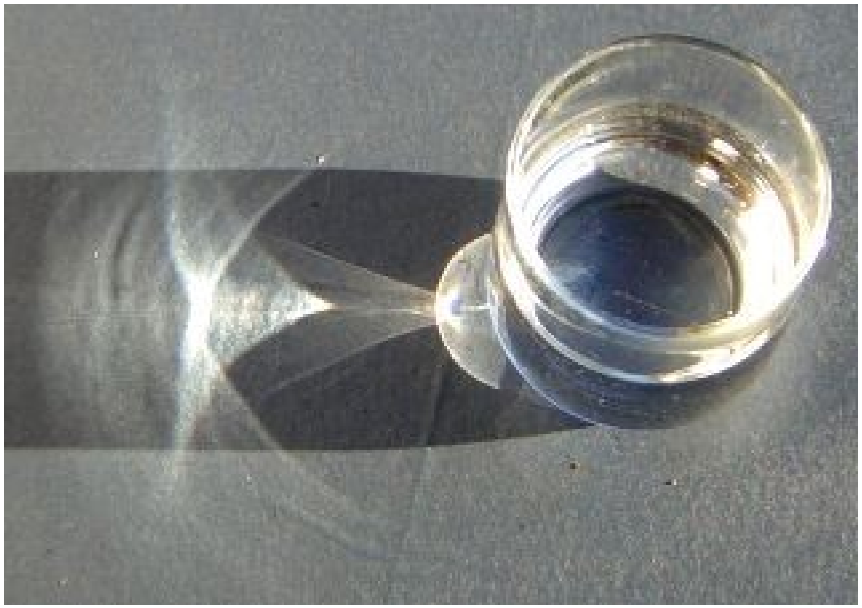
Adding lenses to the system results in an increase in the complexity of the microlensing effect:

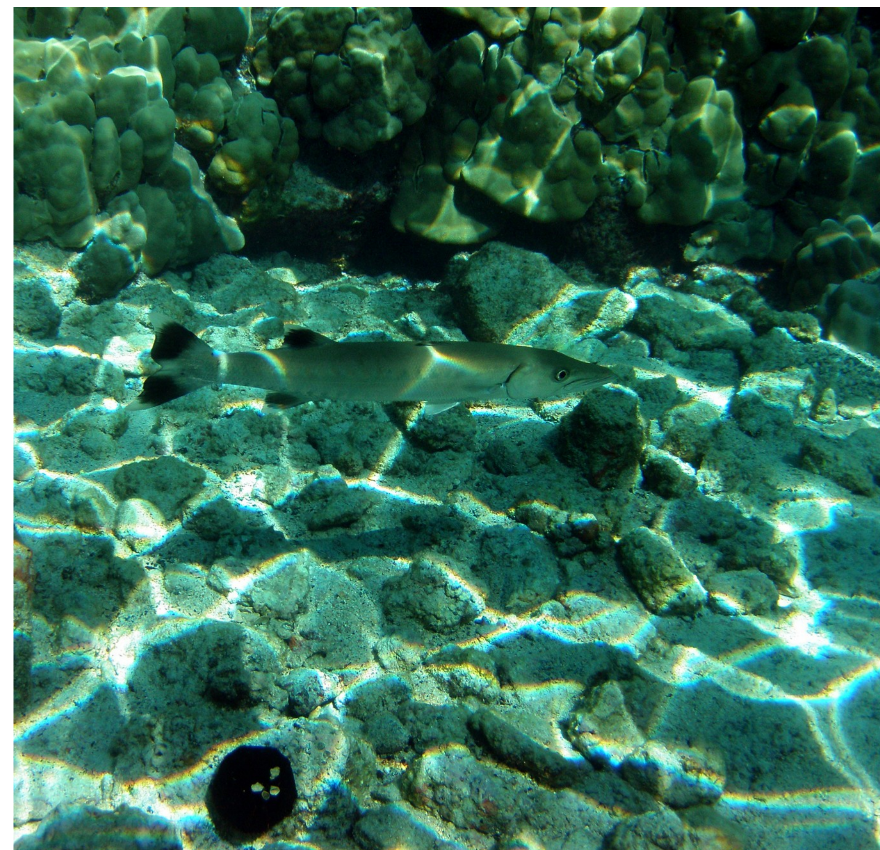
- **More (unresolvable) images are produced**. For an $N$-lens system the maximum number of images produced is $N^2+1$. So for a 2-lens (binary lens) system up to 5 images of a background star may be produced by their combined microlensing effect.

- The lensing geometry is **no longer circularly symmetric** but instead exhibits a form of astigmatism in which the background star light is focused not to a point but along locii. The projections of these locii back onto the plane of the source are referred to as **caustics**.

- Whenever a point source lies at the position of a caustic the magnification diverges. We saw this for the single lens PSPL case when $u$ = 0. In the PSPL case the caustic is a point located exactly behind the lens. For binary lenses the caustics are generally closed concave curves.

# Everyday caustics



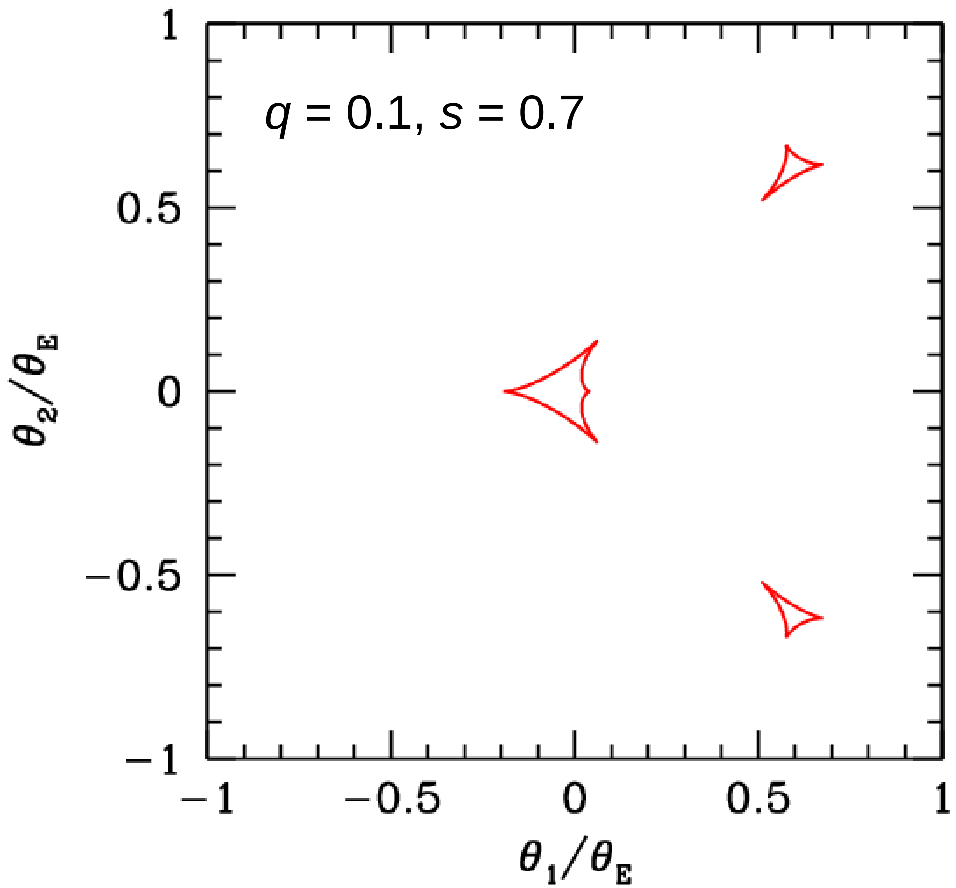


Optical caustics arising in familiar settings. These are analogues of gravitational lensing caustics

# Binary caustics

$q = m_p / M_*$

*s* is binary separation in units of Einstein radius



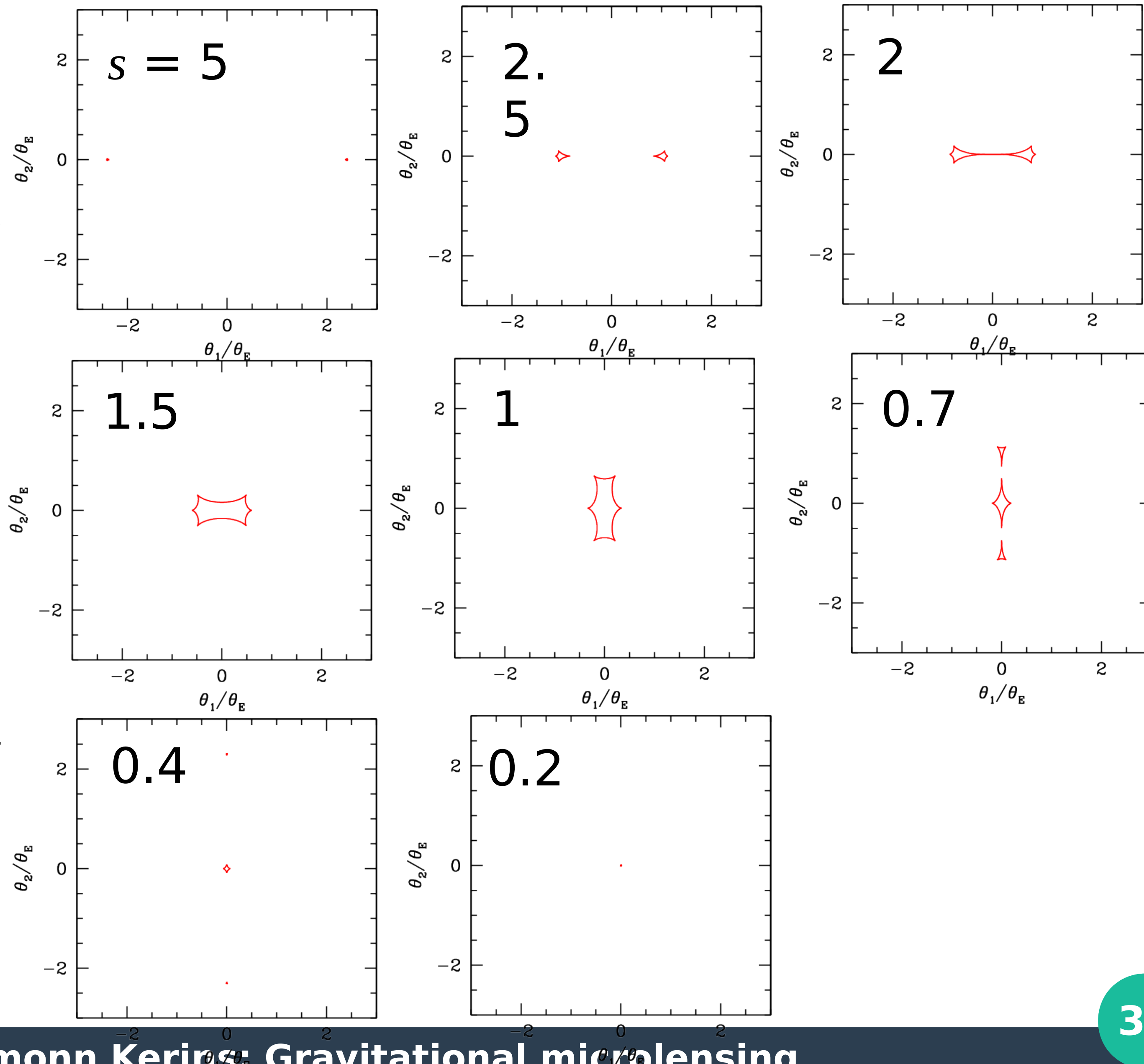

Plots generated from online binary simulator by M. Dominik

# Caustics vs binary separation

Caustic shape versus $s$ for an equal mass binary system ($q$ = 1)

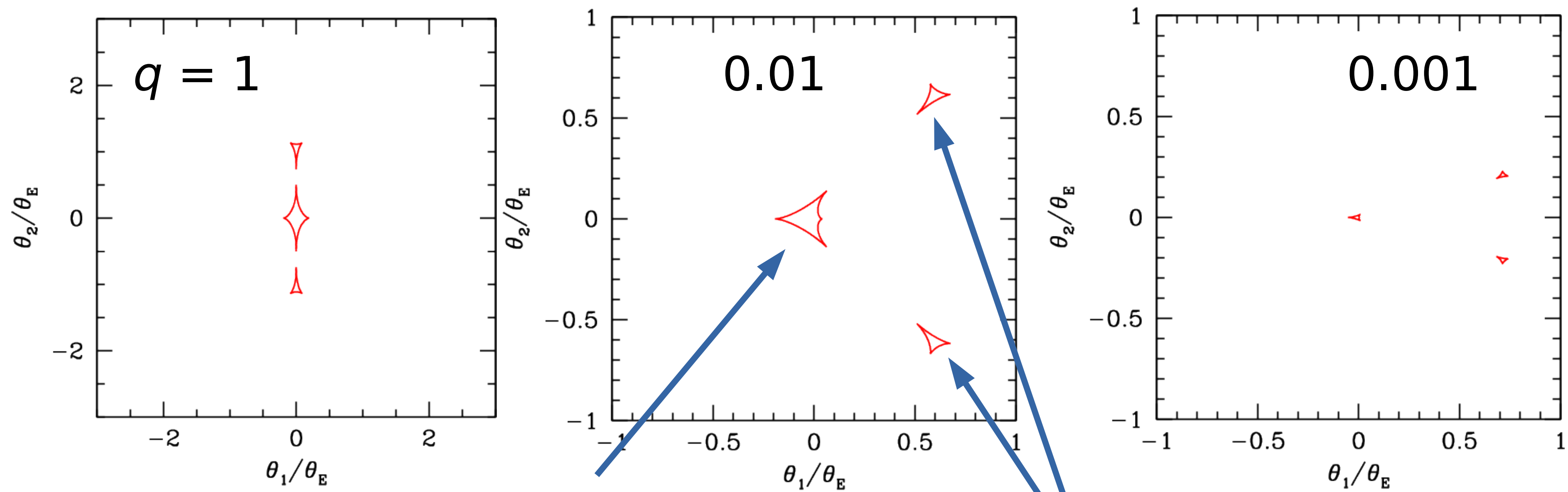
Very large $s$ case looks like 2 single lenses (point caustics).

Very small $s$ case looks like a single lens with double the mass



# Caustics vs planet mass ratio

Caustic shape versus $q$ for a binary separation $s = 1$




Central caustic (always located close to the host star)

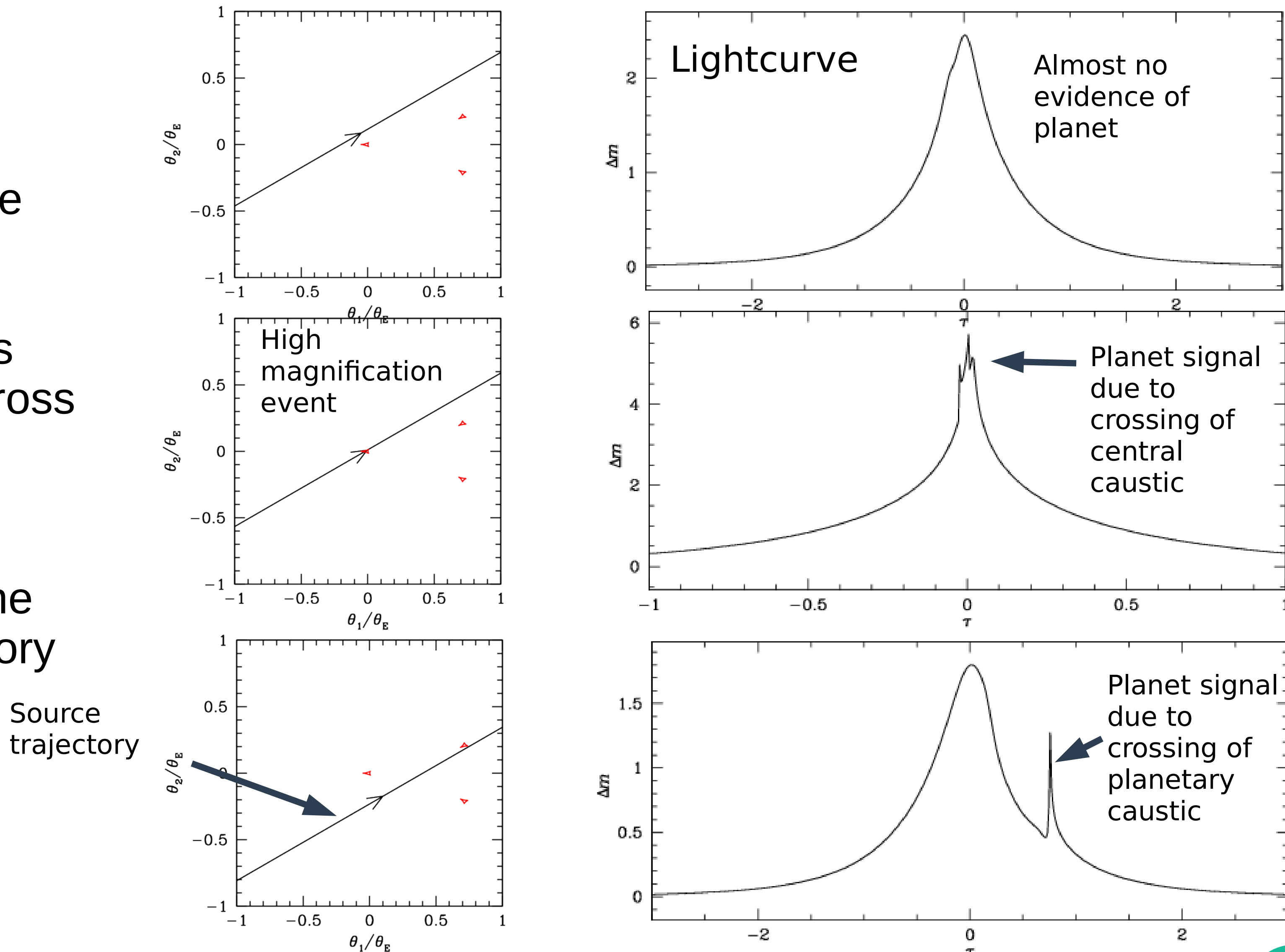
Planetary caustics

# Simulated lightcurves

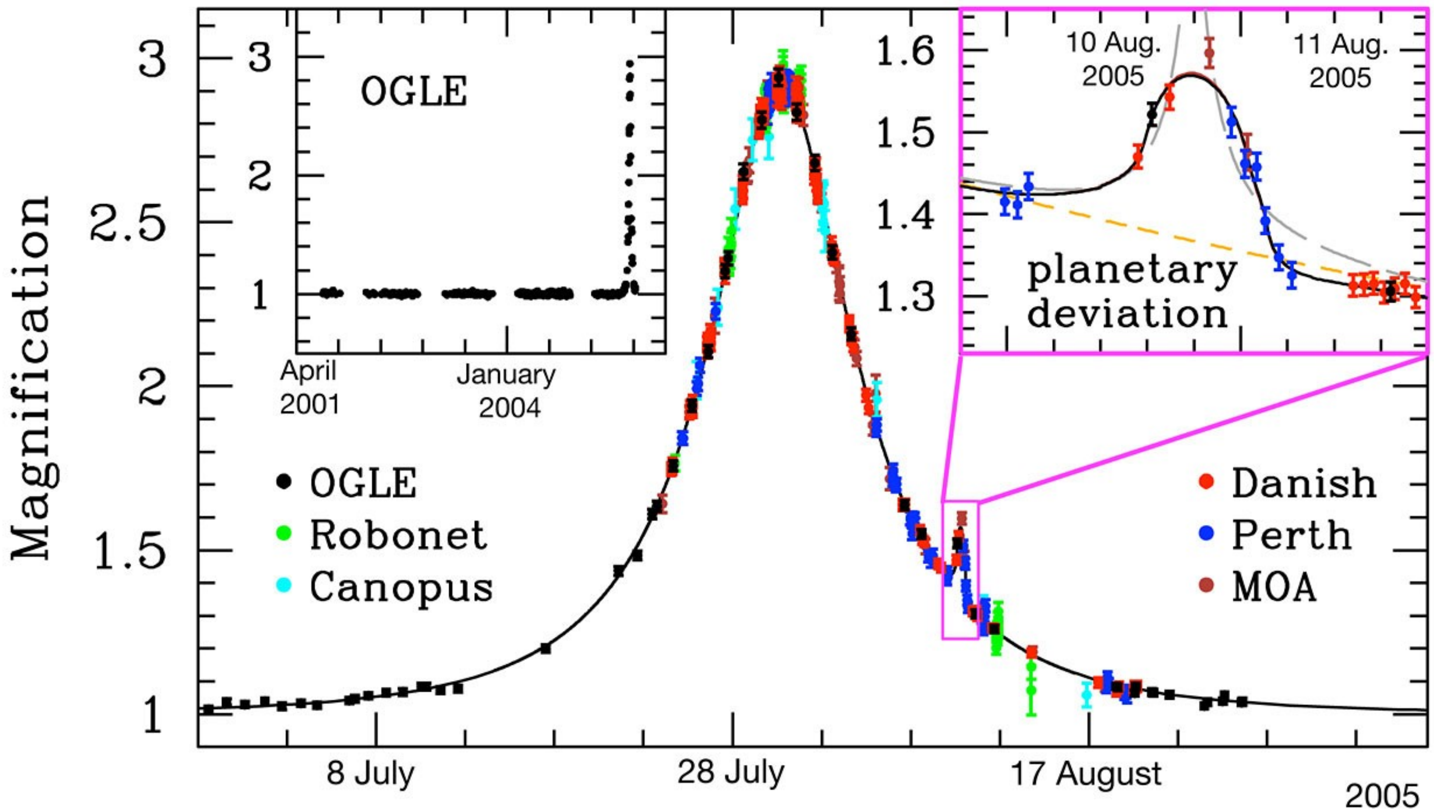
$s = 0.7$
$q = 0.01$

Evidence of a planet from the lightcurve whenever the source passes close to or across a caustic

Lightcurve depends on the source trajectory



# OGLE-2005-BLG-390b

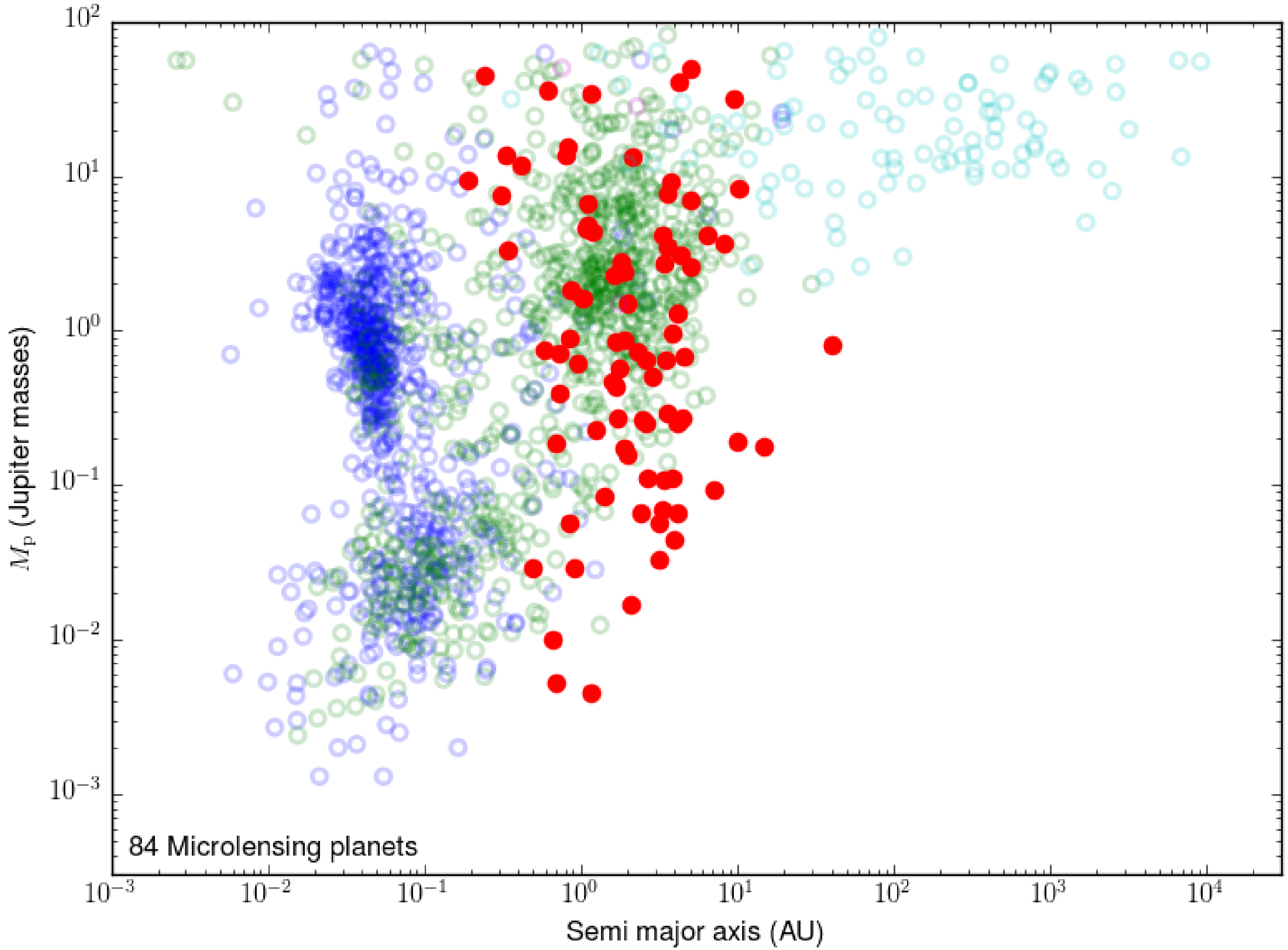



Beaulieu et al (2006)

$$t_{\rm p} \sim t_{\rm E}\sqrt{\frac{M_{\rm p}}{M_*}}$$

Lightcurve modelling usually gives the projected host separation $s$ and the mass ratio $q$ (i.e. not $a$ or $M_p$ directly). Additional information (eg detection of light from the host) can allow $a$ and $M_p$ to be estimated

# Exoplanet demographics



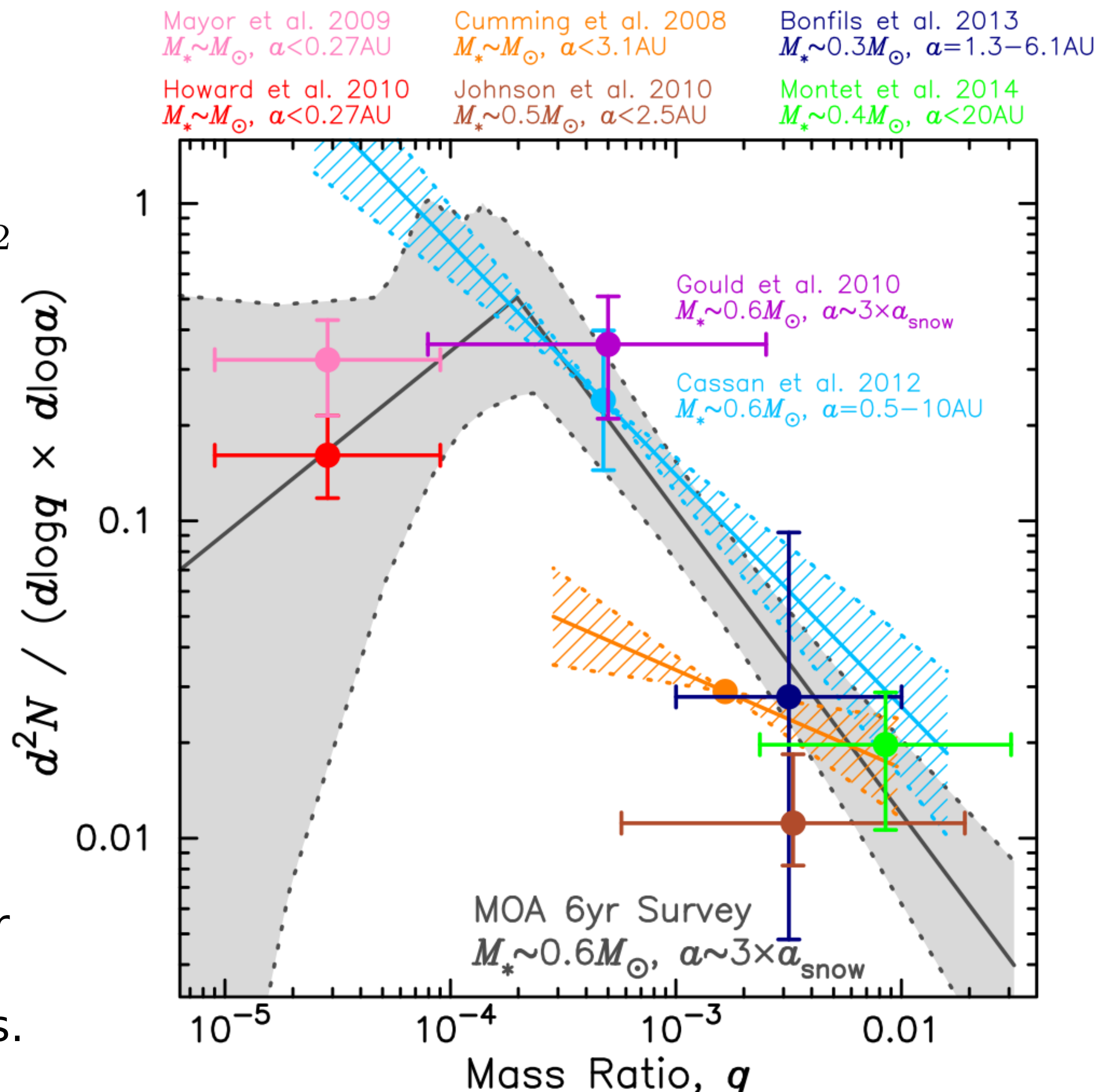
# Planet/host mass ratio

Suzuki et al (2016)

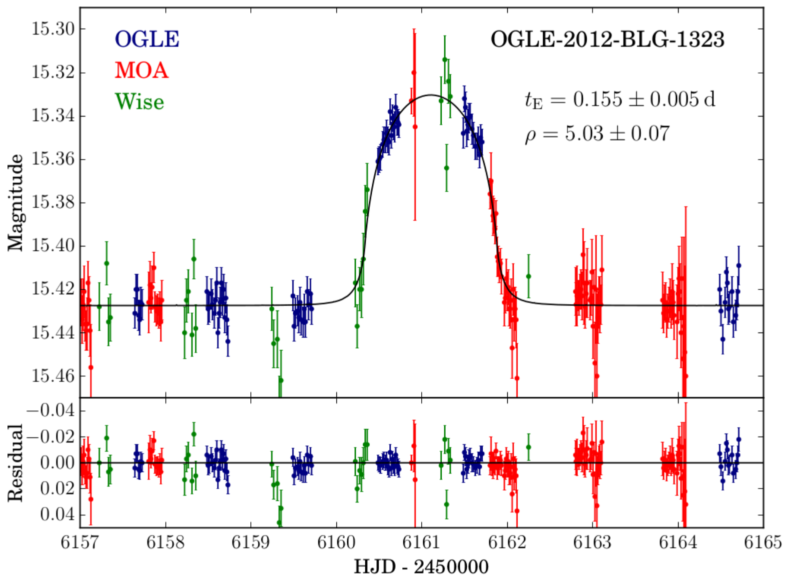
$$\frac{d^2 N}{d\ln q\, d\ln s} = A \left(\frac{q}{q_{\rm br}}\right)^p s^m \;{\rm dex}^{-2}$$

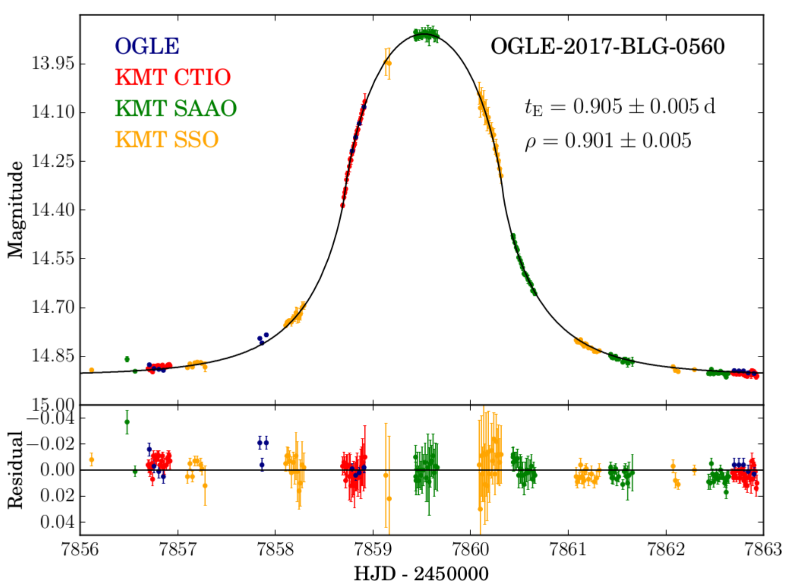
$$A = 0.61^{+0.21}_{-0.16}$$
$$q_{\rm br} = 1.7 \times 10^{-4}$$
$$s = 0.49^{+0.47}_{-0.49}$$
$$p = \begin{cases} 0.6^{+0.5}_{-0.4} & (q < q_{\rm br}) \\ 0.93 \pm 0.13 & (q > q_{\rm br}) \end{cases}$$

Most stars like have Earths/super Earths at host separations comparable to the Einsten radius.



# Free floating planets (FFPs)




Very short timescale lightcurves indicate the possible presence of isolated planetary mass bodies! Dubbed free floating planets (FFPs)

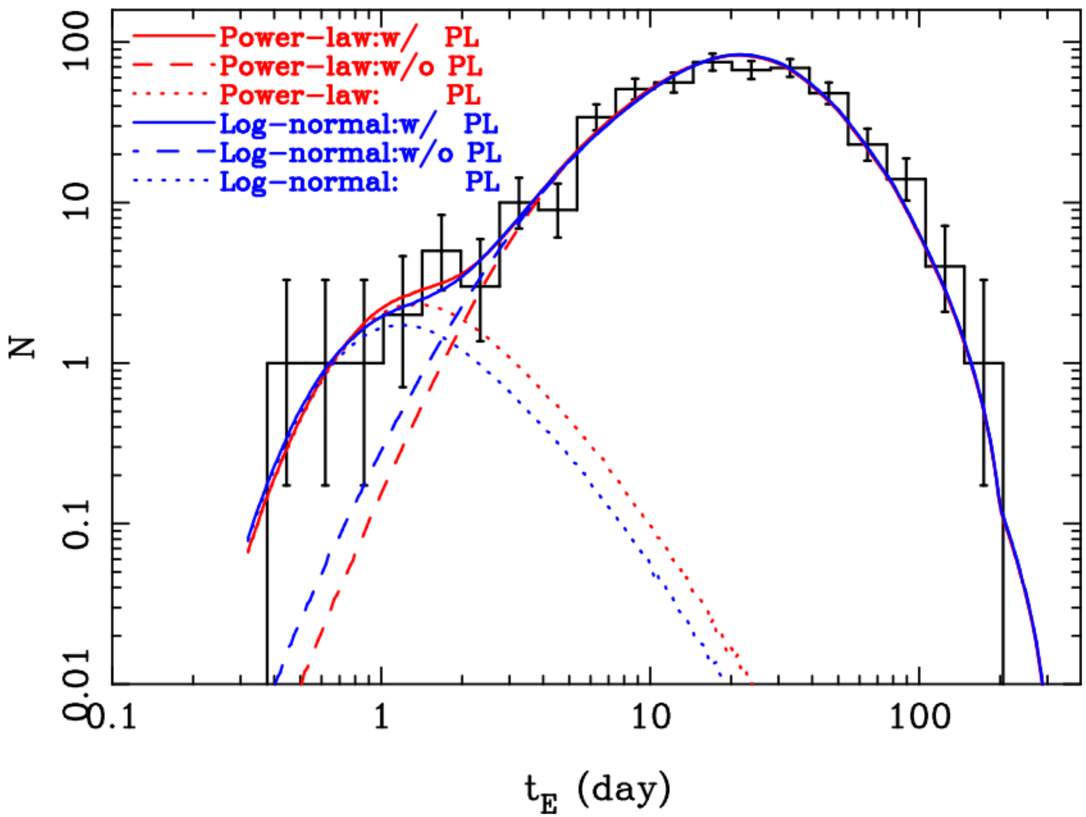
But are they FFPs or are they just widely separation planets?

2 short timescale events from the OGLE team (Mroz et al 2018)

# FFPs in abundance?

The distribution of microlensing event timescales observed by the MOA microlensing survey.

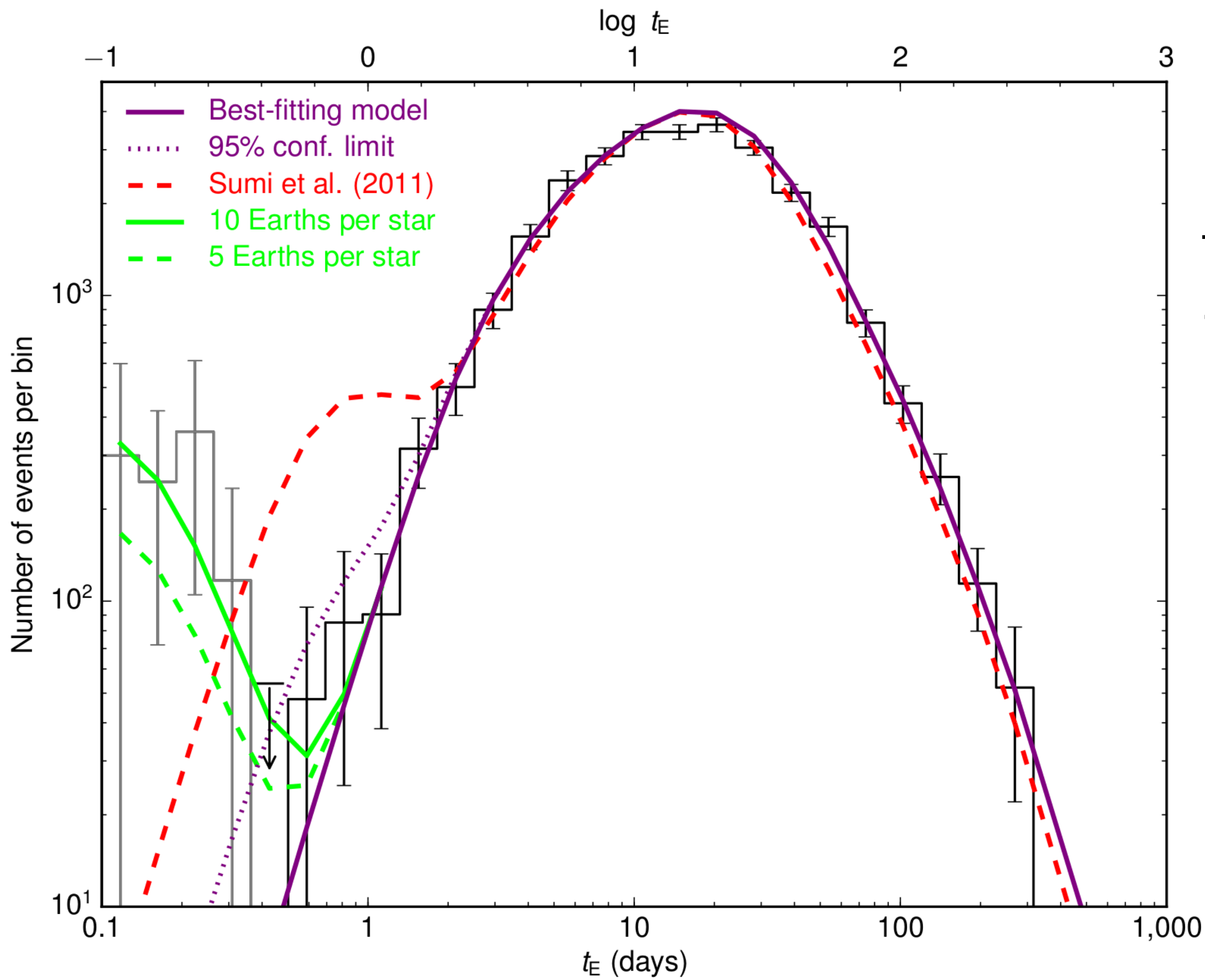
The hump at short timescales is consistent with a large population of Jupiter mass FFPs, but is based on only 10 events.




Sumi et al (2011)

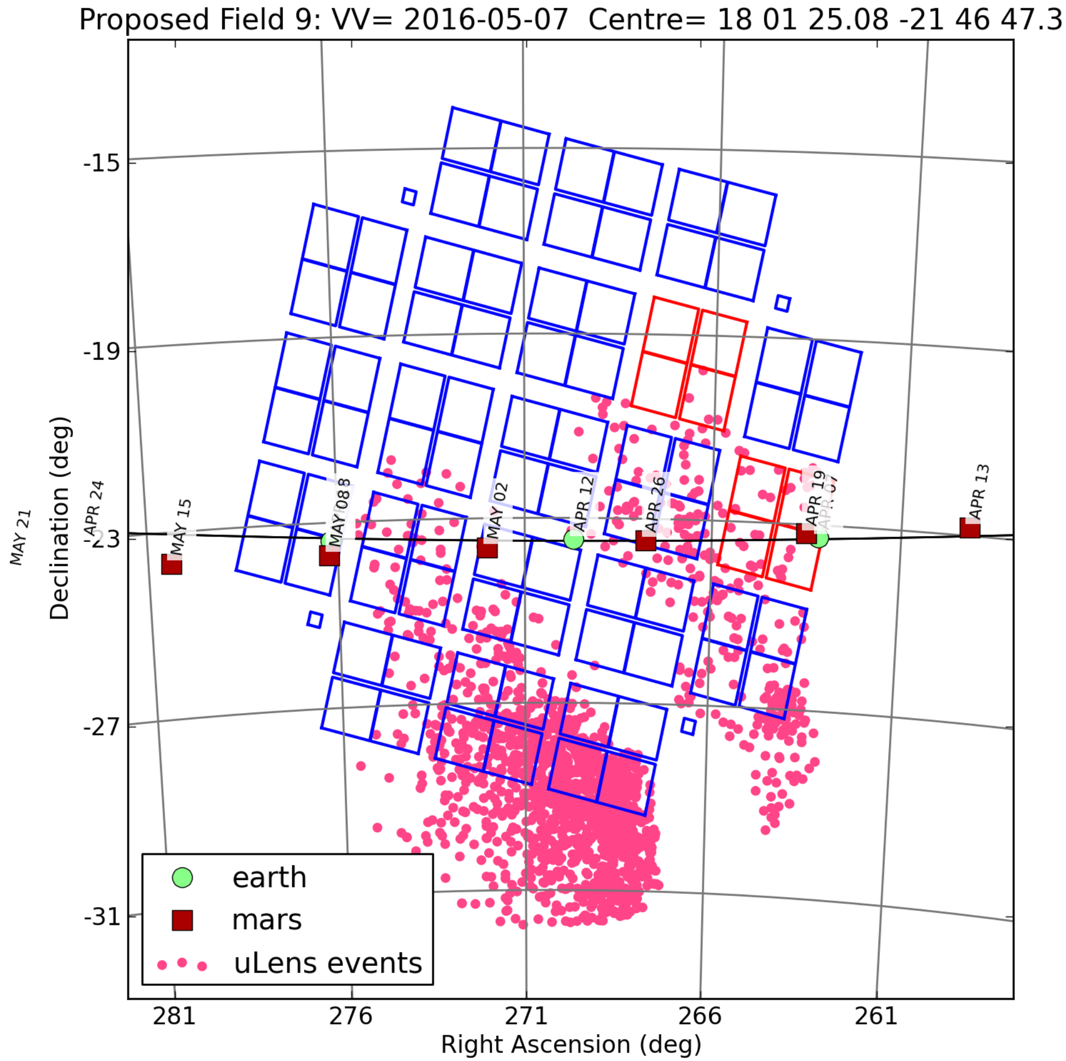
# FFP tension

A recent analysis by the OGLE team finds no significant abundance of Jupiter mass FFPs but **does** find an excess of Earth mass FFPs! More data is needed to resolve the conflict.




The OGLE survey (Mroz et al. 2017)

# Kepler K2 and FFPs

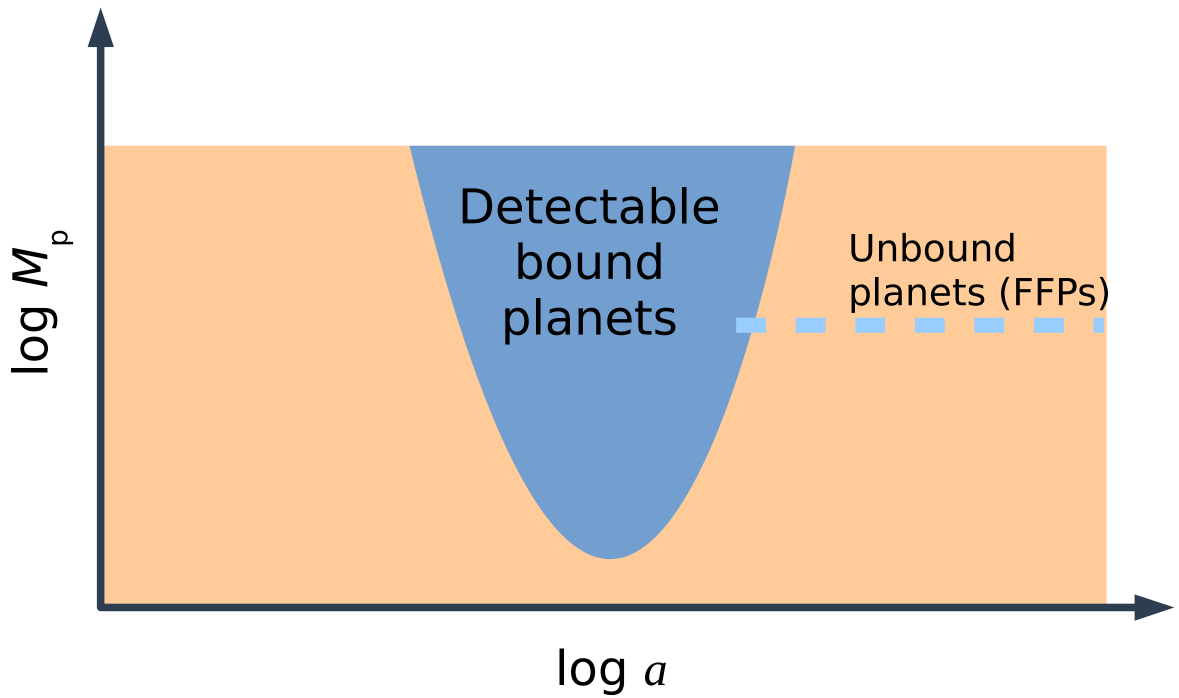



Campaign 9 of the Kepler K2 mission aims to find free-floating planets

# Exoplanet sensitivity

Microlensing is capable of detecting low-mass planets. But its sensitivity declines for planet orbits much larger or smaller that the Einstein radius of the host star.

Planets at very large orbital separation, including unbound (free-floating) planets can be detected as short timescale single-lens events.



# Precision mass measurements

Previously we found

$$\theta_E = \sqrt{\frac{4GMD_{LS}}{c^2 D_S D_L}}$$

impyling

$$M = \frac{\theta_E^2}{\kappa \pi_{\rm rel}}, \qquad \theta_{\rm E} = \mu_{\rm rel} t_{\rm E}$$

$$\kappa = \frac{4G}{c^2 (1\ \mathrm{AU})}, \qquad \pi_{\rm rel} = (1\ \mathrm{AU})(D_{\rm L}^{-1} - D_{\rm S}^{-1})$$

If we want to solve for *M* then we need a way of measuring both $\pi_{\rm rel}$ and $\mu_{\rm rel}$.

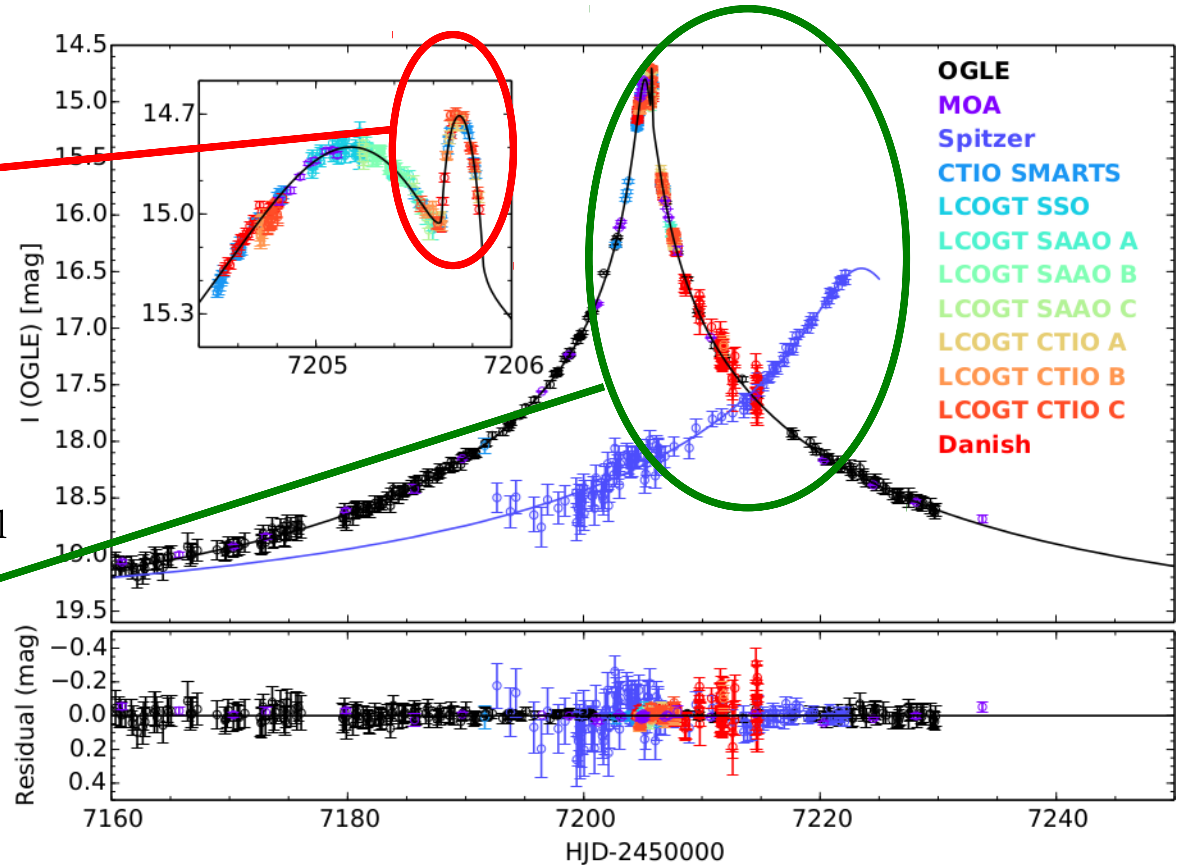
# OGLE-2015-BLG-0966

1. Measure Einstein radius $\theta_E$

Note rounded profile of planet anomaly due to smearing effect of the finite angular size of the background source star. This provides an angular "ruler" to measure the Einstein radius

2. Measure the relative parallax $\pi_{\rm rel}$

The time and magnification differences between observing an event on Earth and by a satellite located far from Earth provides the relative parallax.




Ground + Spitzer observations of microlensing (Street et al 2016) producing a direct planet mass measurement. The anomaly is caused by a Neptune mass planet with a mass of 20±2 Earth masses.