Optical & infrared photometry

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Optical & infrared photometry

- Fundamental limitations
- Ground-based limitations
- Standardising photometry
- Calibrating photometry
- Aperture, annular and PSF photometry
- Absolute and relative photometry
- Specialist techniques

Fundamental limitations

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Photon counting

Photon emission is a *random process*. Consider observation of a photon source over some time interval. Let's split the time interval up into small chunks within each of which either one photon is recorded or no photon is recorded.

For a random time chunk we can say the probability that it will contain a photon is p, in which case the probability it does not contain a photon is 1-p. We therefore expect a series of N small time intervals to contain $n = pN$ photons on average, but the process is random. The probability that they instead collectively contain $k \leq N$ photons is given by the binomial distribution:

$$B(k, N) = \frac{N!}{k!(N-k)!} p^k (1-p)^{N-k}$$

Most of the time chunks are empty and therefore $p \ll 1$ or equivalently $n \ll N$.

Fundamental limitations

Photon counting

Using $p = n/N$ we can re-write B as

$$B(k,N) = \frac{N!}{k!(N-k)!}\left(\frac{n}{N}\right)^k\left(1-\frac{n}{N}\right)^{N-k}$$

$$= \frac{n^k}{k!}\,\frac{N.(N-1)...(N-k+1)}{N.N.....N}\left(1-\frac{n}{N}\right)^{N-k}$$

What happens to B as $N \rightarrow \infty$ for fixed small n?

Fundamental limitations

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$$= \frac{n^k}{k!}\left(\frac{N.(N-1)...(N-k+1)}{N.N.....N}\right)\left(1-\frac{n}{N}\right)^{N-k}$$

$$\lim_{N\to\infty}() = 1$$

What happens to B as $N \to \infty$ for fixed small n?

Fundamental limitations

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$$\lim_{N\to\infty} B(k,N) = \frac{n^k}{k!}\left(1-\frac{n}{N}\right)^{N-k}$$

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$$\lim_{N\to\infty} B(k,N) = \frac{n^k}{k!}\left(1-\frac{n}{N}\right)^{N-k}$$

Use $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$ and set $N = -xn$

What happens to B as $N \to \infty$ for fixed small n?

Fundamental limitations

Photon counting

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$$B(k,N) = \frac{N!}{k!(N-k)!}\left(\frac{n}{N}\right)^k\left(1-\frac{n}{N}\right)^{N-k}$$

$$\lim_{N\to\infty} B(k,N) = \frac{n^k}{k!}e^{-n}$$

The Poisson distribution!

Fundamental limitations

The Poisson limit

Photon emission is the result of a random process and is fundamentally governed by Poisson statistics, with probability

$$P(N, \Gamma\Delta t) = \frac{(\Gamma\Delta t)^N}{N!}\exp(-\Gamma\Delta t)$$

for N emitted photons over a time interval Δt when the *average* emission rate is Γ.

We can **never** know Γ , instead we can only ever measure $\Gamma' = N/\Delta t$.

Fundamental limitations

The Poisson limit

Independent photon measurements N_i, each of duration Δt, are subject to random fluctuations $(\Gamma'_i - \Gamma)\Delta t$ about $\Gamma\Delta t$ with a variance

$$\sigma^2 = \Gamma\Delta t$$

We can **never** know σ^2. Our best estimates of σ^2 are $\sigma_i'^2 = N_i$. Accordingly, for a single observation, we *estimate* that the number of photons we detect is subject to an uncertainty

$$\sigma' = \sqrt{N}$$

such that for a measurement of N detected photons our best estimate of the photon rate uncertainty is

$$\Delta\Gamma' = \sigma'/\Delta t = \sqrt{N}/\Delta t = \sqrt{\Gamma'\Delta t}/\Delta t \propto 1/\sqrt{\Delta t}$$

This uncertainty is fundamental to the photon nature of light and can only be reduced by increasing the observing time Δt.

Fundamental limitations

Signal to noise ratio

The observed number of photons $\Gamma' \Delta t$ is referred to as the *signal* $\mathcal{S}$, whilst the estimated uncertainty $\Delta\Gamma' \Delta t$ is the *noise* $\mathcal{N}$. The quality of an astronomical observation is determined by the signal-to-noise ratio

$$\mathcal{S}/\mathcal{N} = \Gamma' \Delta t/(\Delta\Gamma' \Delta t) = N/\sqrt{N} = \sqrt{N} = \sqrt{\Gamma' \Delta t} \propto \sqrt{\Delta t}$$

Exercise: if we wish to obtain photometric measurements with a precision of 0.1%, what signal-to-noise ratio do we require, and how many photons do we need to collect?

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Exercise: if we wish to obtain photometric measurements with a precision of 0.1%, what signal-to-noise ratio do we require, and how many photons do we need to collect?

Solution:
0.1% precision implies that the noise level is 1/1,000 that of the signal, ie *S/N* = 1000. Since *S/N* = $S^{1/2}$ then *S* = 1,000,000 photons are required.

Fundamental limitations

Signal to noise ratio

In real situations our target is not the only contributor to the photon count in an image. There will be other sources of photons, collectively termed the *background*. For an observed target rate Γ_t and background rate Γ_b we have

$$\mathcal{S}/\mathcal{N} = \frac{\Gamma_\mathrm{t}\Delta t}{\sqrt{(\Gamma_\mathrm{t} + \Gamma_\mathrm{b})\Delta t}}$$

In the limit of a faint target we have

$$\mathcal{S}/\mathcal{N} \simeq \Gamma_\mathrm{t}\sqrt{\frac{\Delta t}{\Gamma_\mathrm{b}}}\left(1 - \frac{1}{2}\frac{\Gamma_\mathrm{t}}{\Gamma_\mathrm{b}}\right) \qquad (\Gamma_\mathrm{t} \ll \Gamma_\mathrm{b})$$

This implies that, for faint targets, the signal-to-noise ratio declines almost linearly with falling target photon rate. This is the *background-dominated* regime.

Fundamental limitations

Resolution

Since we are normally interested in point sources we can minimize background contamination through high spatial resolution θ, allowing us to define a tight aperture around our target source to exclude much of the background. However, for any telescope we are ultimately limited by the diffraction limit

$$\theta \geq \theta_{\rm min} = 1.22\lambda/D$$

Where λ is the observation wavelength and D is the telescope aperture.

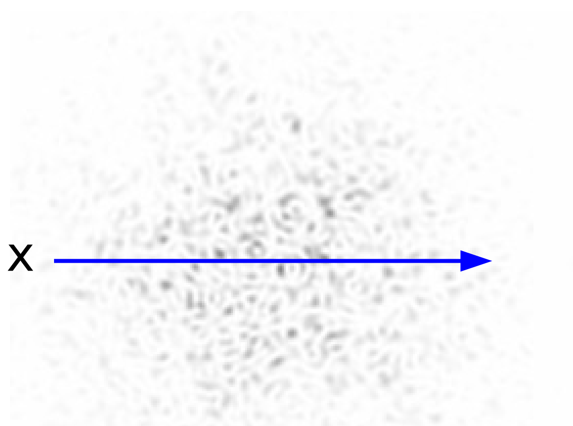
There is no way to make a noise-free or background-free observation!

Ground-based limitations

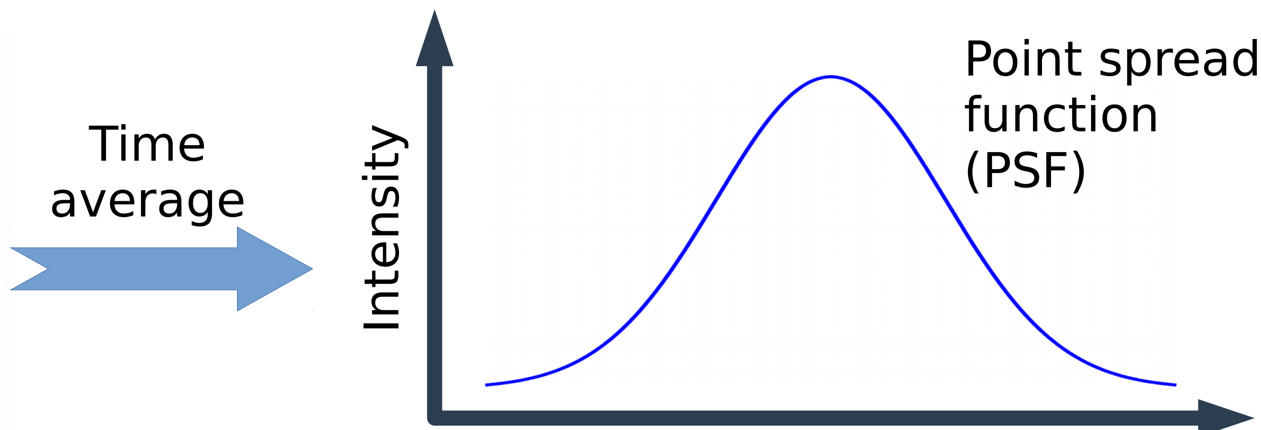
Seeing

Time-averaging of the effects of atmospheric turbulence smears the image of a target to a size larger than the diffraction limit for typical professional telescopes: **seeing**

Seeing effects can be minimised, but not eliminated, using adaptive optics or lucky imaging



Effect of atmospheric turbulence



PSF is usually gaussian-like, though not pure gaussian

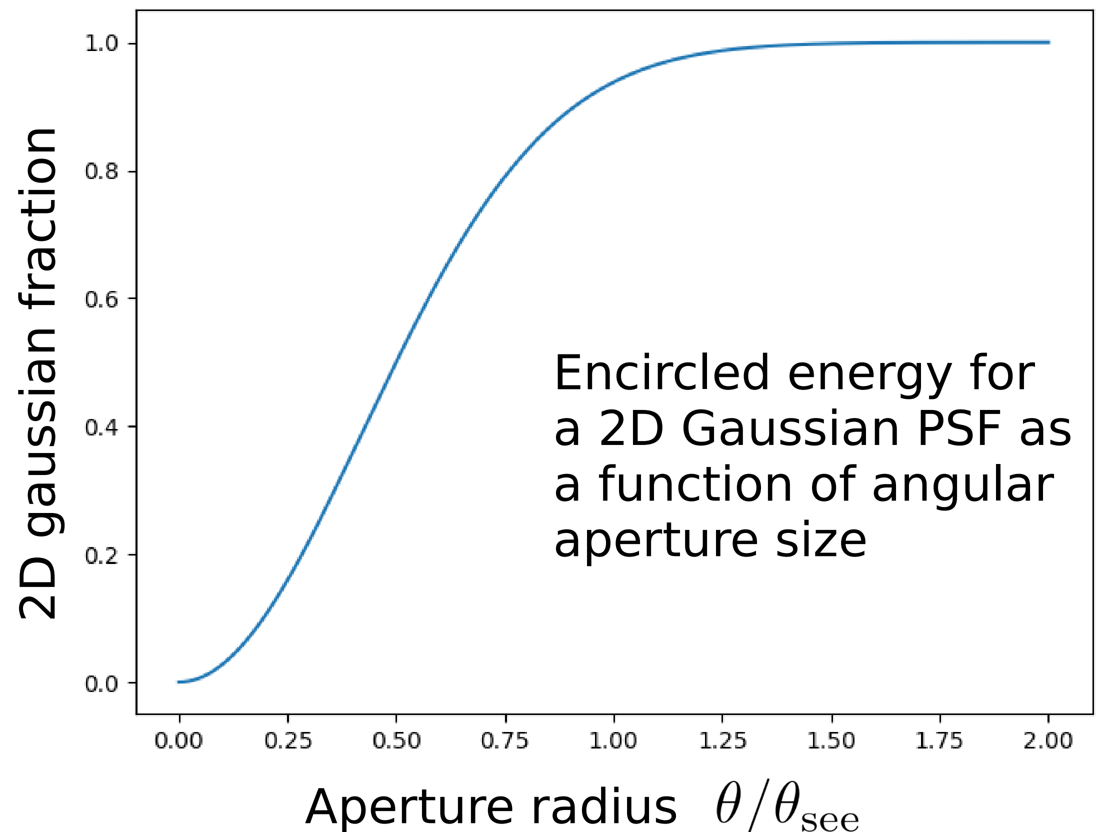
Ground-based limitations

Seeing

The seeing size is defined as the full-width at half maximum (FWHM) of the PSF. In the limit of a pure Gaussian PSF the seeing size is directly related to the Gaussian standard deviation:

$$\theta_{\rm see} = 2\sqrt{2\ln 2}\sigma \simeq 2.35\sigma$$

A circular aperture with a diameter equal to $\theta_{\rm see}$ will only contain around half of the total Gaussian PSF flux!



Ground-based limitations

Seeing and airmass

Airmass refers to the column of atmosphere through which the target is observed, relative to the optimal case of a target overhead (at zenith). In the plane-parallel atmosphere approximation

Airmass = cosec(h),

where h is the elevation angle of the target above the horizon. Usually, we do not perform observations at airmass >2 ($h < 30$ deg) as seeing degrades with airmass. In general

Seeing $\propto$ (airmass)$^{0.6}$ (wavelength)$^{0.2}$

Ground-based limitations

Seeing classification

Photometric: cloudless sky. Target flux is a dependent only on airmass

Non-photometric: some light cloud may be present that can cause some change to the observed flux. If the field of view is small then it may be the case that all sources in the field are equally afftected, allowing relative photometry to be performed.

Ground-based limitations

Throughput

The number of detected photons between wavelengths (λ_1, λ_2) depends upon several factors. For ground-based observations we have

$$N(\lambda_1, \lambda_2) = \Delta t \int_{\lambda_1}^{\lambda_2} D(\lambda)T(\lambda)A(\lambda)M(\lambda)\frac{\mathrm{d}\Gamma}{\mathrm{d}\lambda}\,\mathrm{d}\lambda$$

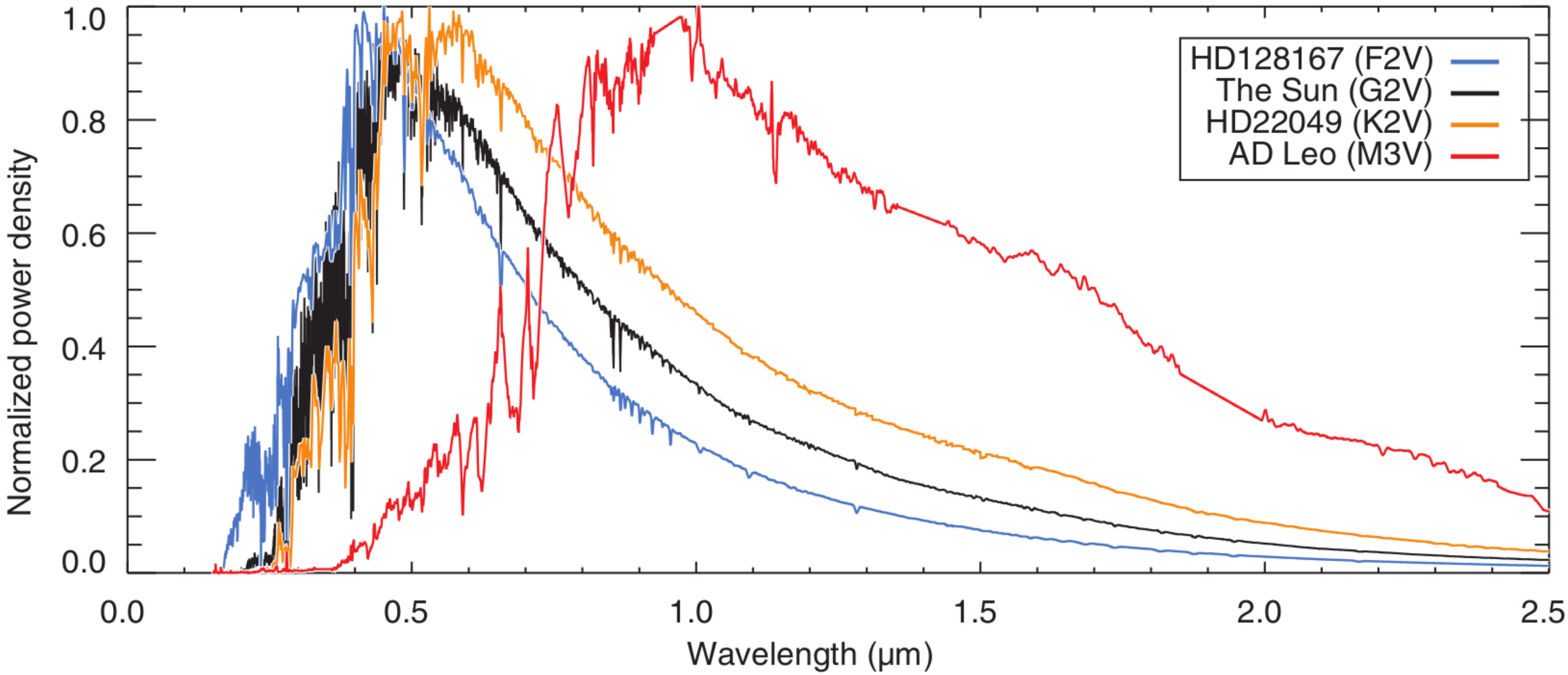
where the photon rate $\Gamma = \Gamma(\lambda).\ \mathrm{d}\Gamma/\mathrm{d}\lambda$ is proportional to the source spectral energy distribution (SED).

D : normalised response of the detector per unit wavelength
T : normalised telescope transmission (mirror surfaces, filters, …)
A : normalised atmospheric transmission
M : normalised transmission through interstellar medium (gas/dust) between the source and Earth

Note that *D*, *T* and *A* will vary with location/observatory!

Ground-based limitations

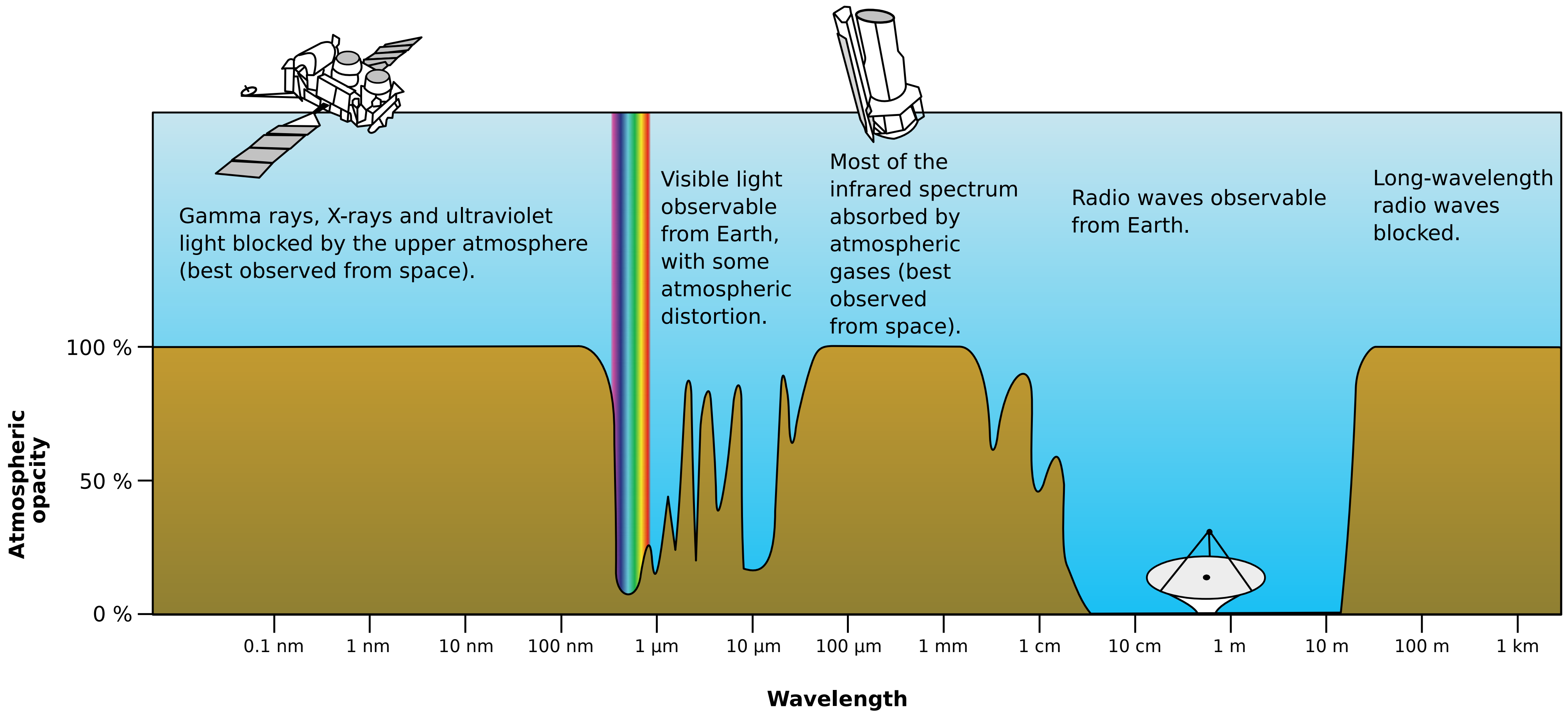
Throughput – Γ term



Stellar SEDs from Shields et al (2013)

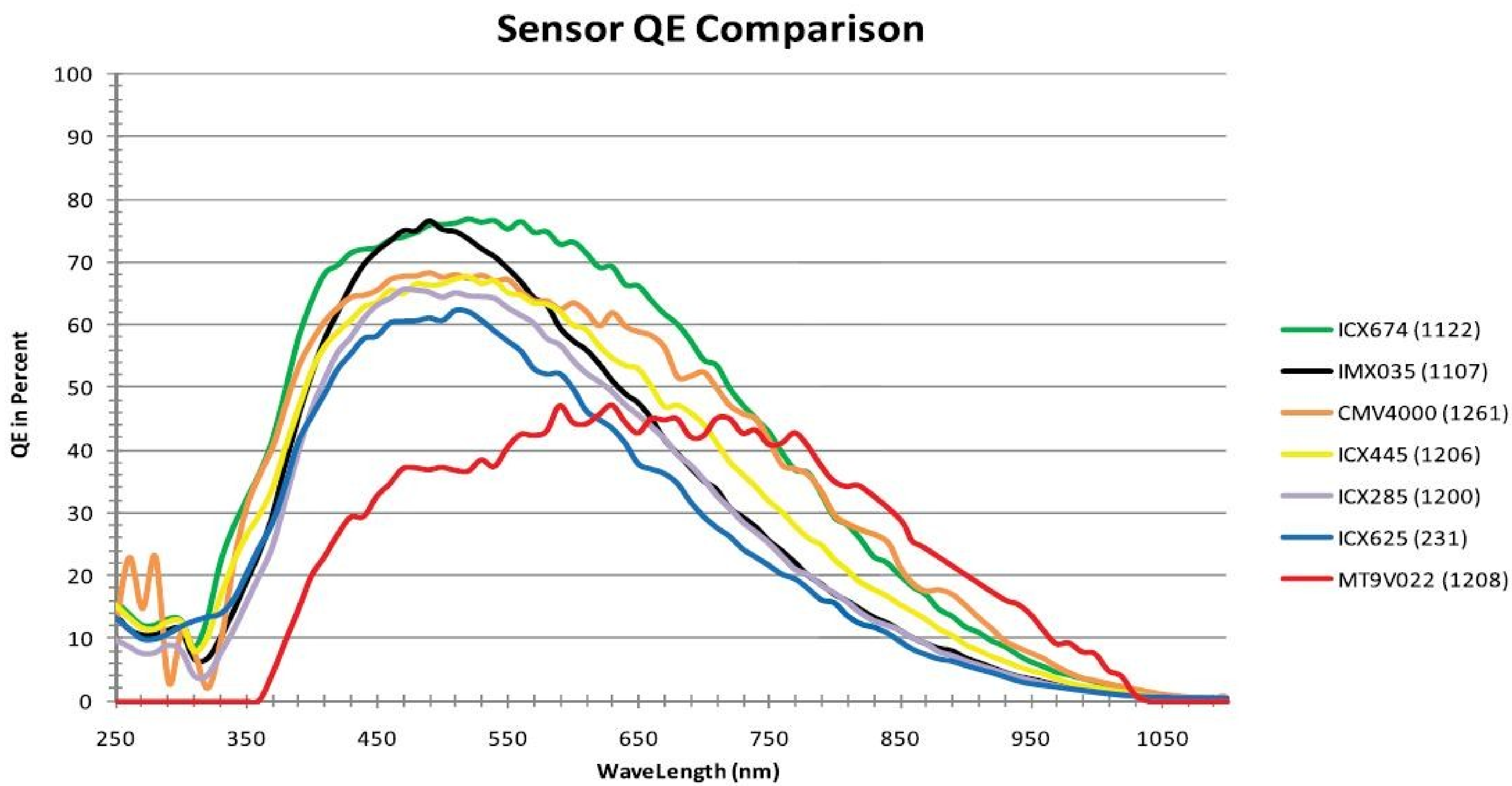
Ground-based limitations

Throughput – *A* term



Ground-based limitations

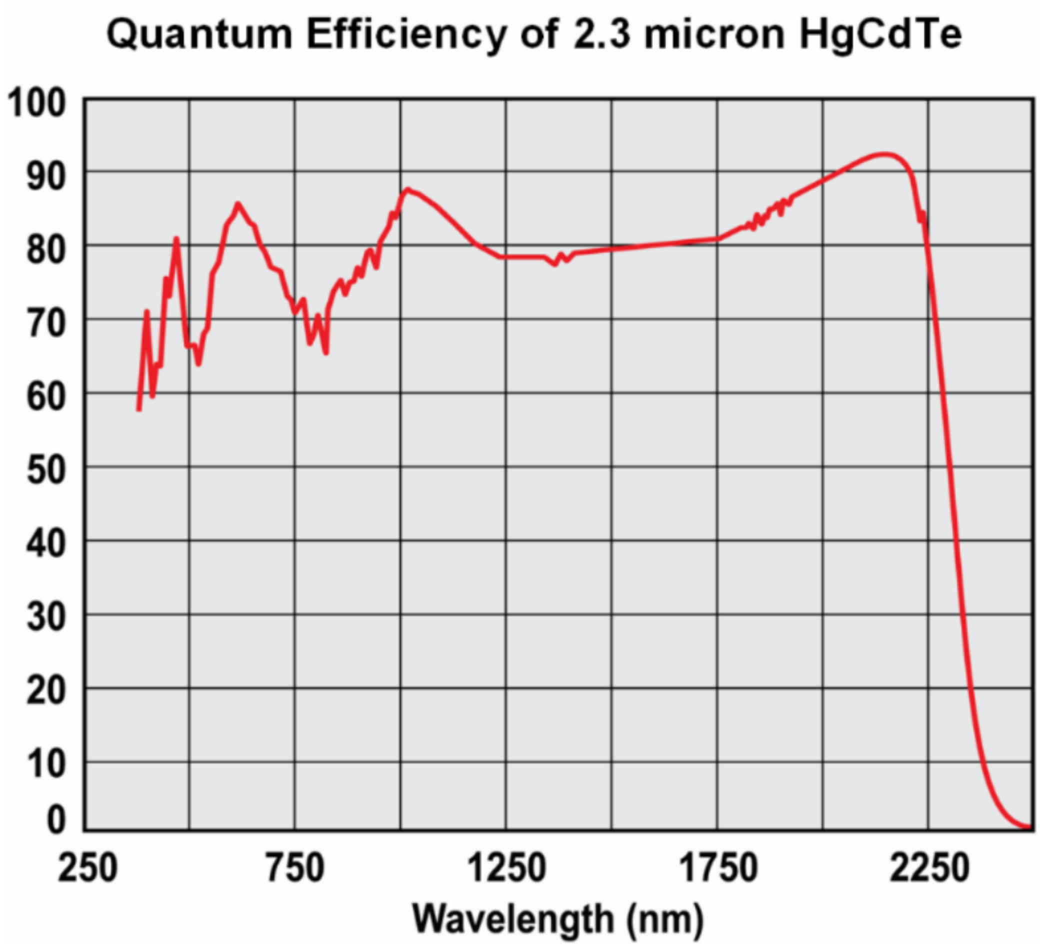
Throughput – *D* term



Throughput for a range of commercial CCD imagers made by Point Grey

Ground-based limitations

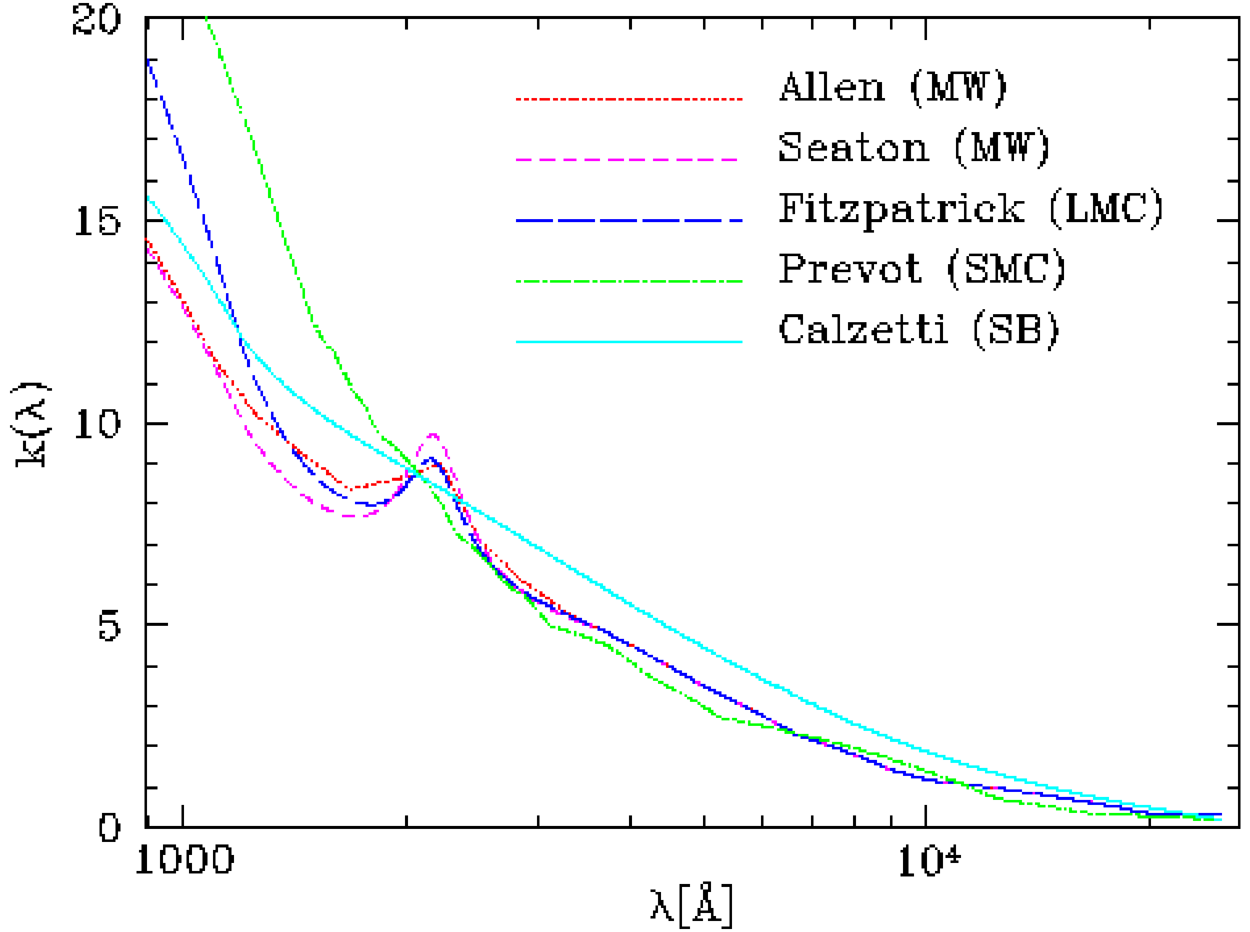
Throughput – *D* term



Throughput for a near-IR optimised array by Teledyne

Ground-based limitations

Throughput – *M* term

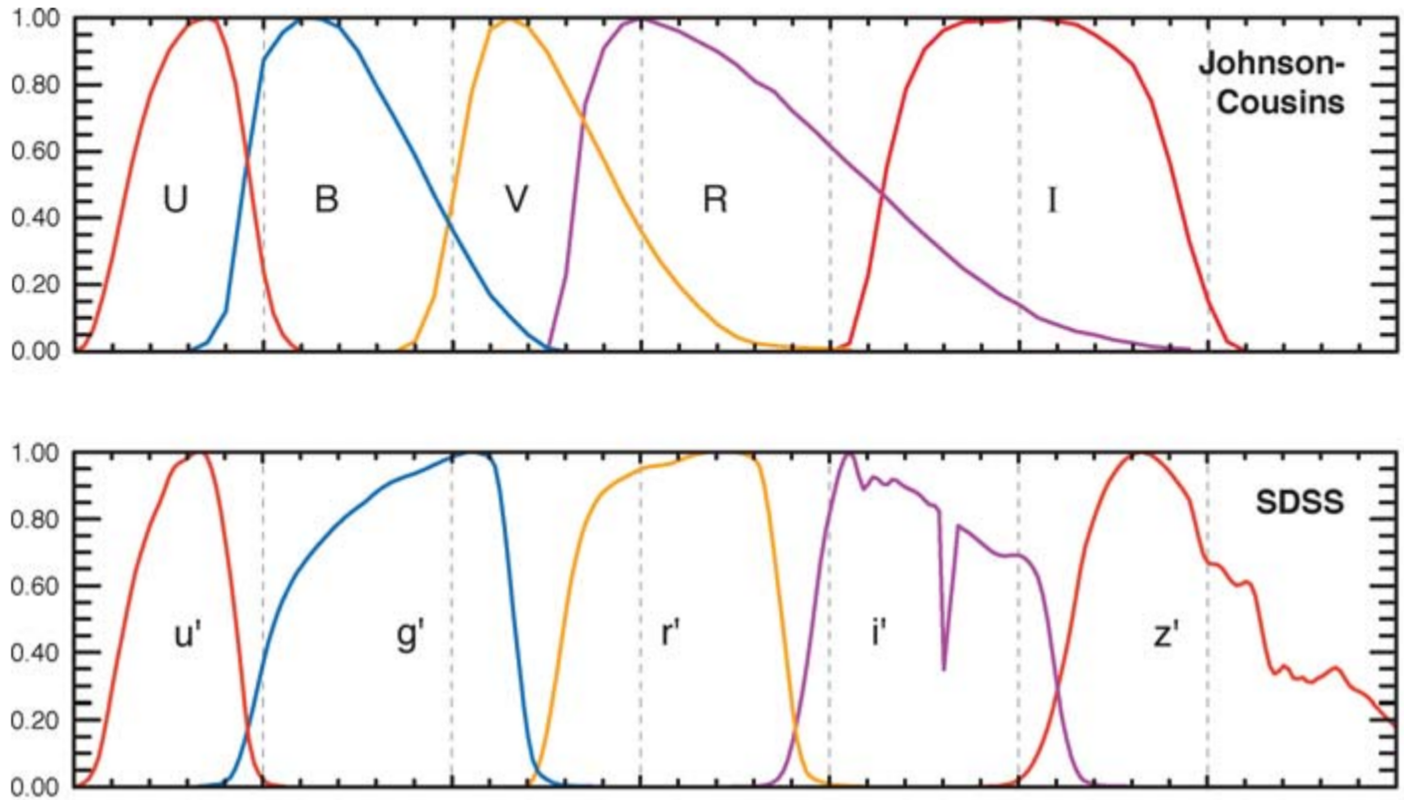


Compilation of extinction curves (Bolzonella et al 2000)

Standardising photometry

Throughput – *T* term

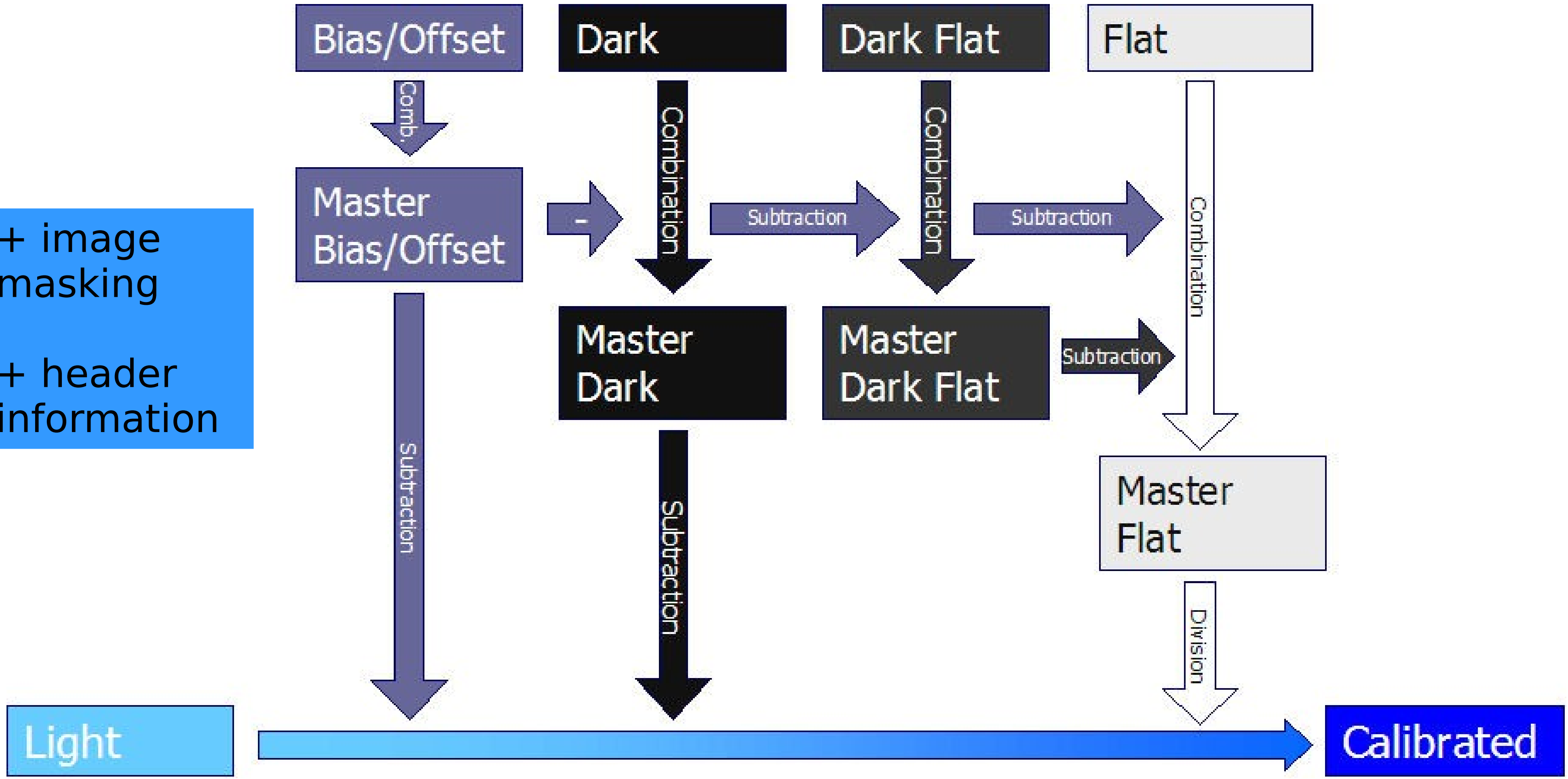
Johnson-Cousin UBVRI system



Bessel (2005)

Sloan system

Image pre-processing



+ image masking

+ header information

Pre-processing pipeline within the popular freeware Deep Sky Stacker astrophotography package.

Image pre-processing

- **Flat field correction** – A flat field is used to remove pixel response variations, vignetting, dust, … The flat field frame requires the detector to be uniformly illuminated, eg from an exposure of a twighlight sky which is too bright for stars to be visible (sky flat) or from a screen inside the observatory (dome flat). Flat field correction involves division of the science frame by a flat field which is normalised to have a mean pixel count of unity to conserve total image flux.

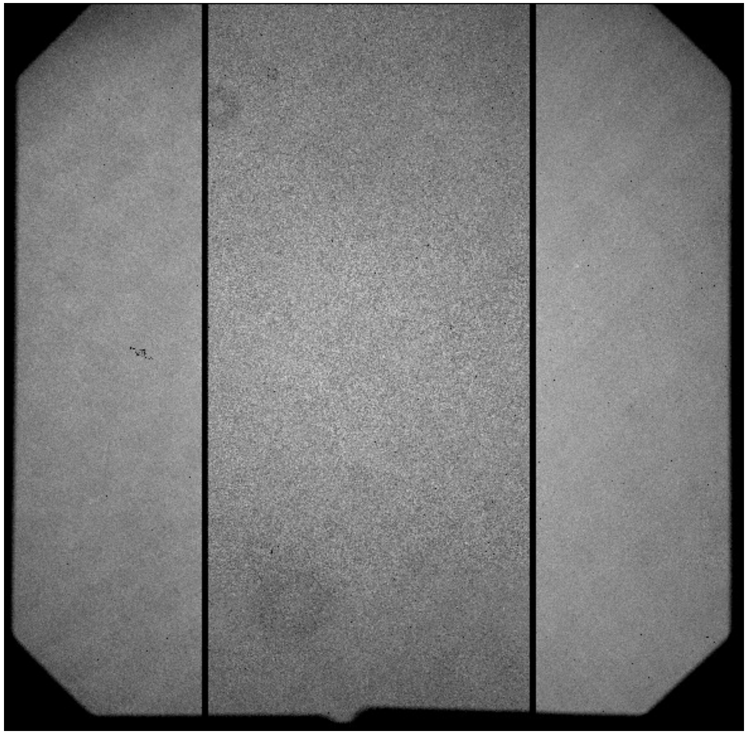
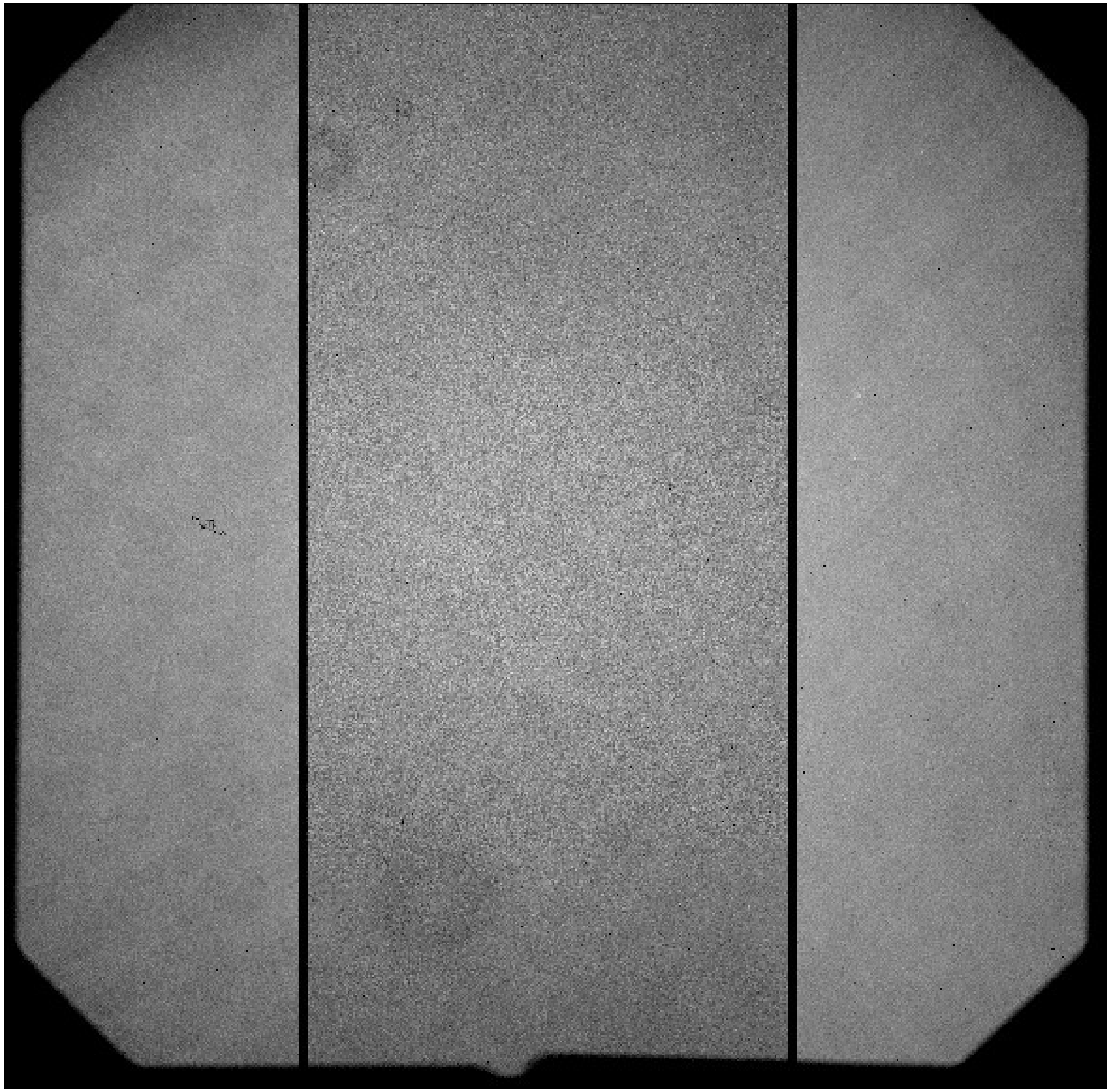


Image pre-processing



Sloan g’ flat field from Gemini South GMOS-S instrument

Greyscale goes from 0.8x to 1.2x mean pixel intensity.

Detector limitations

Standard imaging detectors such as optical CCDs comprise pixels whose size provides another limit to the spatial resolution $\theta_{\rm pix}$. Depending on the location and the equipment in use, observations may be *seeing-limited* ($\theta_{\rm pix} < \theta_{\rm see}$) or *detector limited* ($\theta_{\rm pix} > \theta_{\rm see}$). For detector-limited observations we say that the PSF is **under-sampled**.

Some photometry techniques (PSF fitting, DIA photometry – see later) require a well-sampled PSF where $\theta_{\rm see} > 2.5\,\theta_{\rm pix}$.

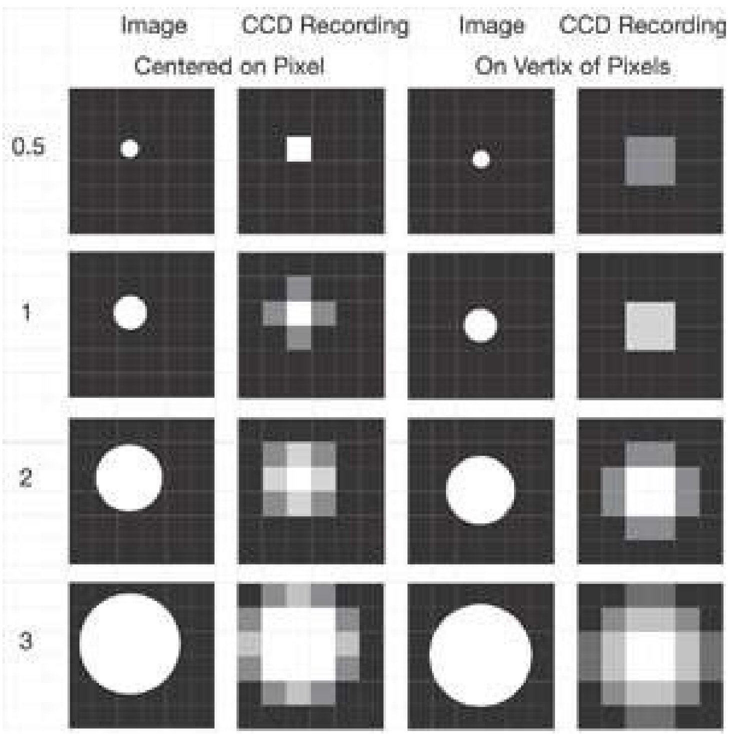


Image from Oxford Instruments (andor.oxinst.com)

Detector limitations

- Detectors have a maximum dynamic range determined by how many distinguishable intensity levels each pixel can record.

- Standard **16-bit** detectors can record 2^{16} = 65536 intensity levels. This is often refered to as the *full-well depth*.
  c.f. Earth transiting the Sun blocks out 1 part in 12,000 of the Sun's light!

- CCDs are designed to have a linear response to photon flux, but this typically breaks down as the detector counts approach the full-well depth. Typically detectors have their most linear response at around half the full-well depth (half-well depth).

- Above ~90% of the full-well depth the response is highly non-linear and unreliable. We say the detector pixel has reached its **saturation level**.

Exercise: What kind of detector dynamic range do we require to detect a transit of an Earth-sized planet around a Sun-like star if we want a signal-to-noise ratio S/N = 10? You may ignore background sources and consider only Poisson noise.

Detector limitations

Exercise: What kind of detector dynamic range do we require to detect a transit of an Earth-sized planet around a Sun-like star if we want a signal-to-noise ratio S/N = 10? You may ignore background sources and consider only Poisson noise.

Solution:
In the Poisson-limited regime $N = S^{1/2}$ so $S/N = 10$ implies $S = 100$ photons.

This number refers to the number of photons that the Earth-like planet *must block out* whilst transiting its Sun-like host.

If the contrast level is 12,000 : 1 for an Earth-Sun transit then the host will be emitting 100x12,000 = 1,200,000 photons.

Ideally this should match the half-well depth of our detector, implying a maximum dynamic range of 2,400,000 counts, requiring a 21-bit detector!

Such detectors don't exist, so what do we do?

Detector limitations

21-bit detectors don’t exist, so what do we do?

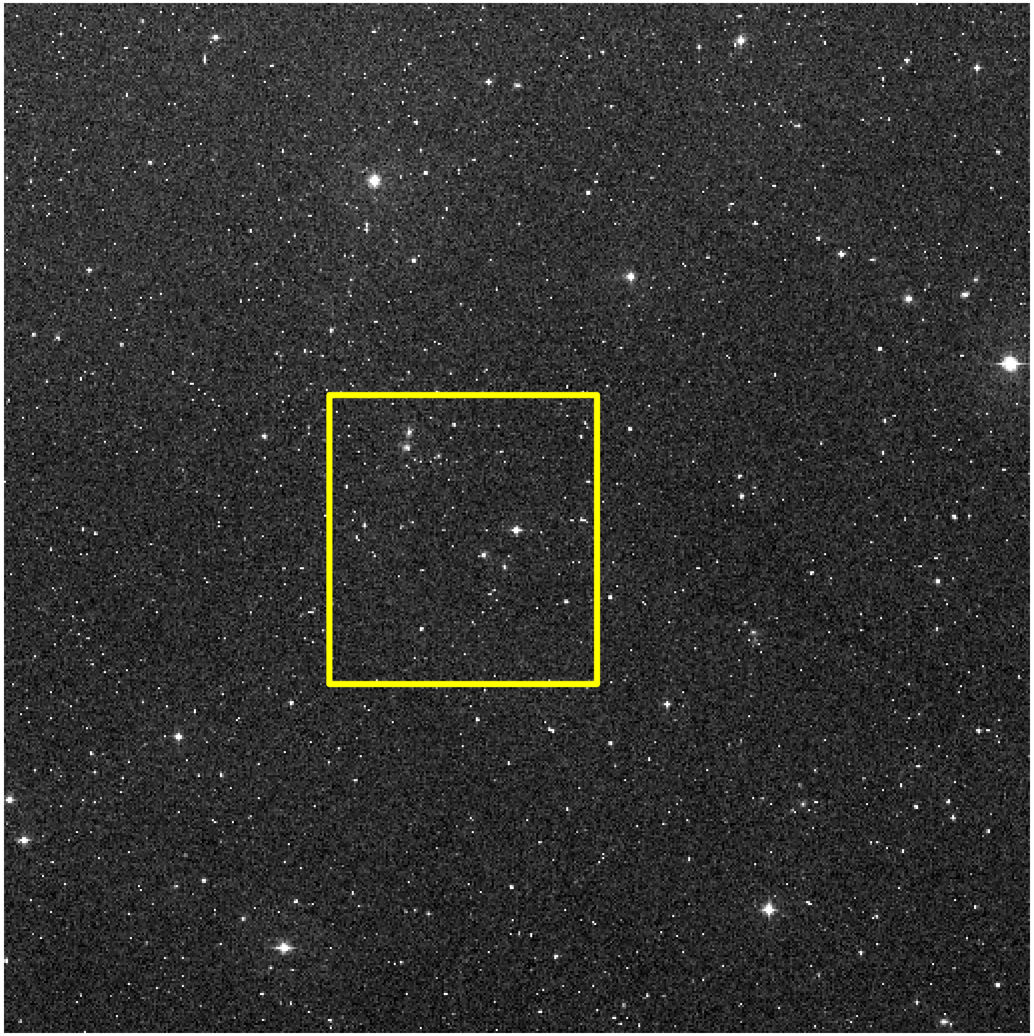
Options:

- Take multiple shorter exposures. eg 36 exposures at around half-well depth with a 16-bit detector, with exposure times 1/36 as long as required to give 1.2 million photons.

- Use a camera with finer pixel resolution. Eg a camera with 6-times finer pixel resolution so that each pixel covers 1/36 the area and so receives 36 times less photons for the same exposure. This would give a typical pixel count matching the half-well depth and a total count of 1,200,000 photons across all pixels within the target PSF.

- Blur the image so that each PSF covers more pixels. Eg blurring to 6 times the original PSF size will spread the counts over 36 pixels as above. This is the basis for **defocussed photometry** – see later.

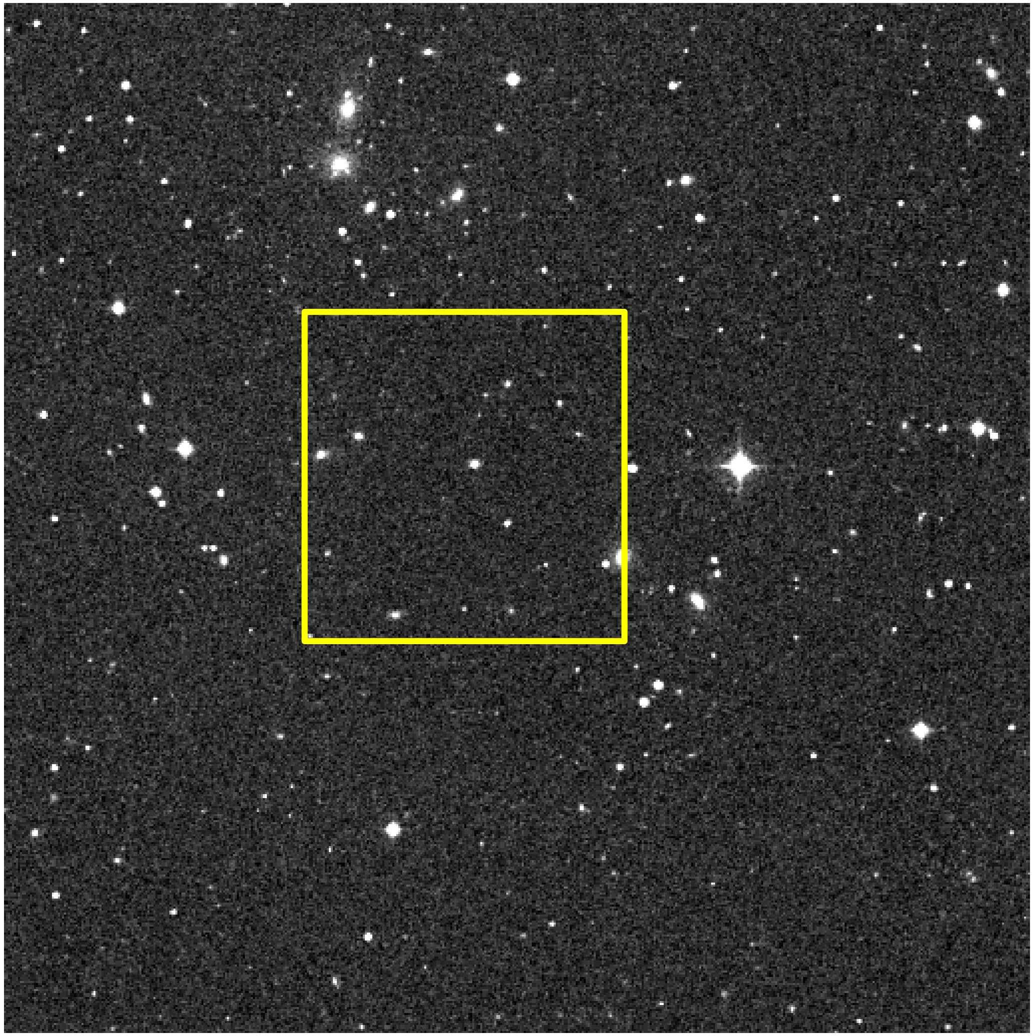
Aperture, annular and PSF photometry



1 sq deg view from DSS-2 (red)

RA: 01 01 01
Dec: 00 00 00
(J2000)

Aperture, annular and PSF photometry



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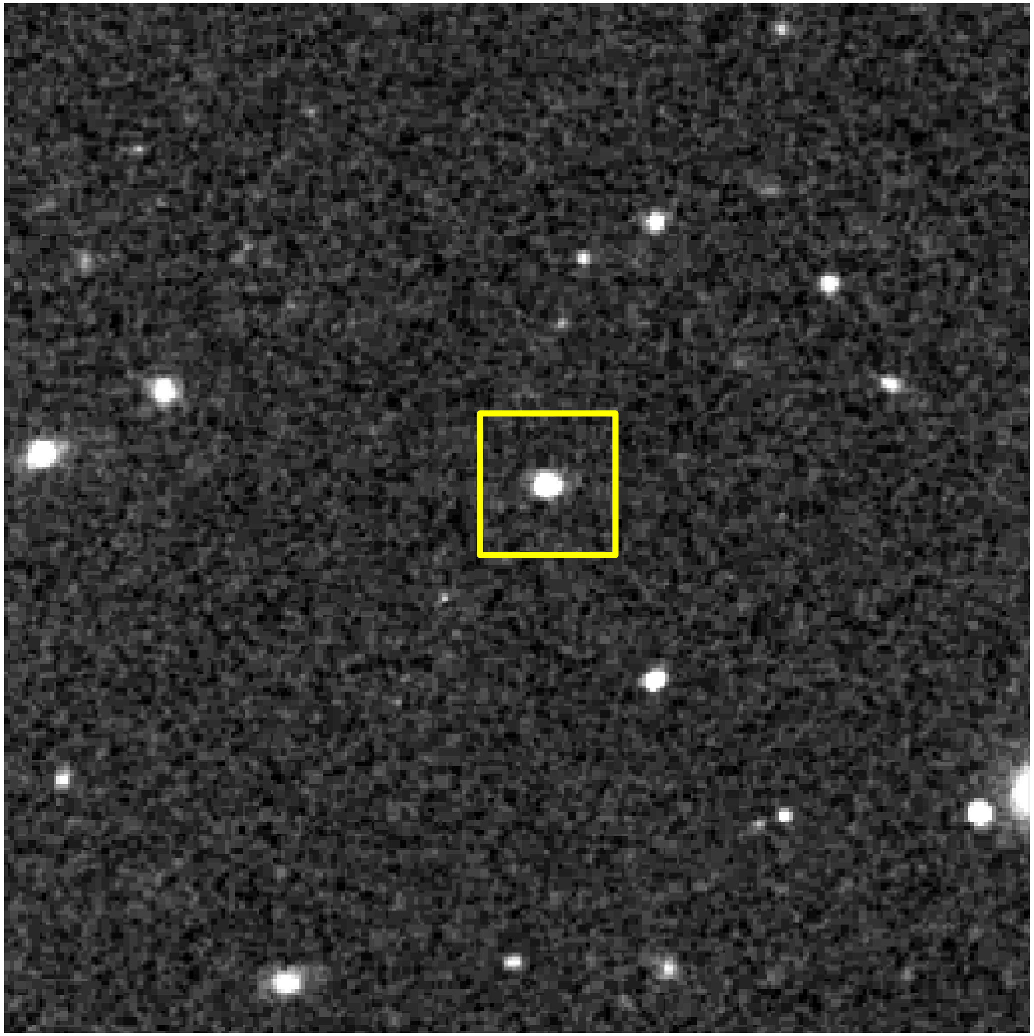

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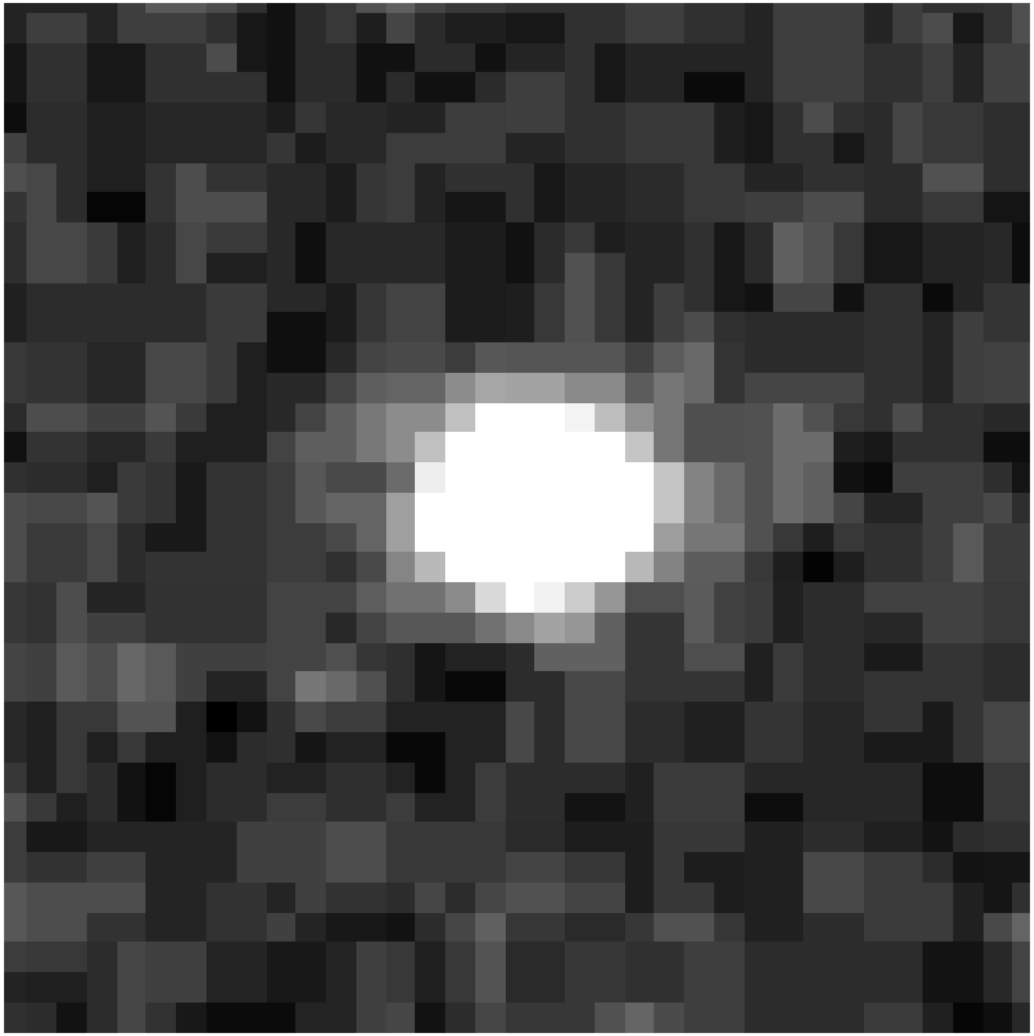
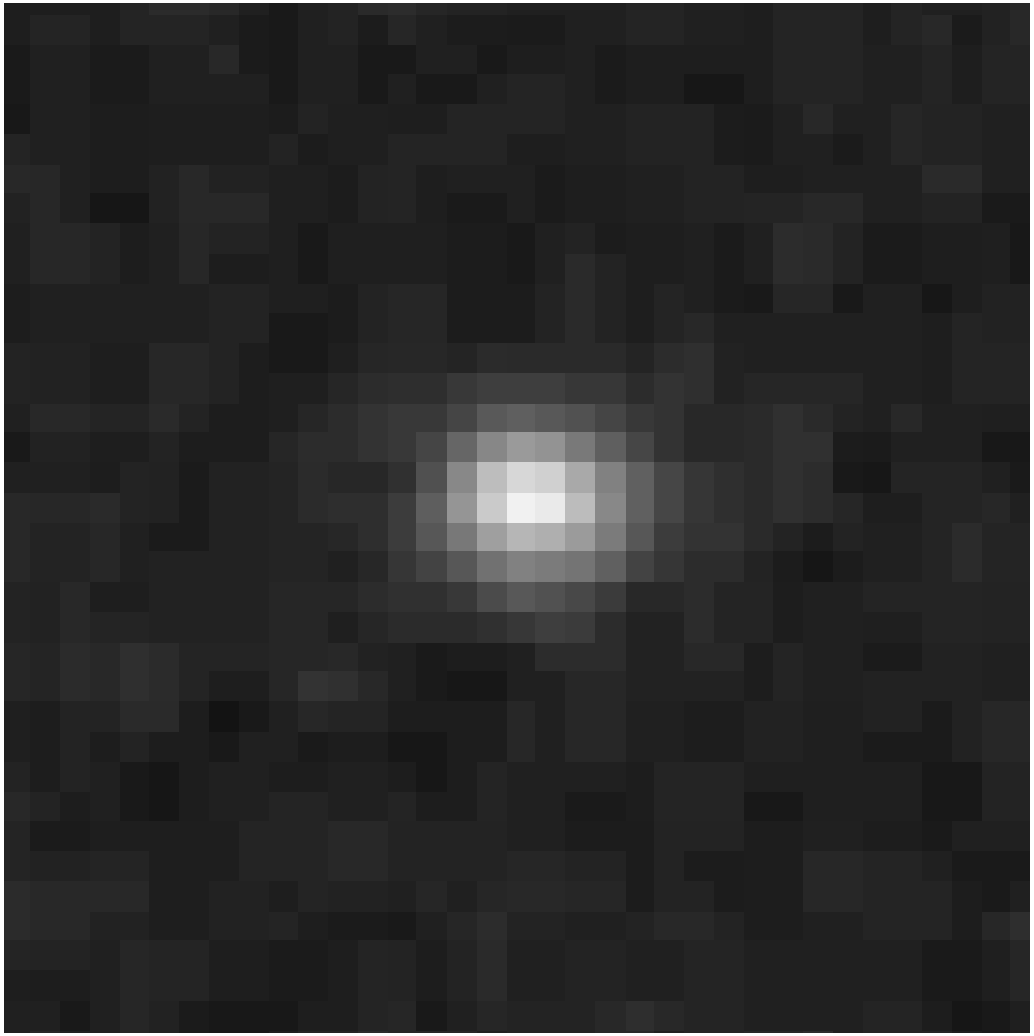
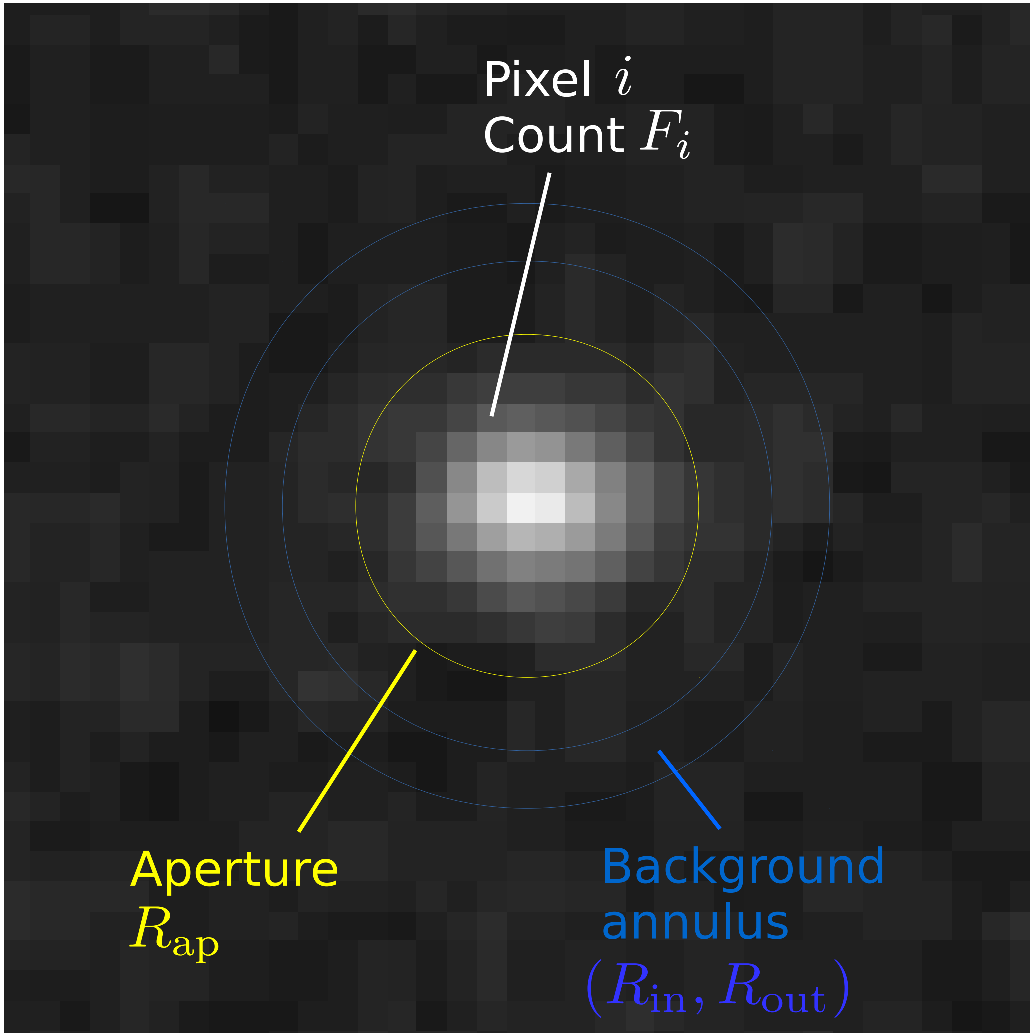


Aperture, annular and PSF photometry

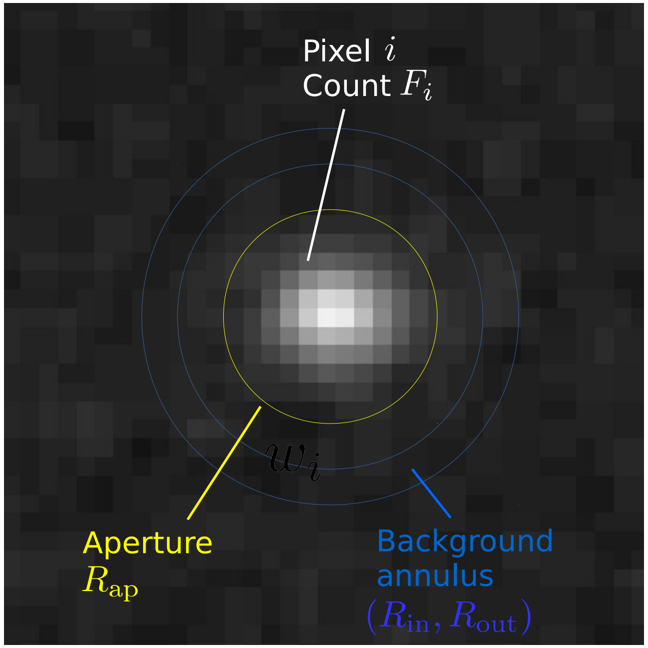


Aperture, annular and PSF photometry



Aperture, annular and PSF photometry

Aperture photometry



$$F_{*,\rm ap} = \overbrace{\sum_{i(\rm aperture)} w_i F_i}^{\text{Source}} - \overbrace{\frac{R_{\rm ap}^2}{(R_{\rm out}^2 - R_{\rm in}^2)} \sum_{i(\rm annulus)} w_i F_i}^{\text{Background}}$$

w_i: fraction of pixel within aperture or annulus

Aperture, annular and PSF photometry

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Potential problems with aperture photometry:

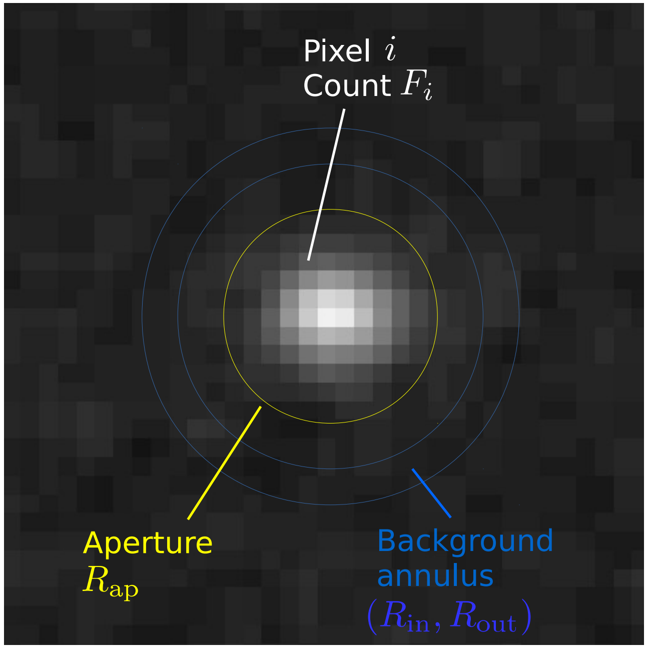
- The PSF may be elliptical rather than circular, or perhaps even more complex. *Solution:* use an appropriate aperture shape or widen it so that the shape has a small affect (but more noisy).
- Seeing variations between epochs will change the fraction of the PSF lying inside a fixed aperture. *Solution:* scale the aperture size to a fixed multiple of the seeing or widen it so that the affect is minimised (but more noisy)
- The background annulus may have stars or defects in it. *Solution:* try to pick a clear annulus and/or use the median count in the annulus.

$$F_{*,\mathrm{ap}} = \overbrace{\sum_{i(\mathrm{aperture})} w_i F_i}^{\text{Source}} - \overbrace{\frac{R_{\mathrm{ap}}^2}{(R_{\mathrm{out}}^2 - R_{\mathrm{in}}^2)} \sum_{i(\mathrm{annulus})} w_i F_i}^{\text{Background}}$$

w_i: fraction of pixel within aperture or annulus

Aperture, annular and PSF photometry

PSF photometry



$$F(x,y) = F_{*,\rm PSF}\, P(x,y) + B(x,y)$$

Aim is to fit for $F_{*,\rm PSF}$ subject to

$$\int_x \int_y P(x,y)\, {\rm d}x\, {\rm d}y = 1$$

$$\int_x \int_y F(x,y)\, {\rm d}x\, {\rm d}y = \text{target counts}$$

PSF model *P*(*x*,*y*) needs to be flexible, e.g. the Moffat function:

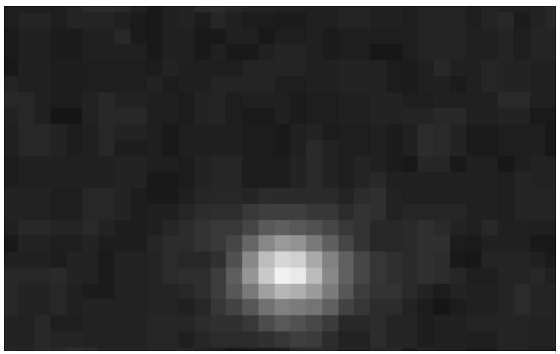
$$P(x,y) = \frac{\beta - 1}{\pi\alpha^2}\left\{1 + \left[\frac{(x-x_0)^2 + (y-y_0)^2}{\alpha^2}\right]\right\}^{-\beta}$$

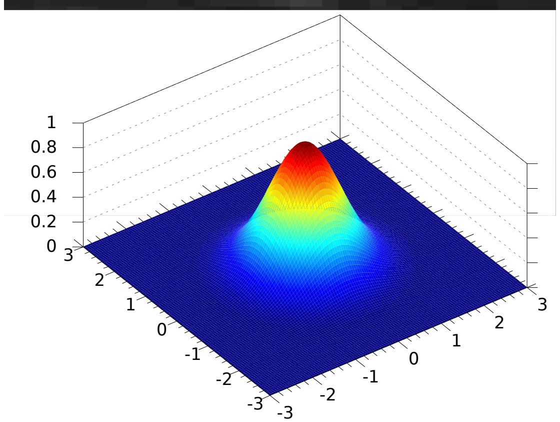
The background model *B(x,y)* is usually taken to be a constant over the star. So in this case we fit for 6 free parameters

$$(F_{*,\rm PSF},\ B,\ \alpha,\ \beta,\ x_0,\ y_0)$$

Aperture, annular and PSF photometry

PSF photometry





- PSF fitting requires that the number of pixels exceeds the number of free parameters in our model. It is also sometimes assumed that $P(x,y)$ is constant or linearly varying across a pixel. The PSF **must be well sampled** for such an approach to be valid.

- Fitting may use a weighting scheme (eg inverse variance weighting) to account for the fact that pixels near the peak have lower relative noise.

- An alternative to supplying a model is to attempt to build a numerical model of the PSF by using the pixel intensity distribution of other stars in the image. Difficult – requires lots of other stars and knowledge of how differential focus across the detector plane may alter the PSF profile.

Aperture, annular and PSF photometry

MANCHESTER 1824
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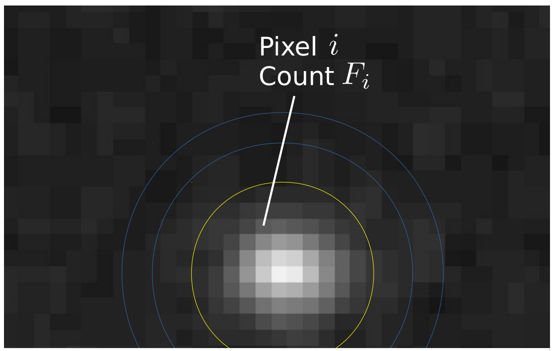
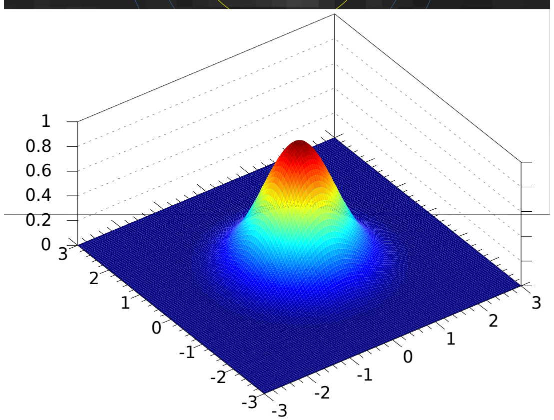
Annular photometry





Annular photometry employs both aperture and PSF methods to allow photometry estimation even from stars that may be saturated on an image. Procedure in outline:

- Pick a number of fainter (unsaturated) stars on the image and perform PSF photometry of them or use them to build a PSF model.
- Define an annular region centred on the target star within which the pixel counts are all below the saturation level.
- Use the PSF model determined from fainter stars to solve for the PSF amplitude of the bright star, but using only data from the unsaturated annular region in the fitting.

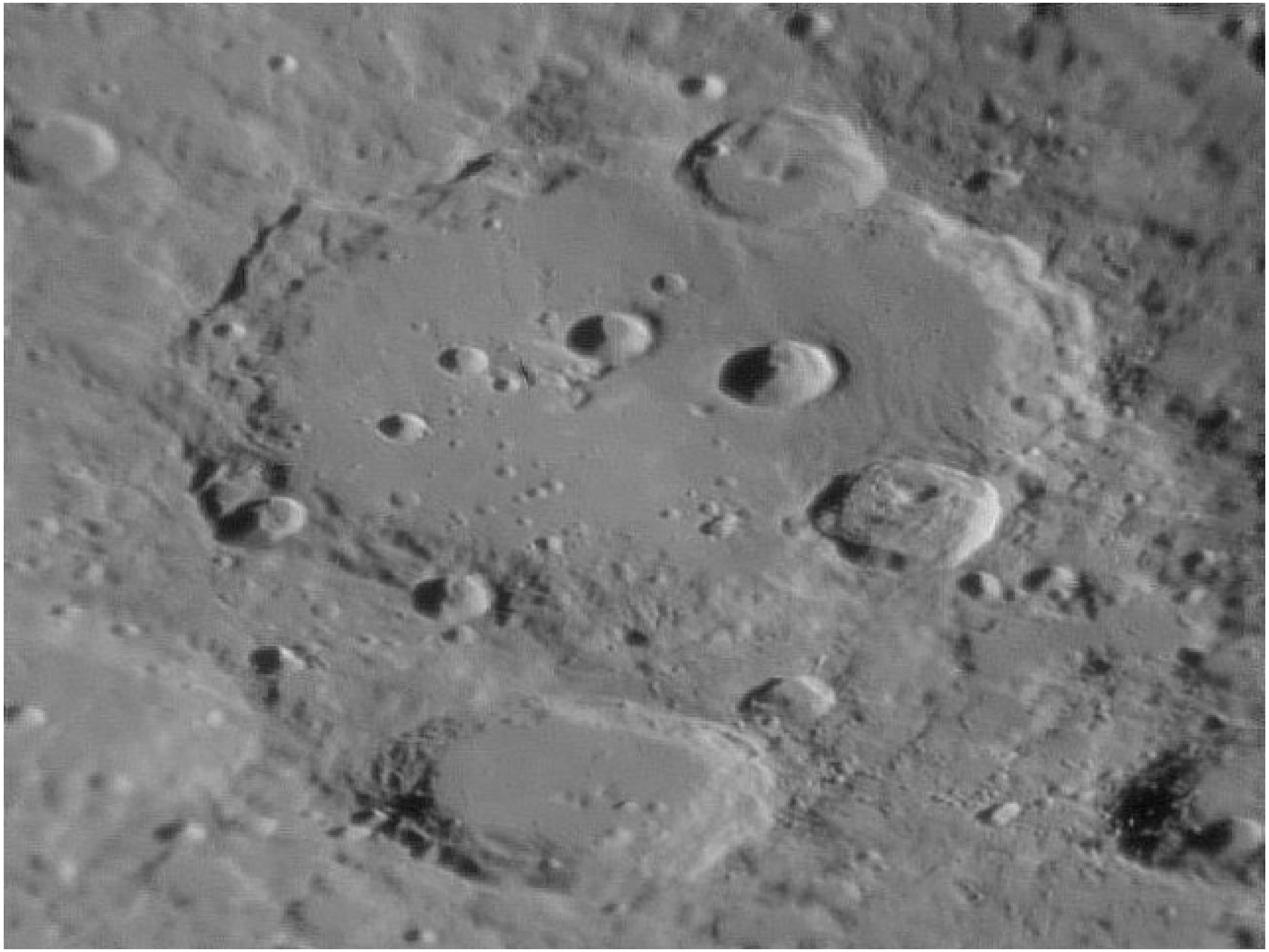
Absolute and relative photometry

- **Absolute photometry** refers to photometry that is calibrated onto a standard flux or magnitude scale, eg the UBVRI filter system.
- **Relative photometry** refers to photometry that is calibrated either relative to other uncalibrated stars in the same image or to the same target observed at a different epoch with the same instrument.
- Exoplanet photometry is often only concerned with relative photometry from a time series of photometric measurements.
- Absolute photometry acheived via relative photometry with respect to a *standard star* – a star with accurately calibrated photometry.
- Absolute photometry from an uncommon filter system may require use (or construction) of colour-magnitude and colour-colour equations to transform magnitudes to a standard filter system.

Lucky imaging

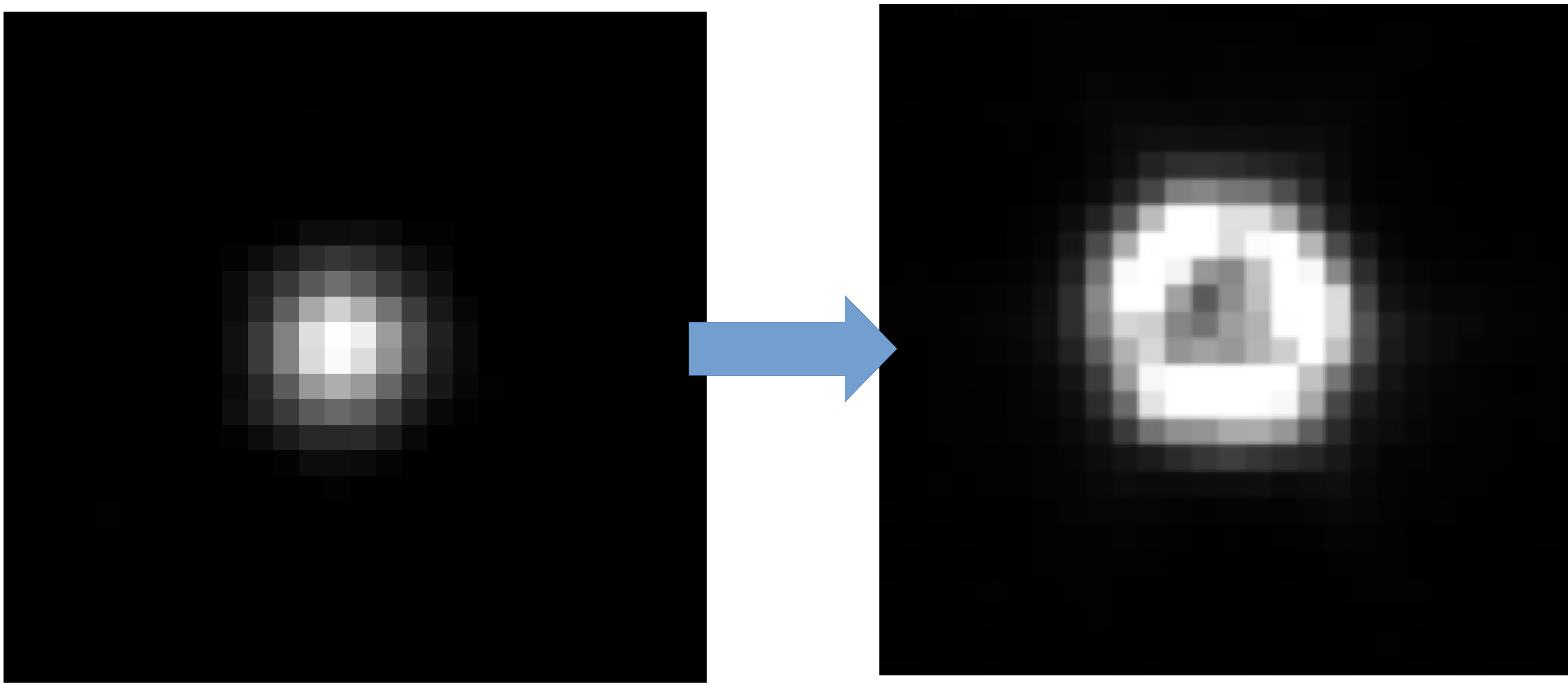
Source: Image by Wah! Via https://www.ast.cam.ac.uk/research/lucky

Lucky imaging

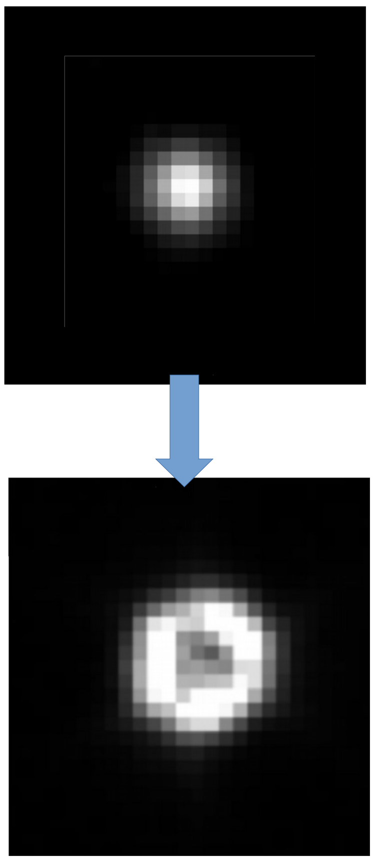


Source: Image by Wah! Via https://www.ast.cam.ac.uk/research/lucky

Defocussed photometry



Defocused photometry

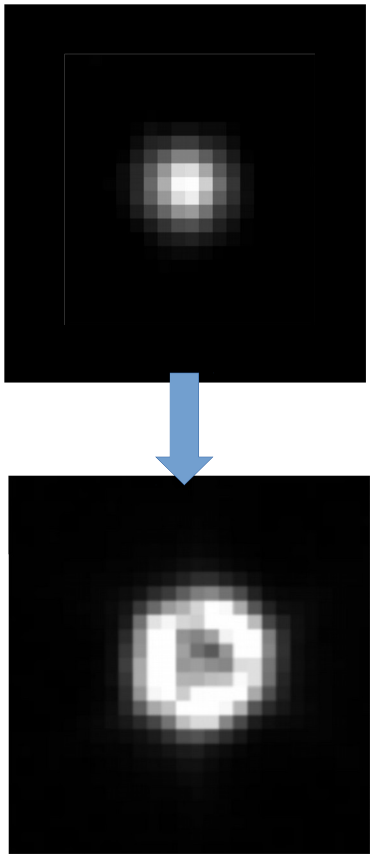


Deliberately defocus image to smear out target over more pixels

- **Why do this?**

Have a think about it and let’s discuss!

Defocused photometry



Deliberately defocus image to smear out target over more pixels

- **Why do this?**
  - Brighter stars can be targetted without reaching the saturation level.
  - For the right amount of defocusing the maximum allowed source signal scales with the area of the defocused PSF, whilst the total noise only increases with PSF radius
  - The total PSF count can be much higher than for focused photometry and therefore it is possible to achieve very good photometric accruacy per exposure.
  - Helps to minimize residual flat fielding errors as the result depends on the response of a larger number of pixels.

Optimal source brightness depends on telescope / instrument. Lose advantage if source is too bright or too faint.

DIP: photometry for crowded fields

Difference Image Photometry (DIP) attempts to obtain optimal differential photometry by removing the effects of seeing and background light variations.

We wish to obtain relative photometry between two epochs:

- A **reference** epoch with an image *R* obtained in good seeing
- A **target** epoch with an image *T* obtained in poorer seeing

We assume that most stars do not vary between *R* and *T* except bacause of seeing and background changes.

In principle we can find a **kernel function** *K* that can be convolved with *R* to blur it to the same seeing characteristics as *T*. We can also add in a **background model** *B* for the difference in background between R and *T*.

Finally we can obtain a difference image *D*:

$$D = (R \otimes K) - T + B$$

DIP: photometry for crowded fields

The Kernel *K*

$$D = (R \otimes \boldsymbol{K}) - T + B$$

To allow for complex PSF shapes *K* can be a sum independent kernel functions *k*:

$$K = \sum_{i=1}^{N_{\rm PSF}} k_i$$

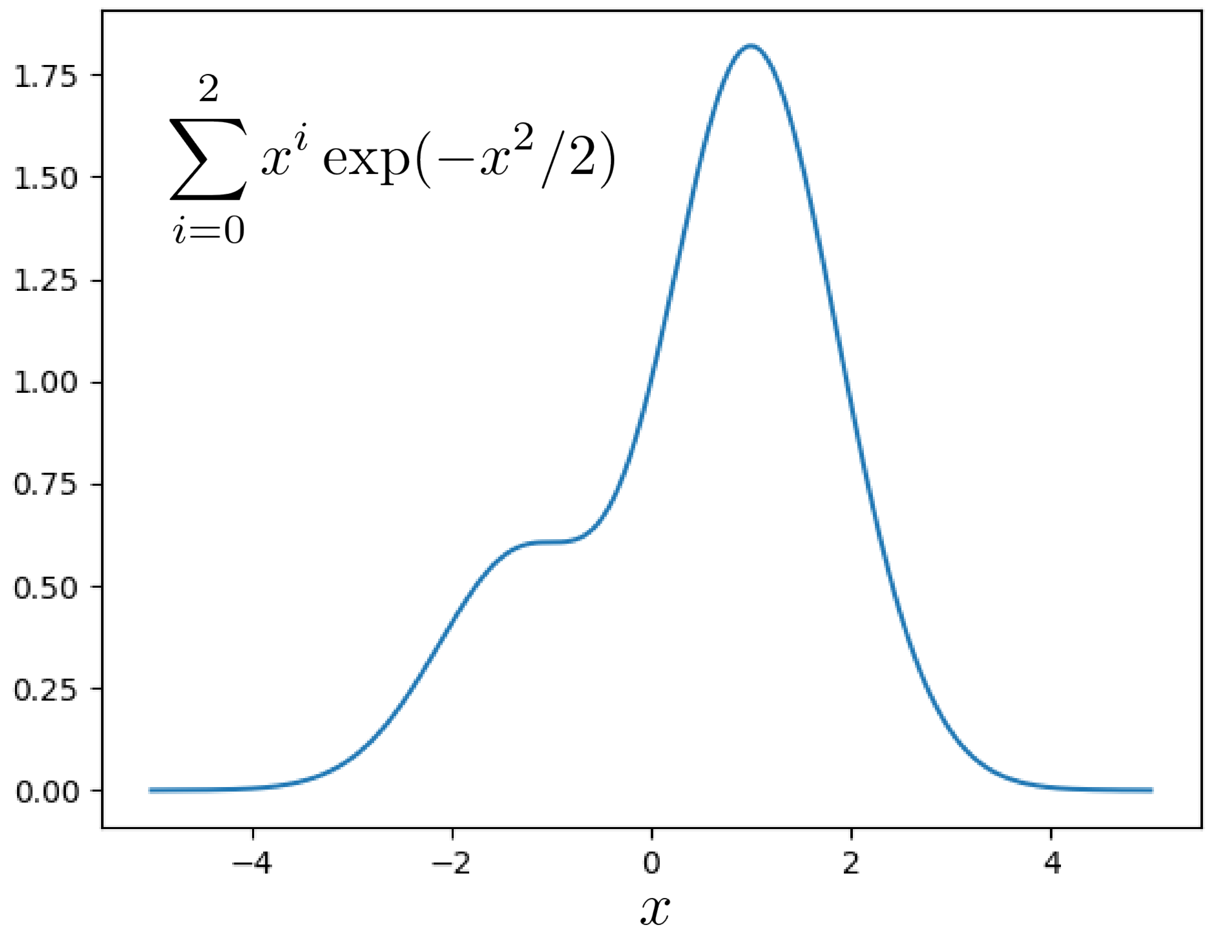
Each kernel k_i is the product of a 2-D Gaussian of width σ_i and a 2-D polynomial of order N_i as a function of position (*x*,*y*):

$$k_i = \sum_j a_j x^{n_j} y^{m_j} \exp\left[-\left(\frac{x^2+y^2}{2\sigma_i^2}\right)\right] \qquad (n_j \leq N_i; m_j \leq N_i - n_j)$$

K_i has $(N_i+1)(N_i+2)/2$ terms

DIP: photometry for crowded fields

Gives very good flexibility for PSF shape



DIP: photometry for crowded fields

The differential background *B*

$$D = (R \otimes K) - T + \boldsymbol{B}$$

B can also be expressed as a polynomial function of position (*x*,*y*):

$$B = \sum_i b_i x^{n_i} y^{m_i} \qquad (n_i \leq M; m_i \leq M - n_i)$$

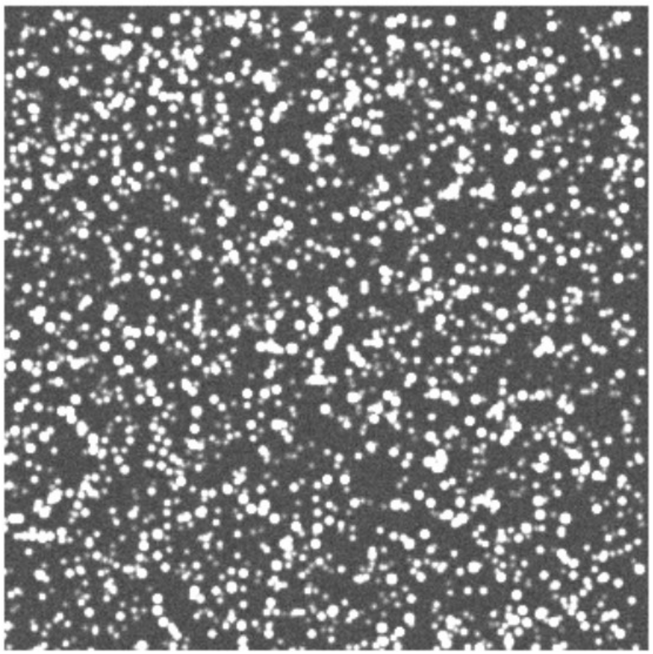
B has $(M+1)(M+2)/2$ terms.

If we supply N_i, σ_i and *M* then the free parameters are the coefficients a_i, b_i. Potentially this is a lot of free parameters, e.g for $N_{PSF} = 3$ with $N_i = (3,4,5)$, $M = 3$ we have 56 free parameters! But we can use standard least squares methods to solve form them.

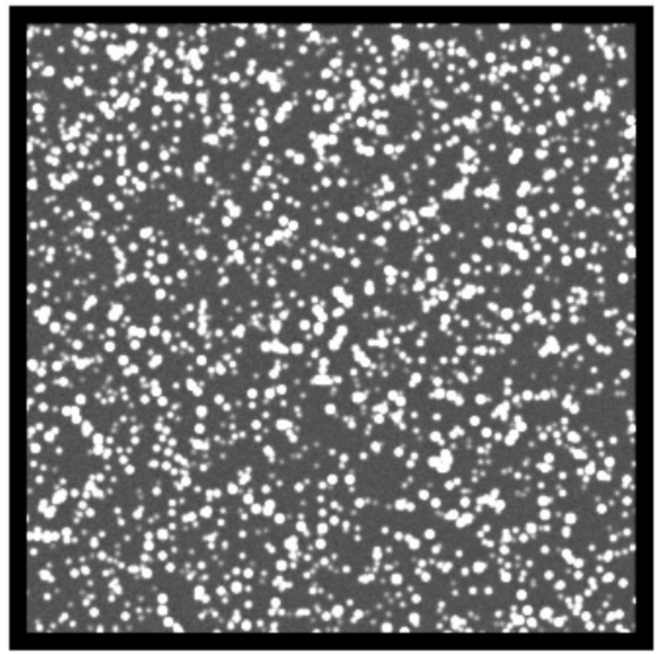
Solution should be reasonably robust to user inputs.

DIP: photometry for crowded fields

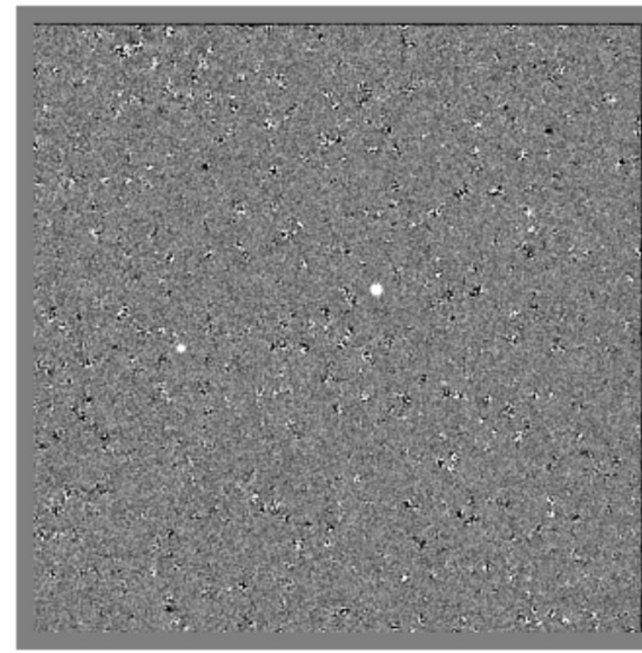
DIP for crowded fields towards the Galactic bulge



Target T



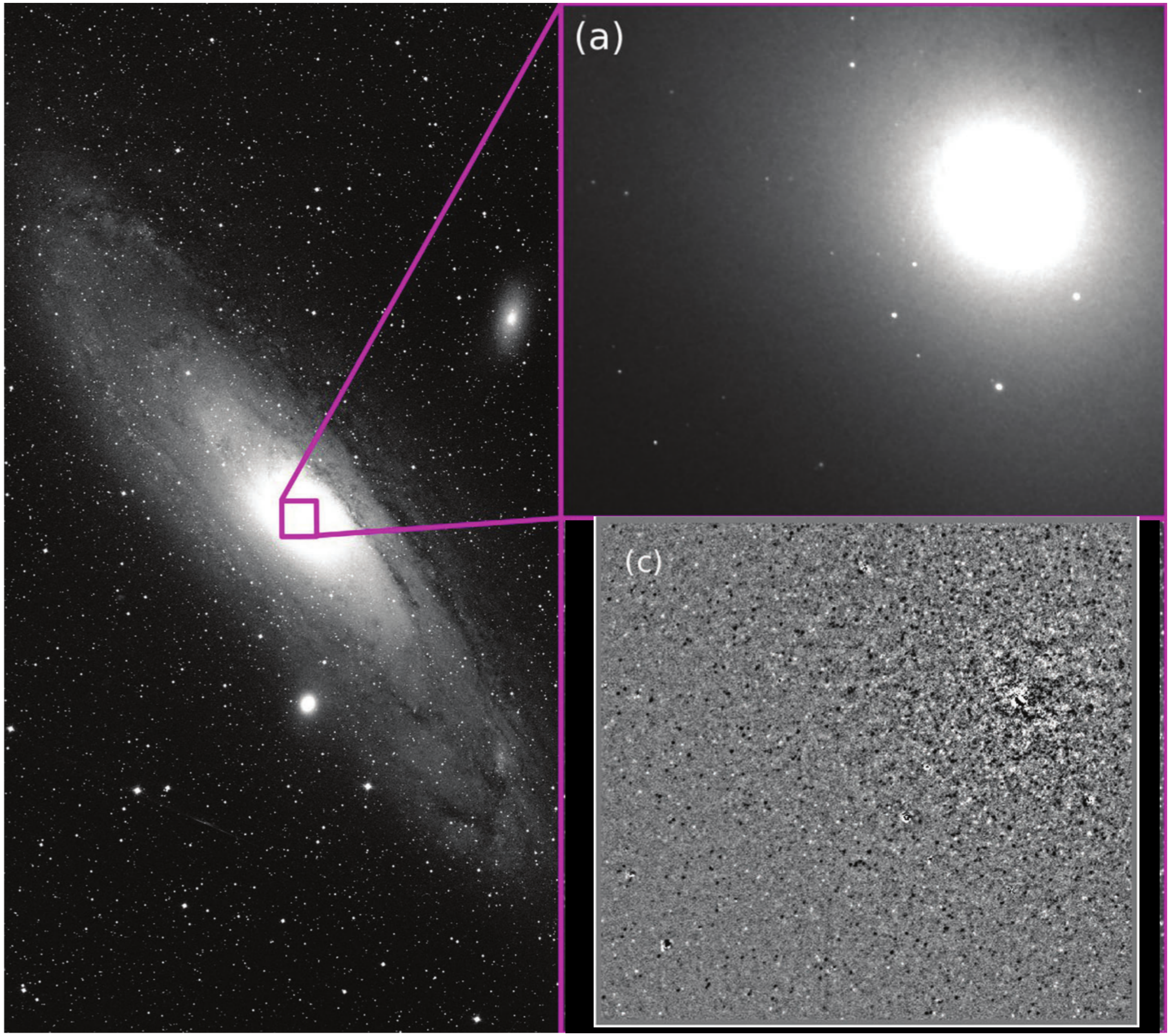
Reference R



Difference image D

DIP: photometry for crowded fields

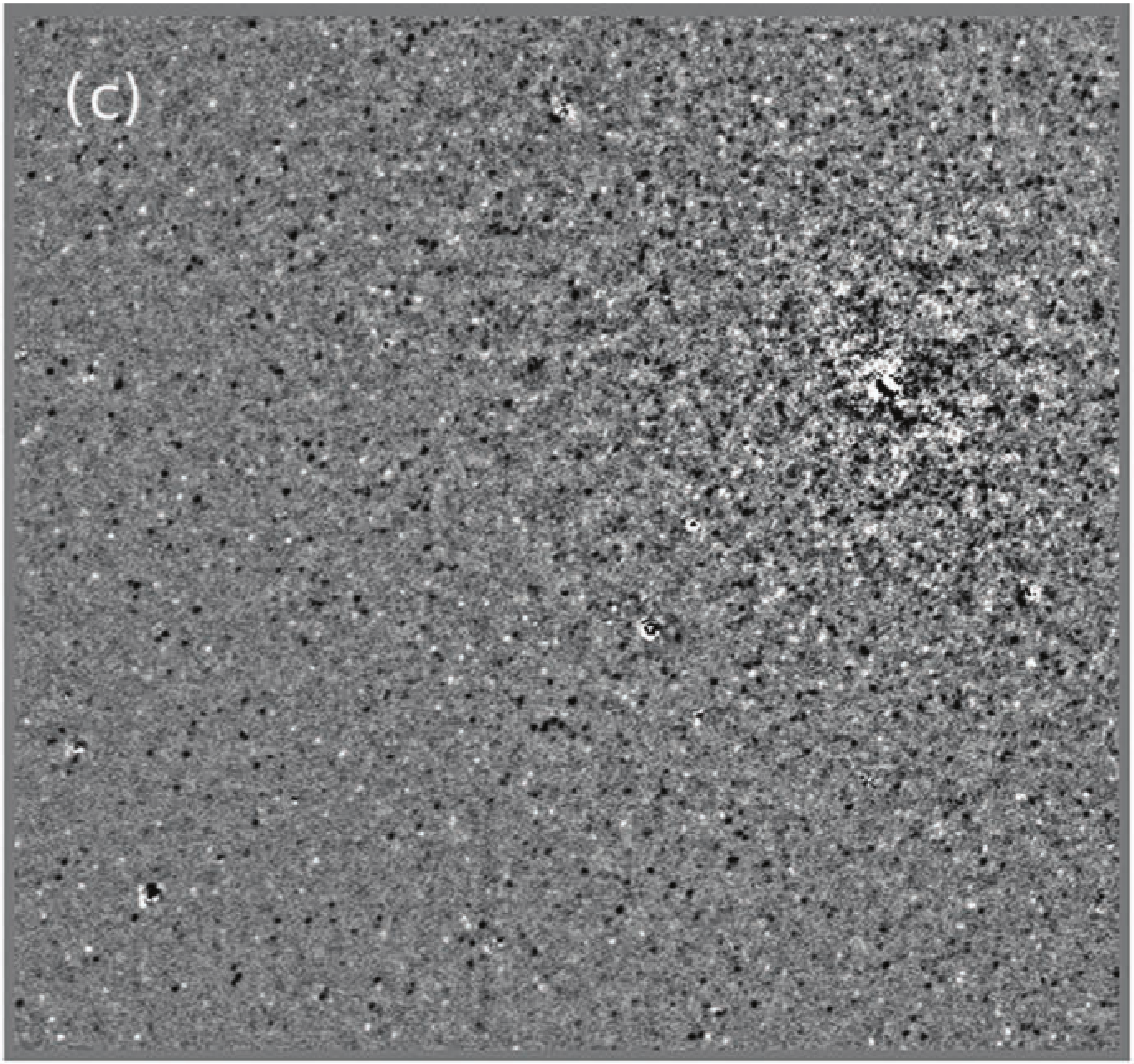
DIP for even more crowded fields in M31



Kerins et al (2010)

DIP: photometry for crowded fields

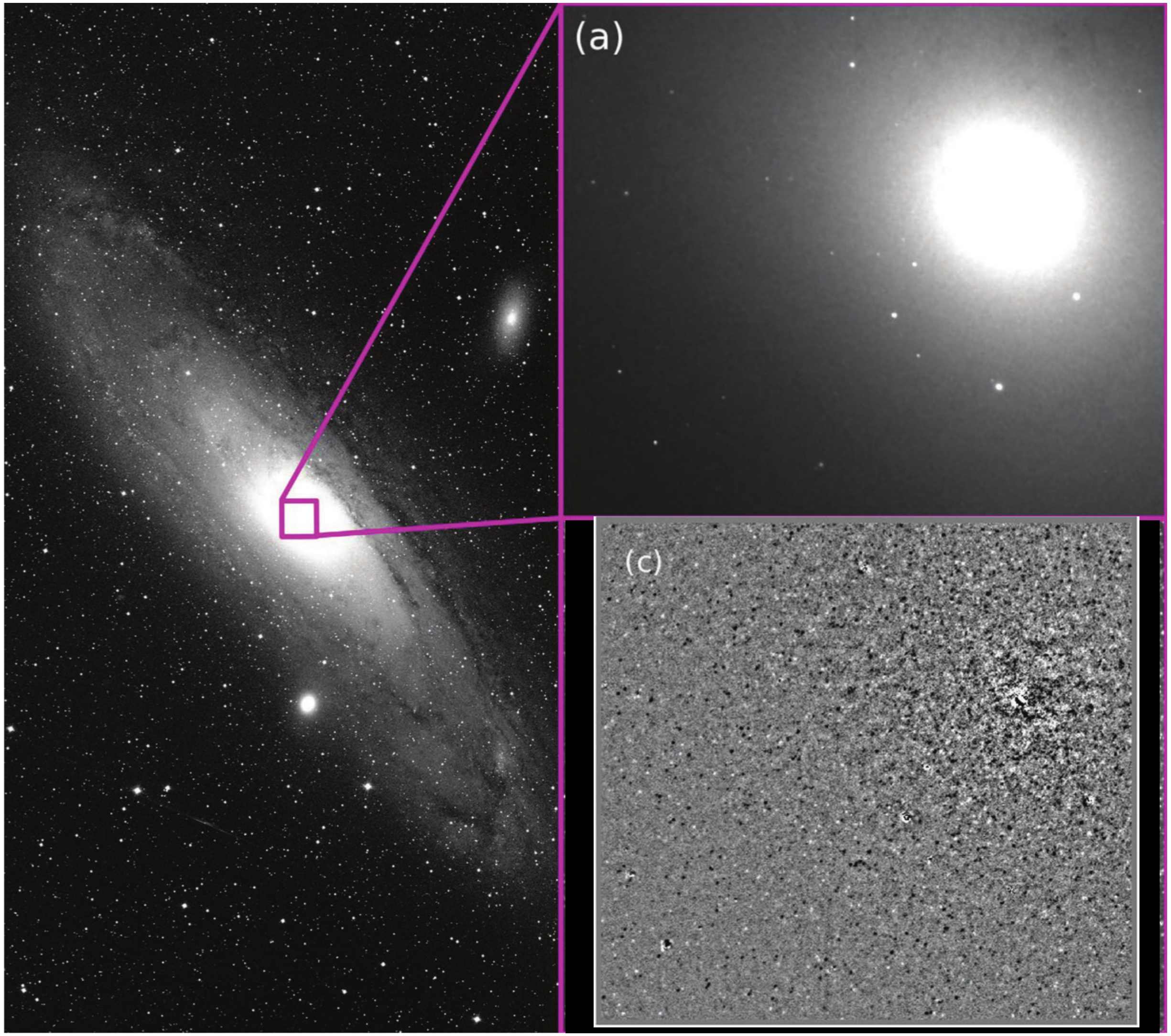
DIP for even more crowded fields in M31



Kerins et al (2010)

DIP: photometry for crowded fields

DIP for even more crowded fields in M31



Kerins et al (2010)

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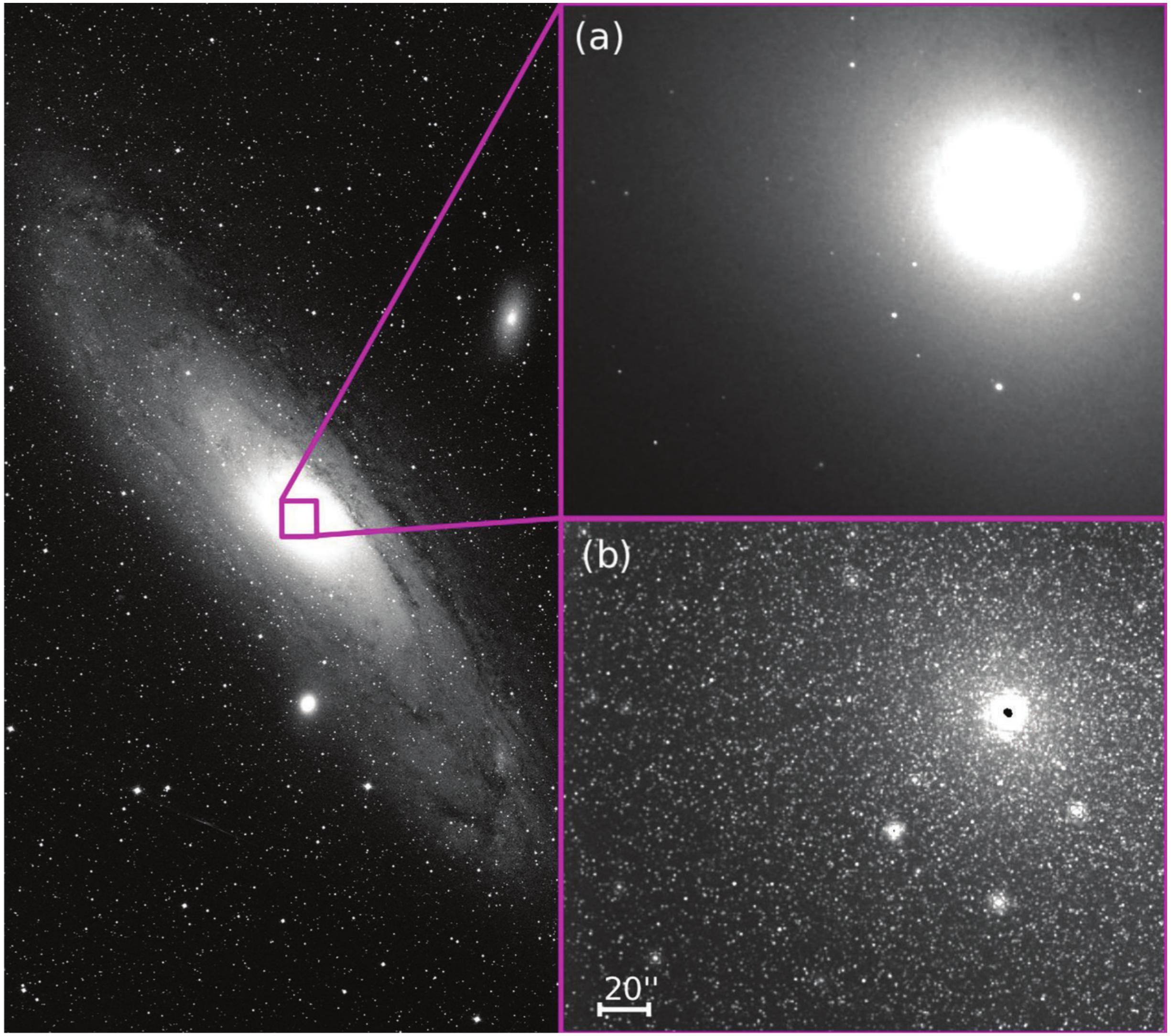
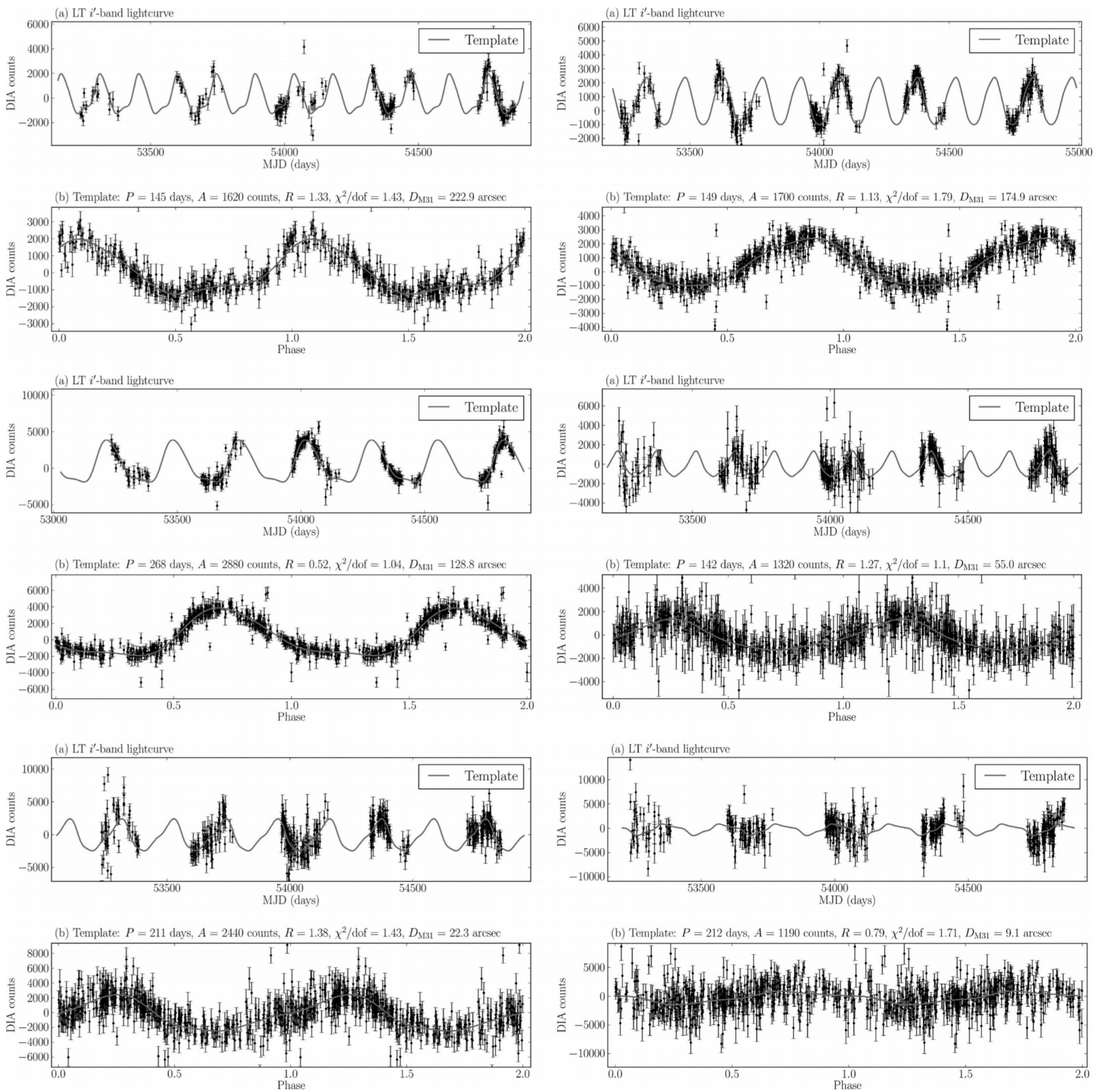


Kerins et al (2010)

DIP: photometry for crowded fields



Kerins et al (2010)