Thermal History of the Universe

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Pre-workshop CTA 2019

Outline

- The expanding universe
- A brief history of time
- Geometry and dynamics

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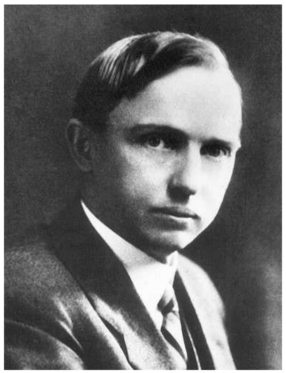
- Geometry and dynamics

Nebulae?

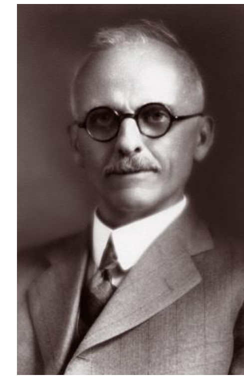
NGC891 discovered by Caroline Herschel in 1784

Great Debate (Shapley–Curtis Debate)

26 April 1920



The Scale of the Universe



Harlow Shapley

- Spiral nebulae are belong to the home galaxy
- the Milky Way is the entirety of the universe
- Van Maanen also claimed that he had observed the Pinwheel Galaxy rotating

Heber Curtis

- Andromeda and other such nebulae were separate galaxies, or "island universes“ (Immanuel Kant)
- He showed that there were more novae in Andromeda than in the Milky Way.

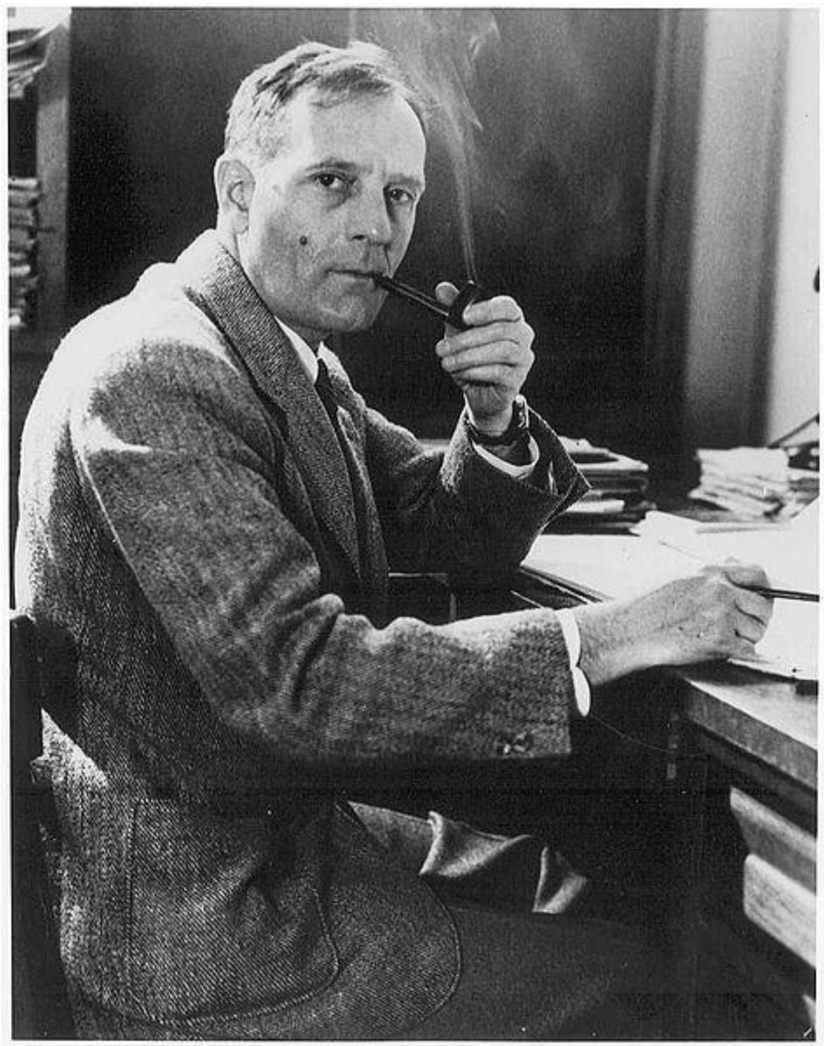
We need “standard candles” to measure the Universe.

Measure the Universe by Cepheid Variable Stars : 1908

Henrietta Swan Leavitt

Credit: HST

Edwin Hubble



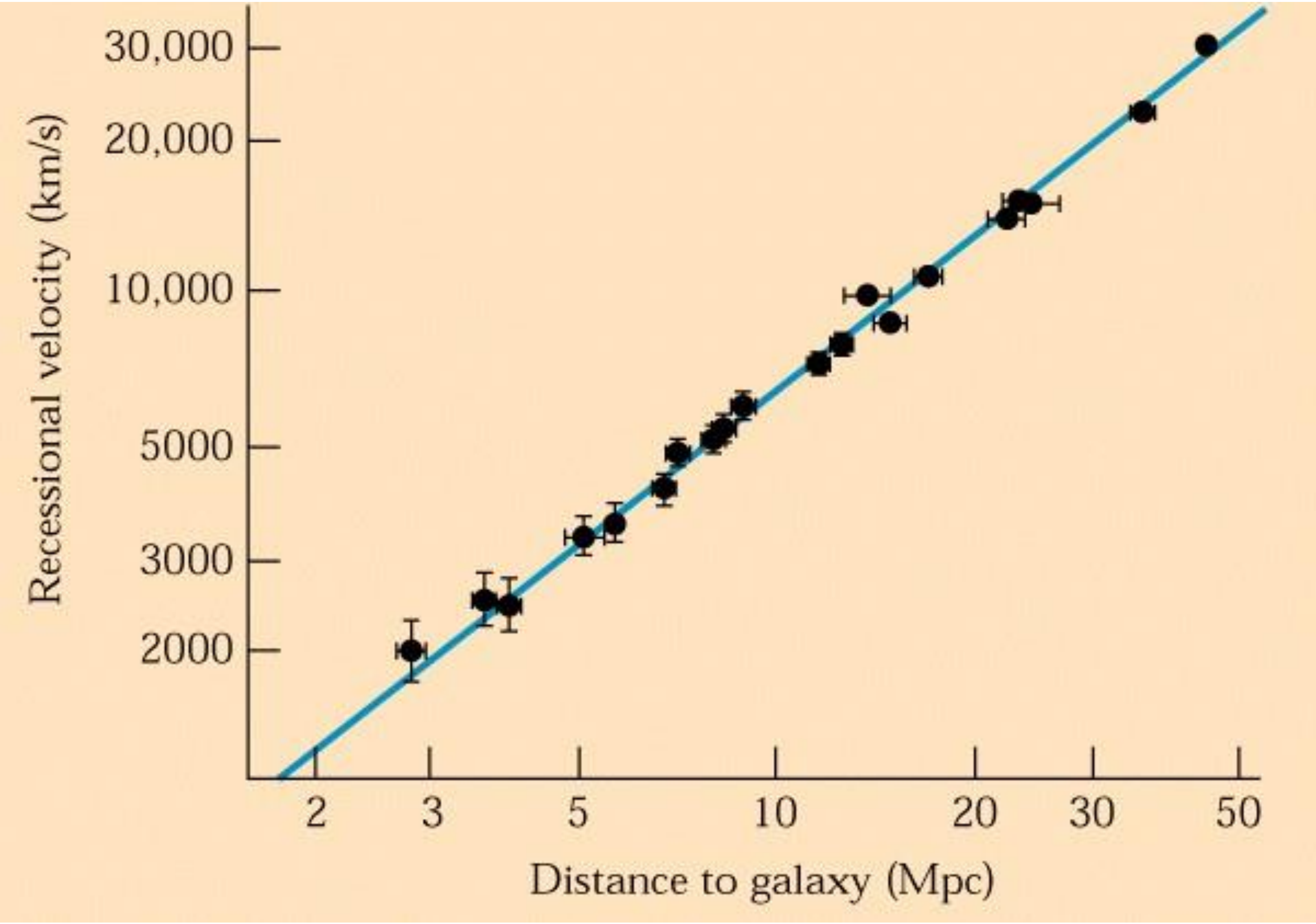
1924: Hubble closed the debate
Galaxy ≠ Universe
& Hubble classified galaxies

Edwin Hubble: Universe is expanding

Hubble–Lemaître law

- Objects observed in deep space—extragalactic space, 10 megaparsecs (Mpc) or more—are found to have a redshift, interpreted as a relative velocity away from Earth.
- This Doppler shift-measured velocity of various galaxies receding from the Earth is approximately proportional to their distance from the Earth for galaxies up to a few hundred megaparsecs away.

Hubble–Lemaître law

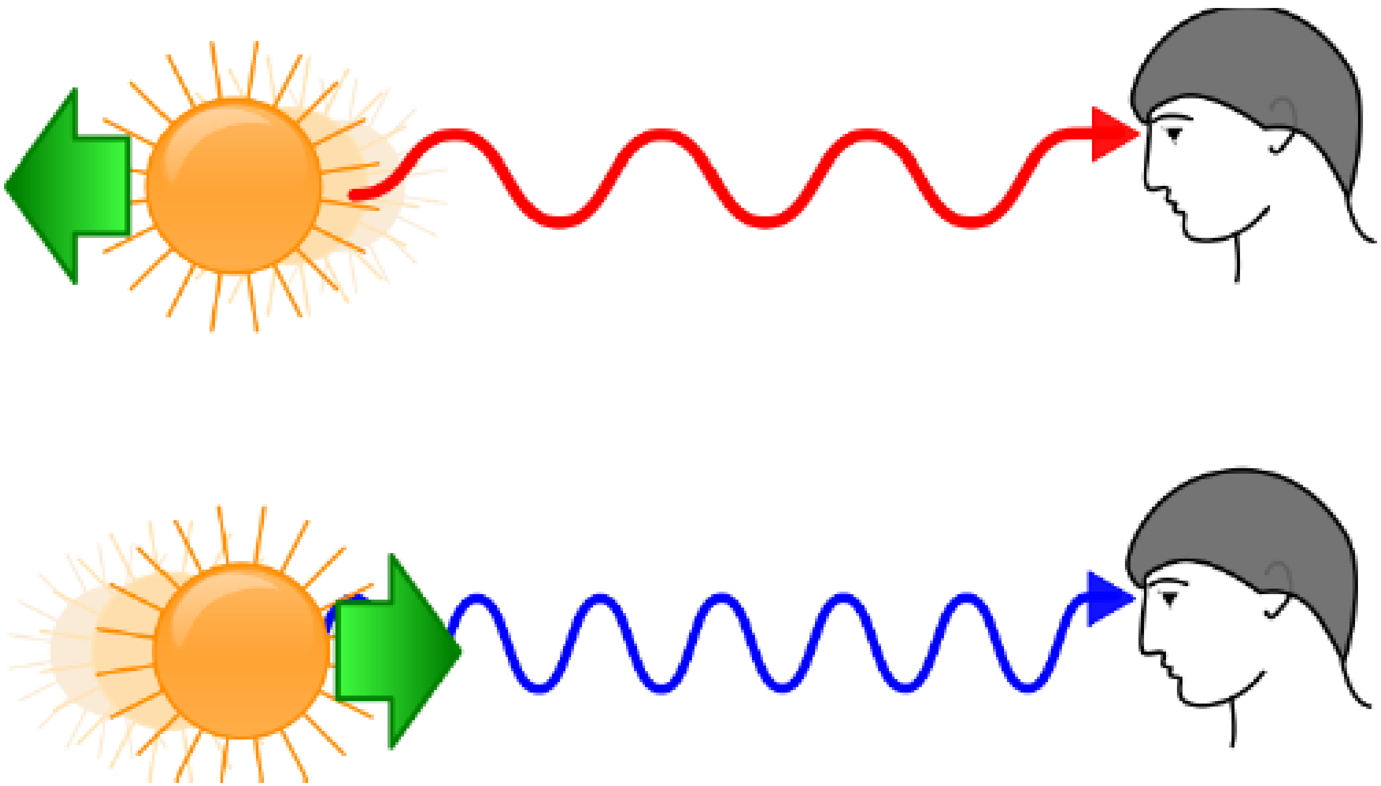


Hubble–Lemaître law

- $\boldsymbol{v = H_0 d}$
- d is the distance of galaxies
- v is velocity
- H_0 is the present Hubble constant

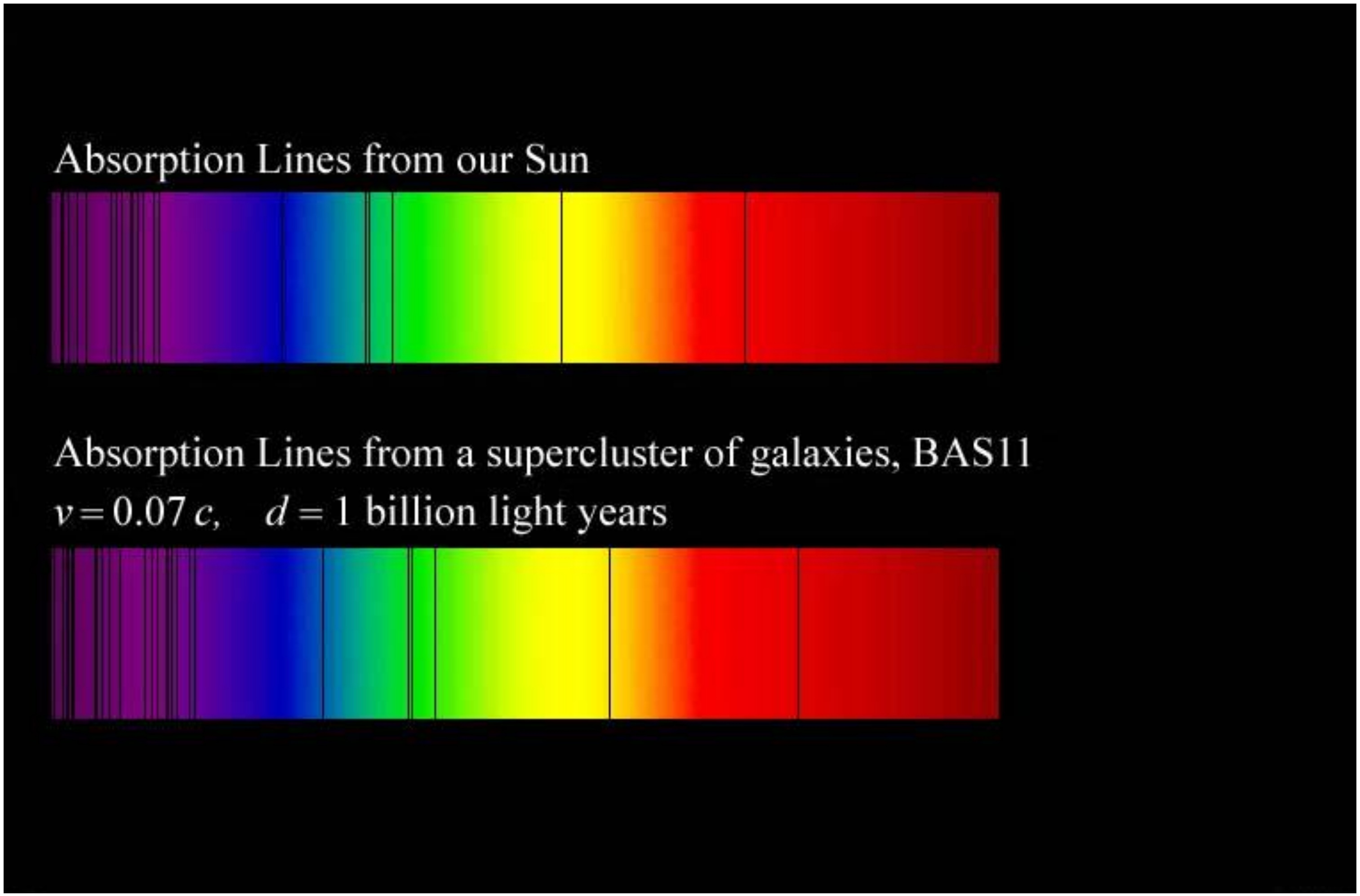
$$H_0 = 67.66 \pm 0.42 \text{ km/s/Mpc}$$

Red Shift z

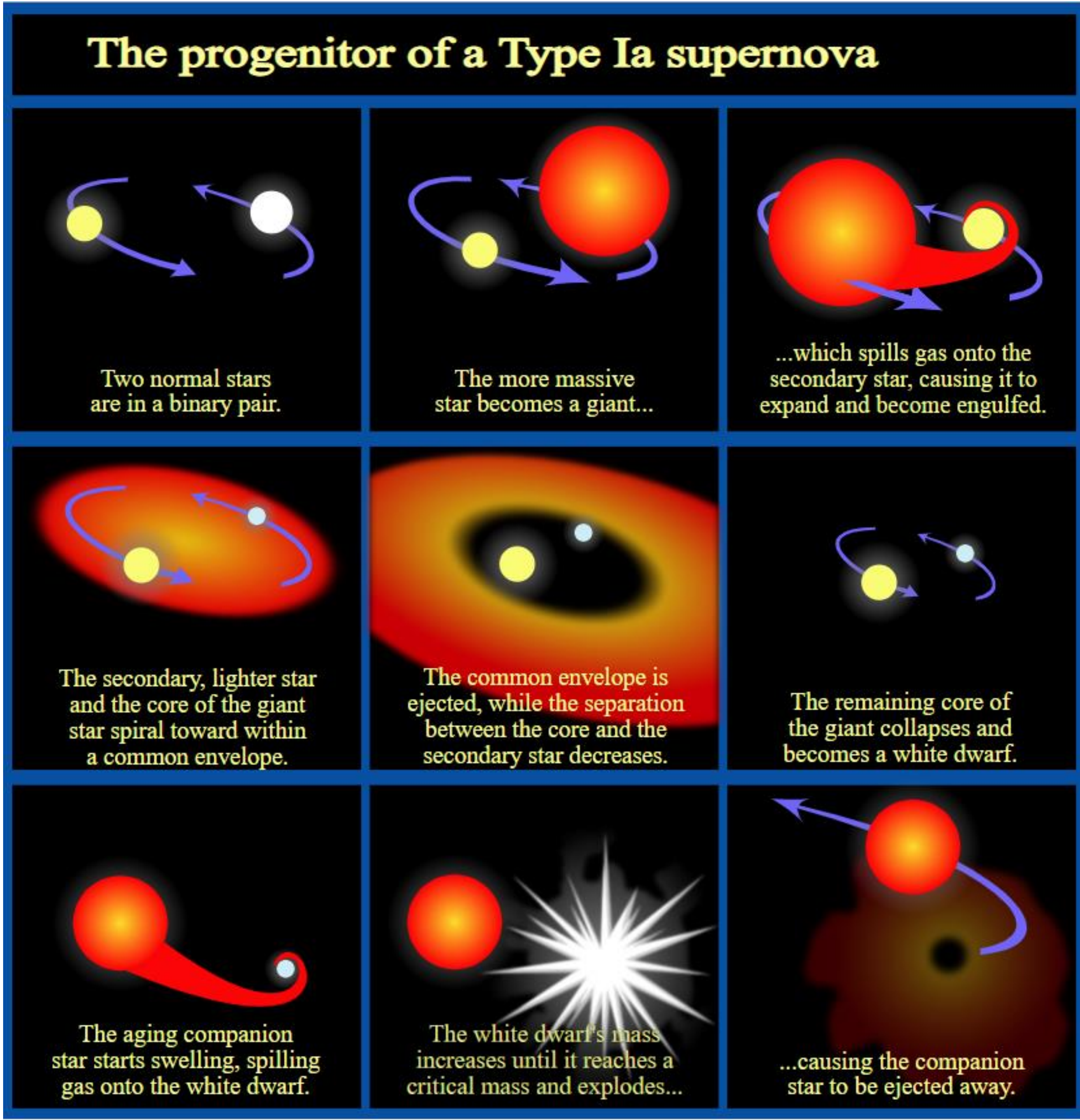


Red Shift (z)

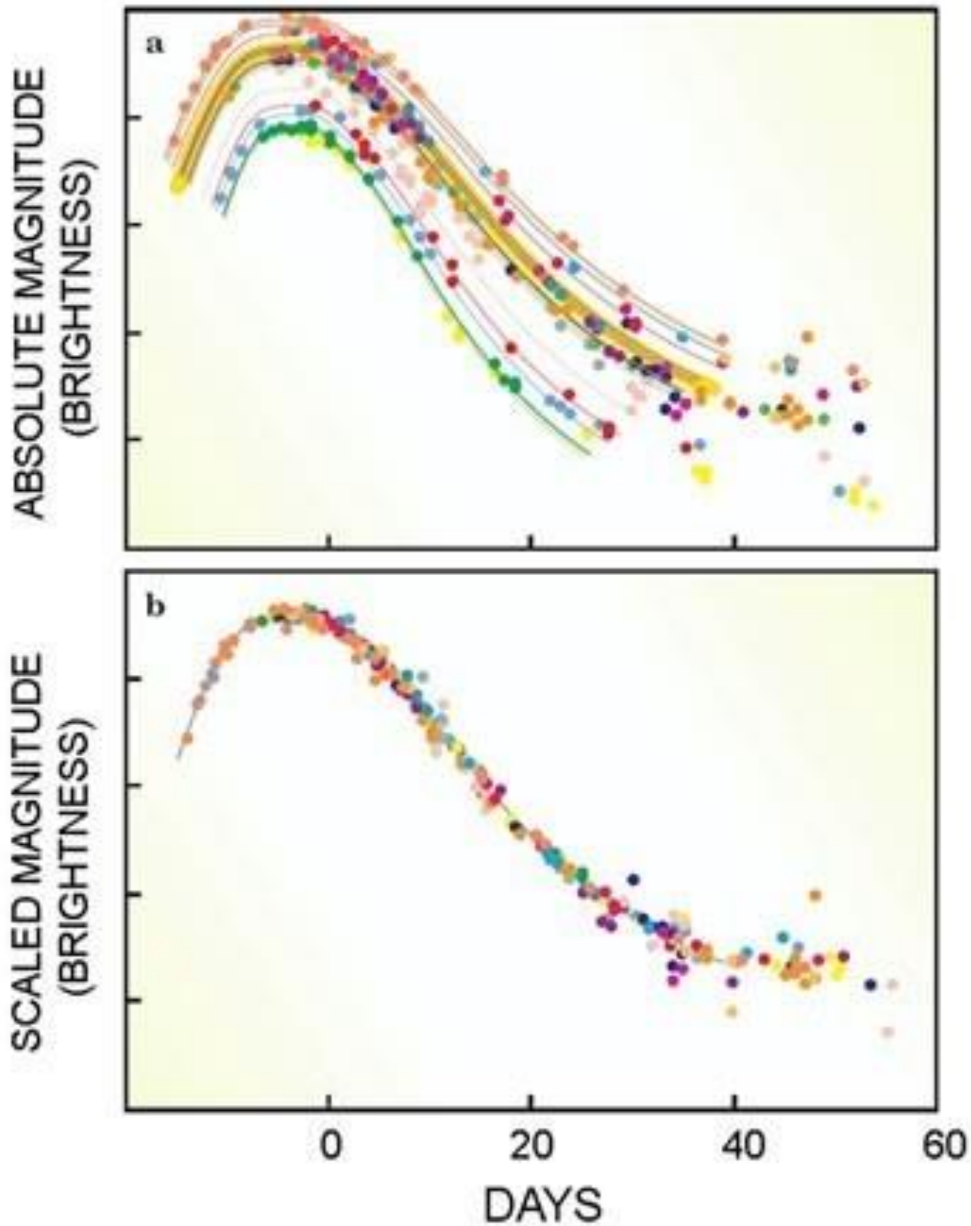
$$z = \frac{\Delta\lambda}{\lambda}$$



Measure the Universe by Supernovae Type 1a



NASA, ESA and A. Feild (STScI); vectorisation by chris



Light curves of Supernovae Type 1a

SOURCE: [BERKELEY LAB]

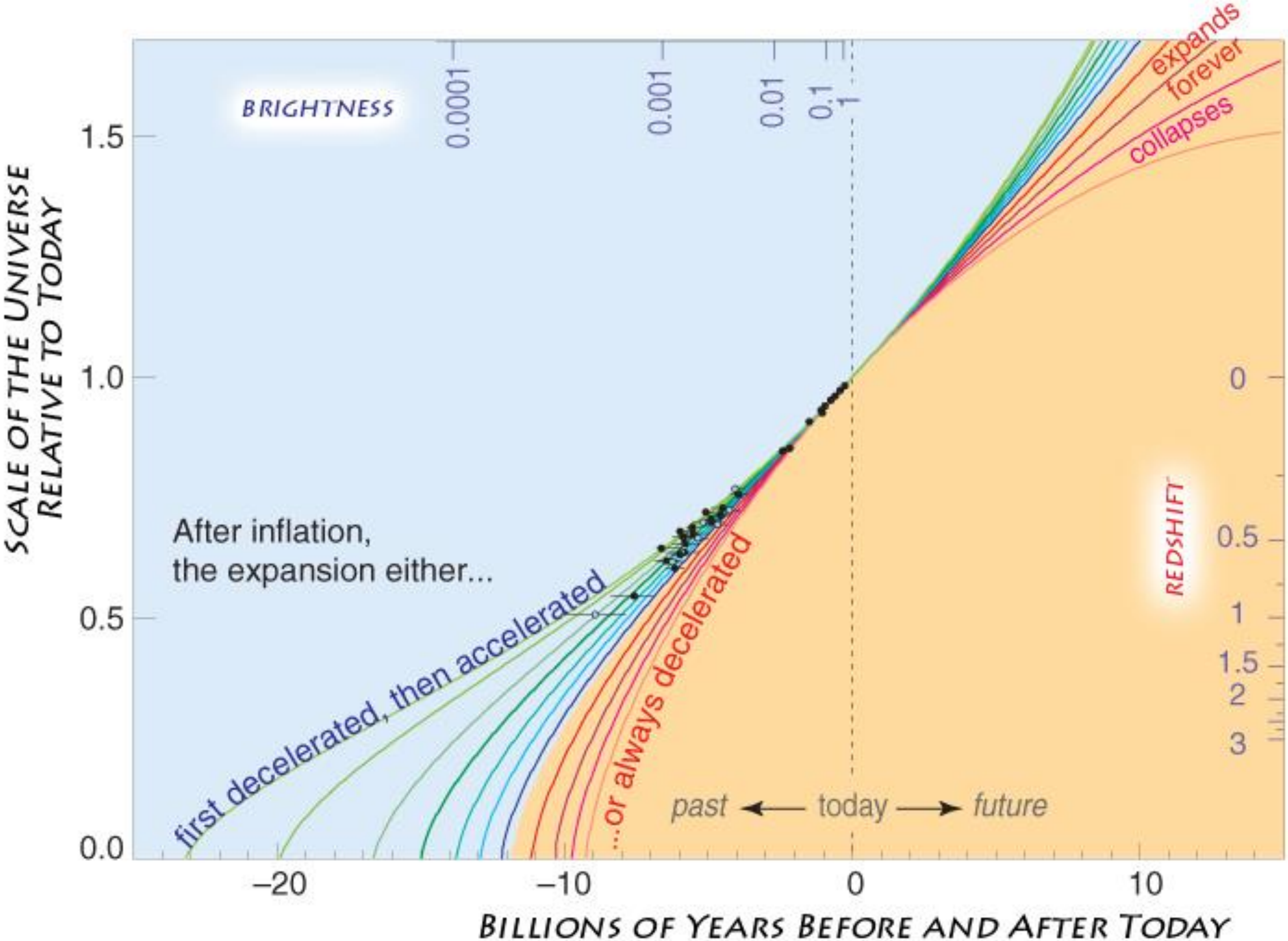
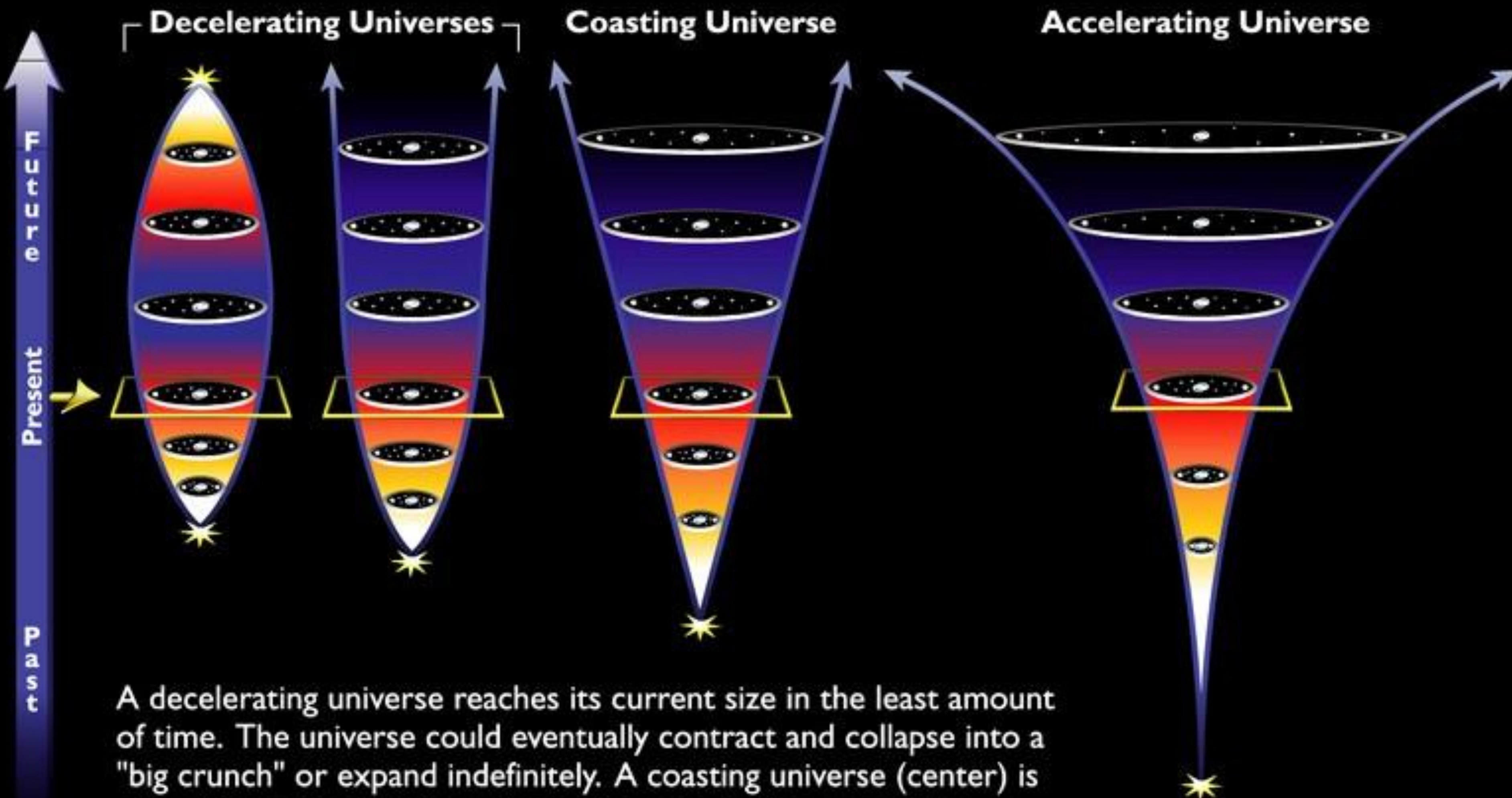


Image credit: Saul Perlmutter / UC Berkeley

Possible Models of the Expanding Universe



A decelerating universe reaches its current size in the least amount of time. The universe could eventually contract and collapse into a "big crunch" or expand indefinitely. A coasting universe (center) is older than a decelerating universe because it takes more time to reach its present size, and expands forever. An accelerating universe (right) is older still. The rate of expansion actually increases because of a repulsive force that pushes galaxies apart.

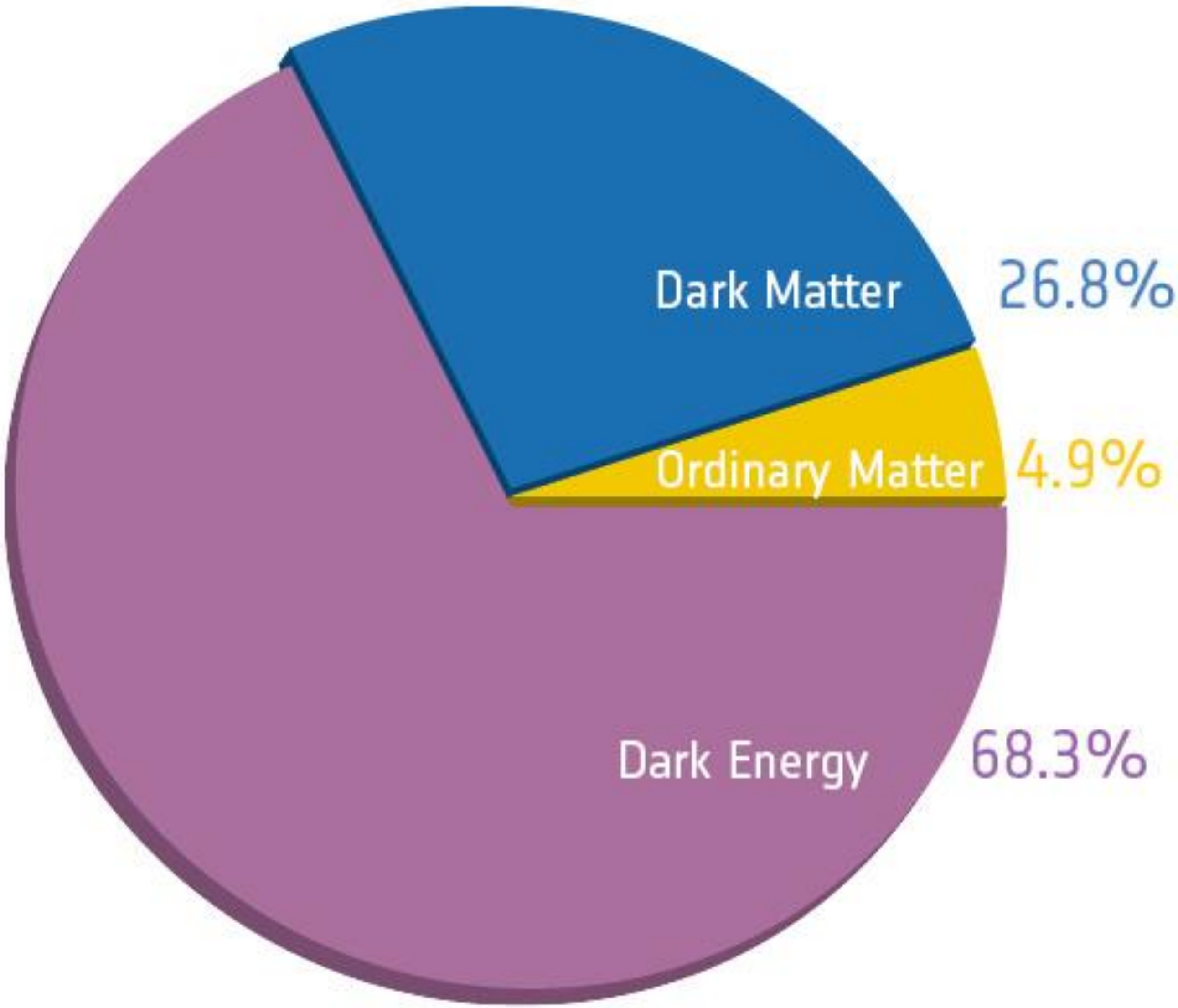
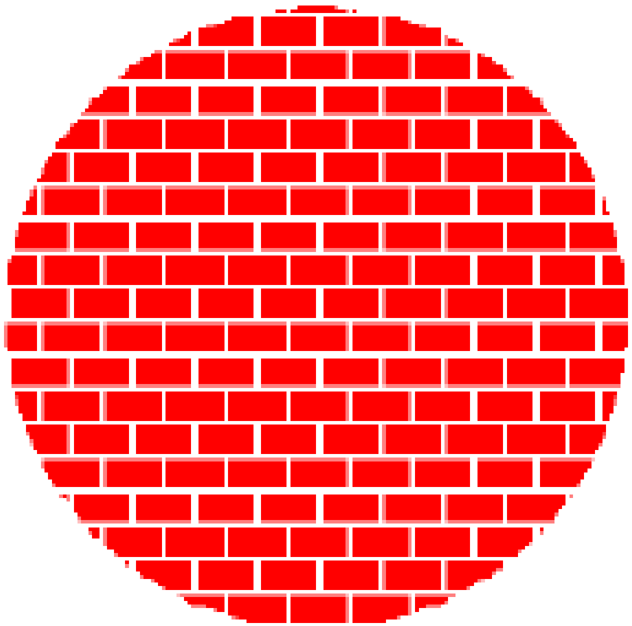


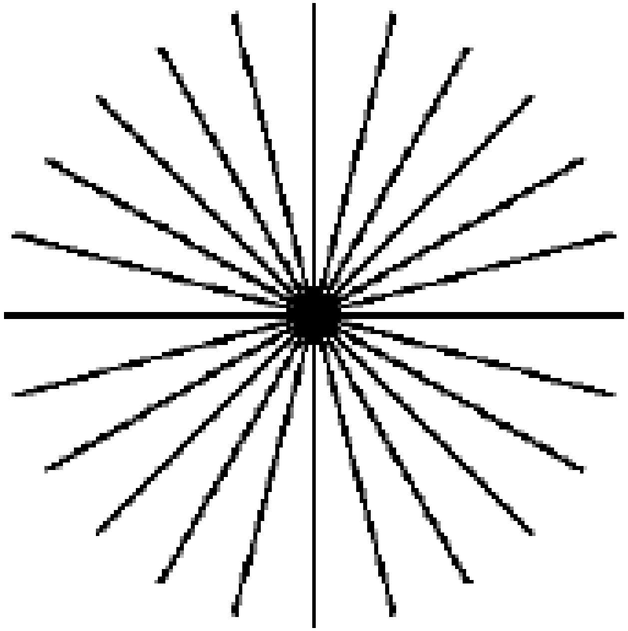
Image credit: Planck

What is the picture of expansion?

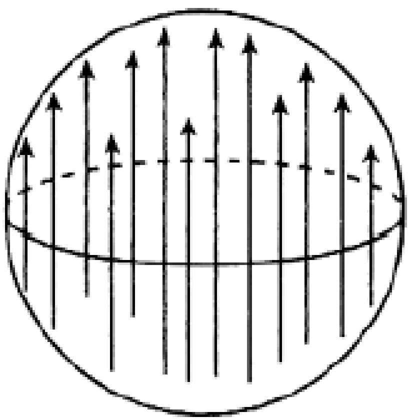
Cosmological Principle

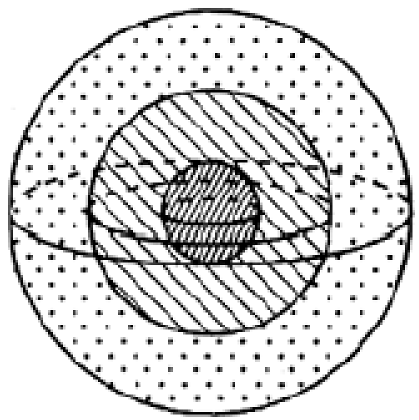
- “Viewed on a sufficiently large scale, the properties of the universe are the same for all observers.”
- The universe is **homogeneous** and **isotropic**

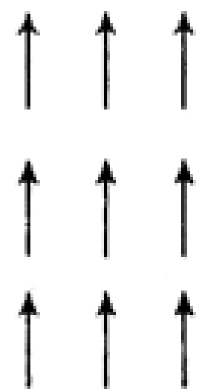


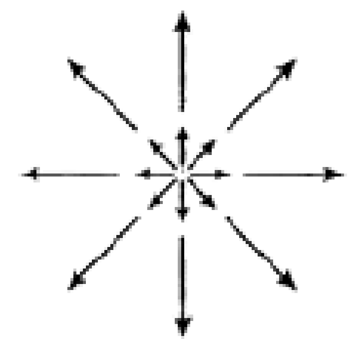


Homogeneous Isotropic



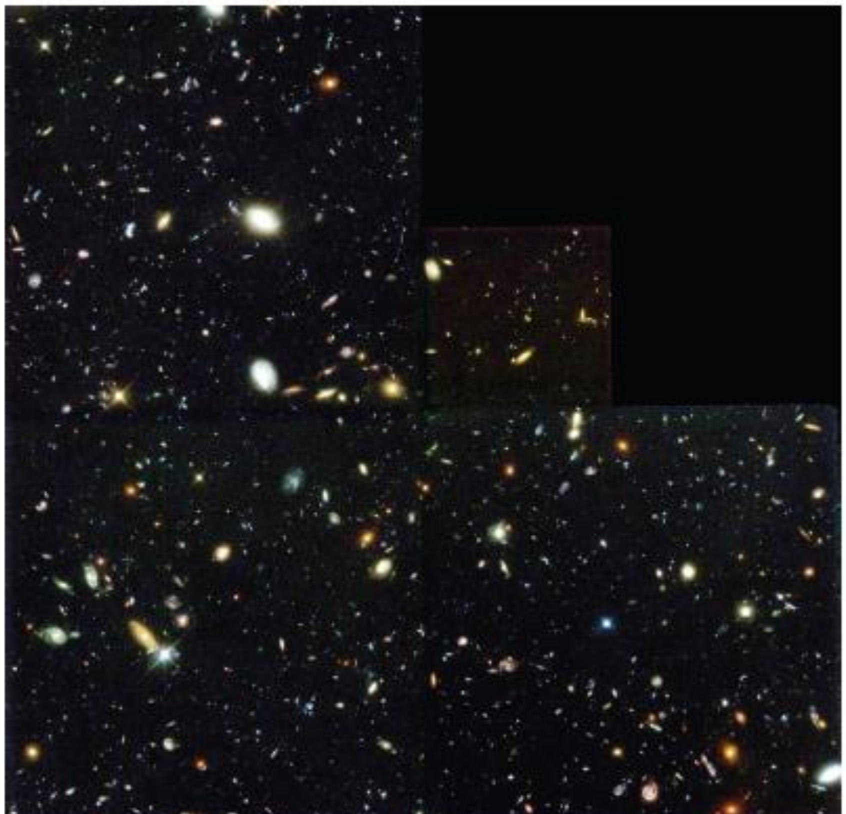






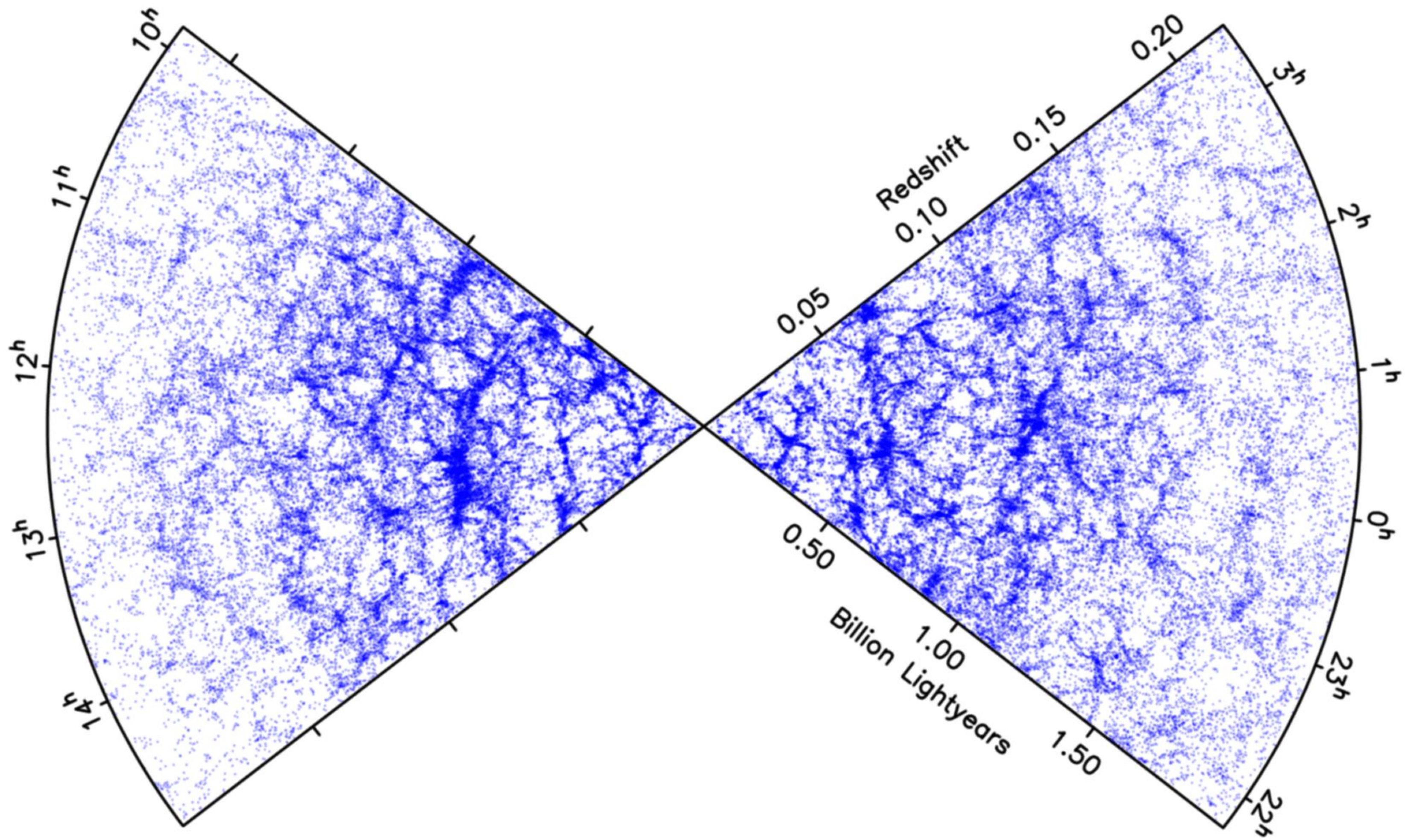
Supported by observations: The Universe is Isotropic—it looks the same in every direction

HDF-North

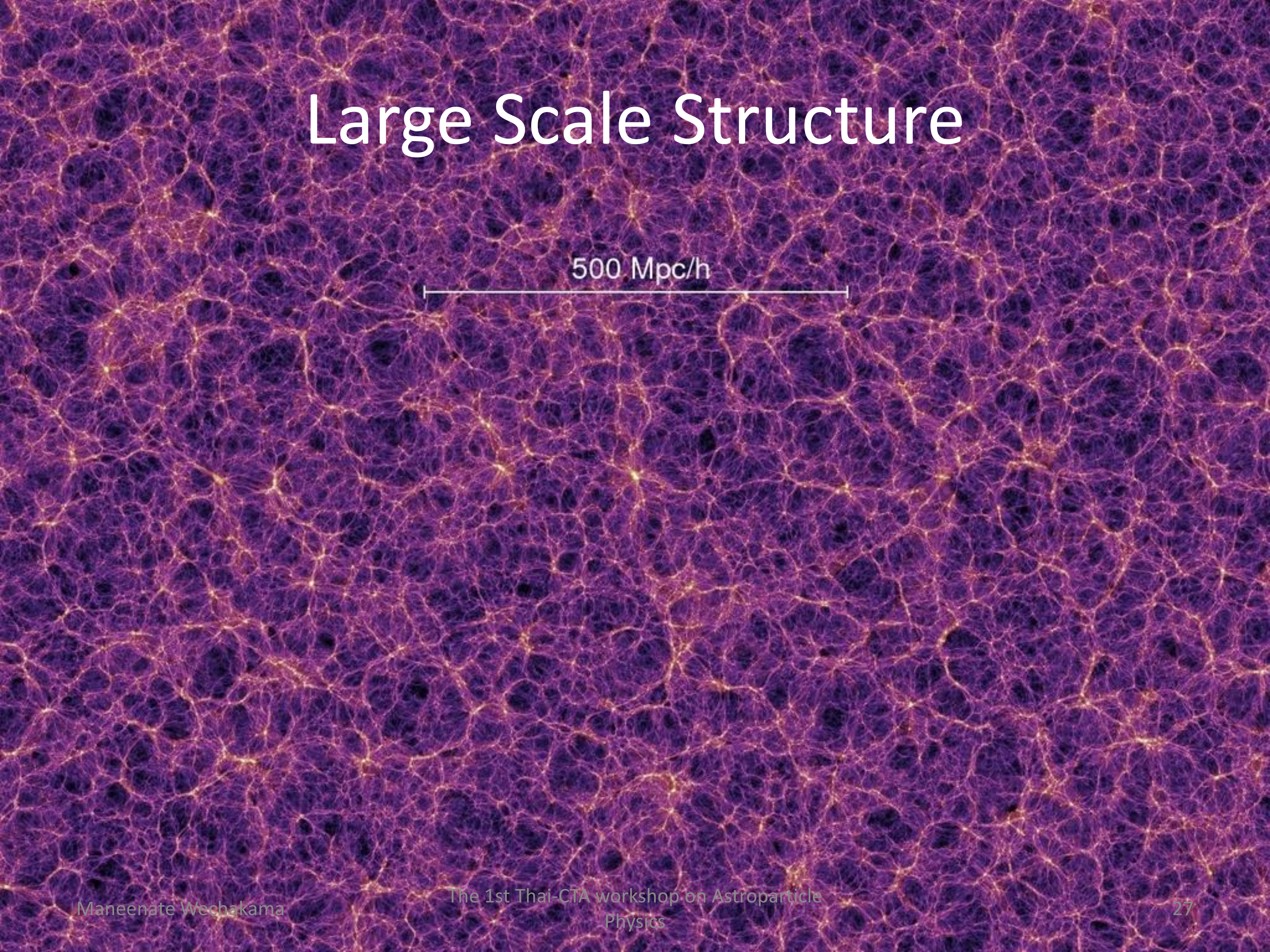


HDF-South

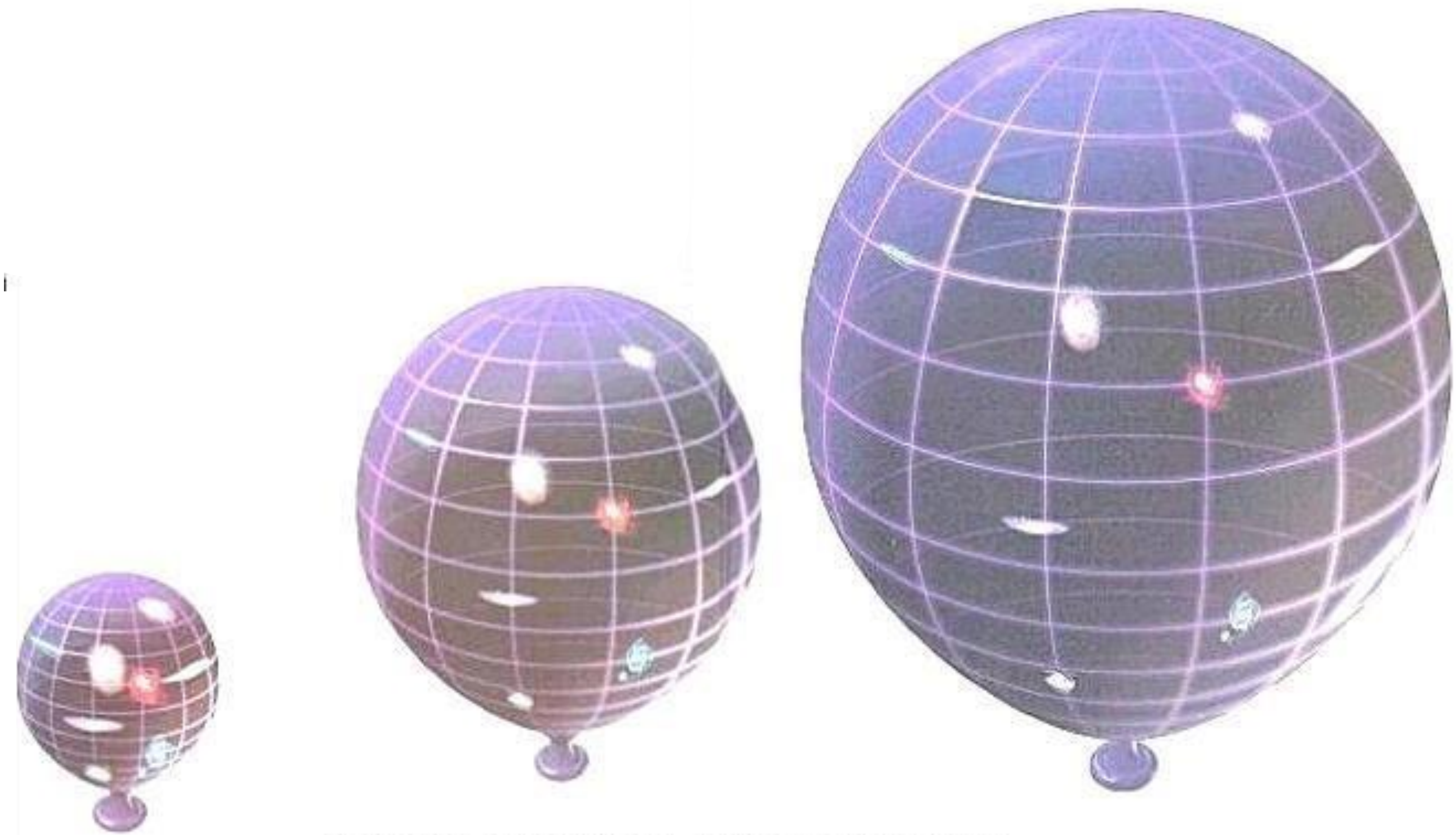
The universe is homogeneous—each volume is about like every other volume on scales larger than about 100 Mpc



Large Scale Structure



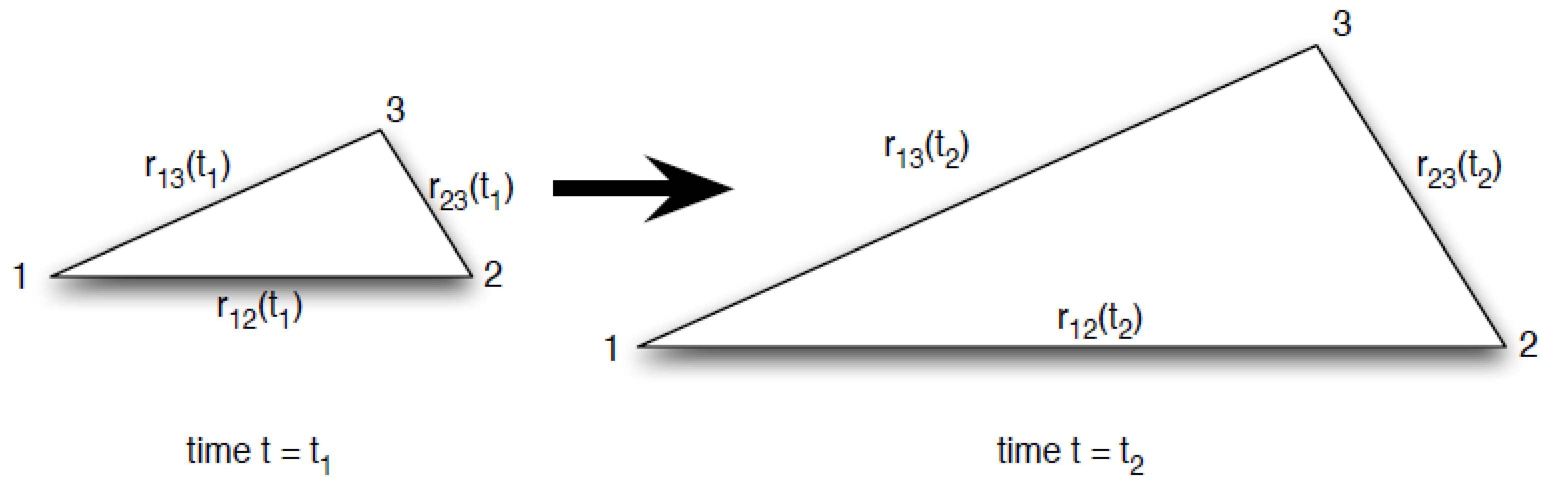
Comoving Coordinate



Expanding distance between galaxies

Comoving Coordinate

- Real galaxies or other objects may have small movements with respect to this coordinate system.
- But on average, due to the homogeneity and isotropy of the Universe , they will be at rest with respect to these coordinates as the Universe expands.
- We call this a “comoving coordinate” system.
- Hypothetical observers expanding along with these coordinates are called “comoving observers”.



- If $r_{ij}(t) = r_i(t) - r_j(t)$ is the distance between points *i* and *j* at a given time, we must scale all of these relative distances as a function only of time, but not position.

- That is

$$\boxed{\begin{aligned} r_{ij}(t_2) &= a(t_2)r_{ij}(t_0) \\ r_{ij}(t_1) &= a(t_1)r_{ij}(t_0) \end{aligned}}$$

- where t_0 is some arbitrary time, which we can take to be the present day.
- We can combine these to get

$$r_{ij}(t_0) = a^{-1}(t_1)r_{ij}(t_1) = a^{-1}(t_2)r_{ij}(t_2) = \text{constant}$$

- So, at a general time, t,

$$a^{-1}(t)r_{ij}(t) = \text{constant}$$

- We can take the derivative:

$$a^{-1}(t)\dot{r}_{ij}(t) - a^{-2}(t)\dot{a}(t)r_{ij}(t) = 0$$

- or

$$\frac{\dot{a}}{a} = \frac{\dot{r}_{ij}}{r_{ij}}$$

- we can write most generally for an arbitrary distance *r*

$$\boxed{\frac{\dot{a}}{a} = \frac{\dot{r}}{r}}$$

- The quantity ***a*****(t)**, the "**scale factor**".
- It describes the evolution of the Universe, and for a homogeneous and isotropic Universe.
- It tells us almost everything we need to know.

- We will sometimes write

$$H = \left.\frac{\dot{a}}{a}\right|_t$$

$$\boxed{H_0 = H(t_0) = \left.\frac{\dot{a}}{a}\right|_{t=t_0}}$$

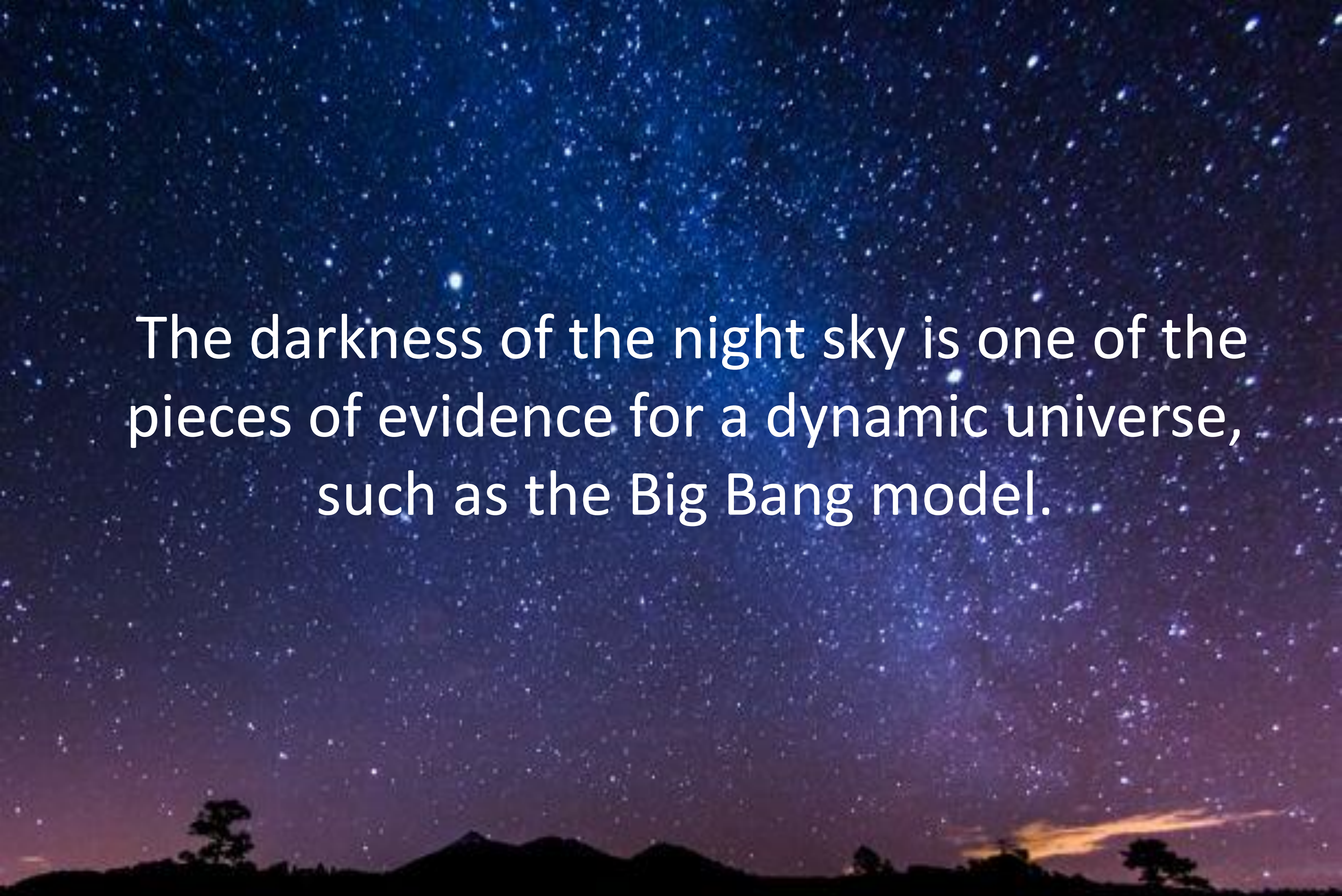
- is the expansion rate today.
- It is one of the most important quantities in cosmology!!

Outline

- The expanding universe
- A brief history of time
- Geometry and dynamics

Olbers’ paradox
“Dark night sky paradox“

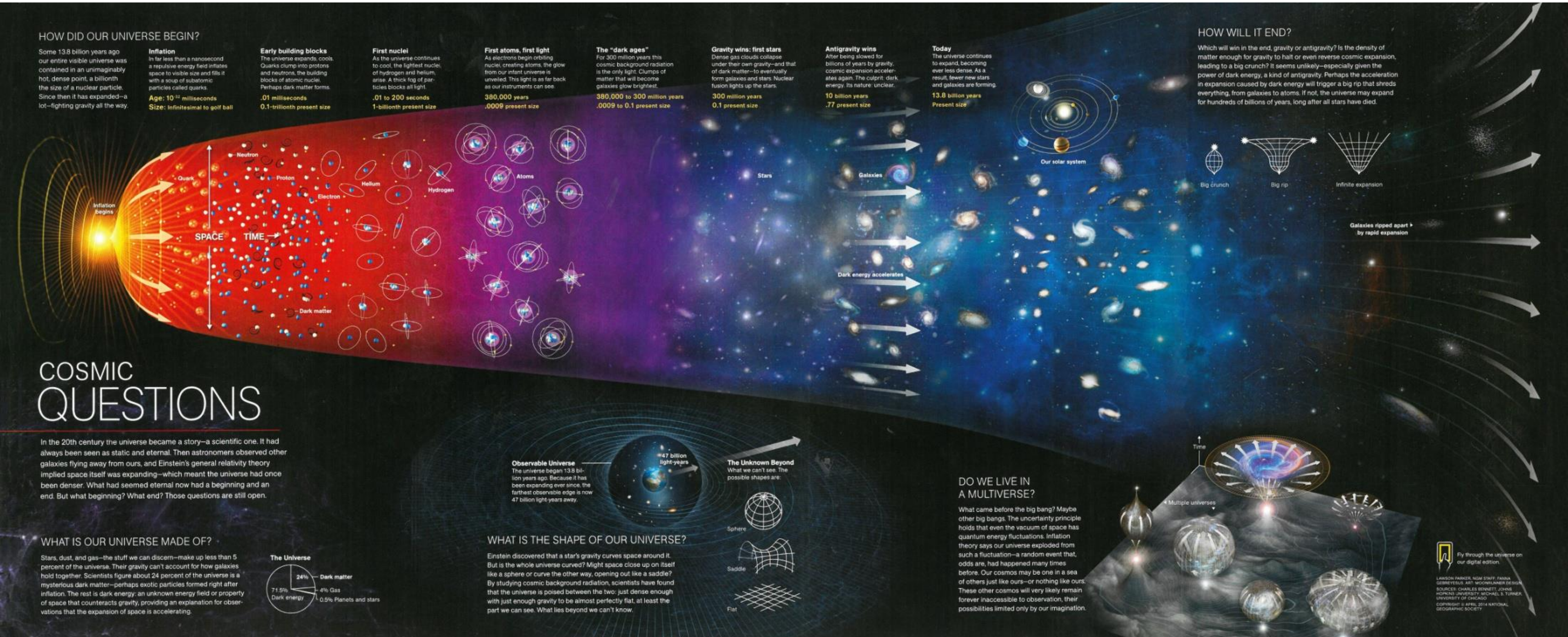
A static, infinitely old universe with an infinite number of stars distributed in an infinitely large space would be bright rather than dark…



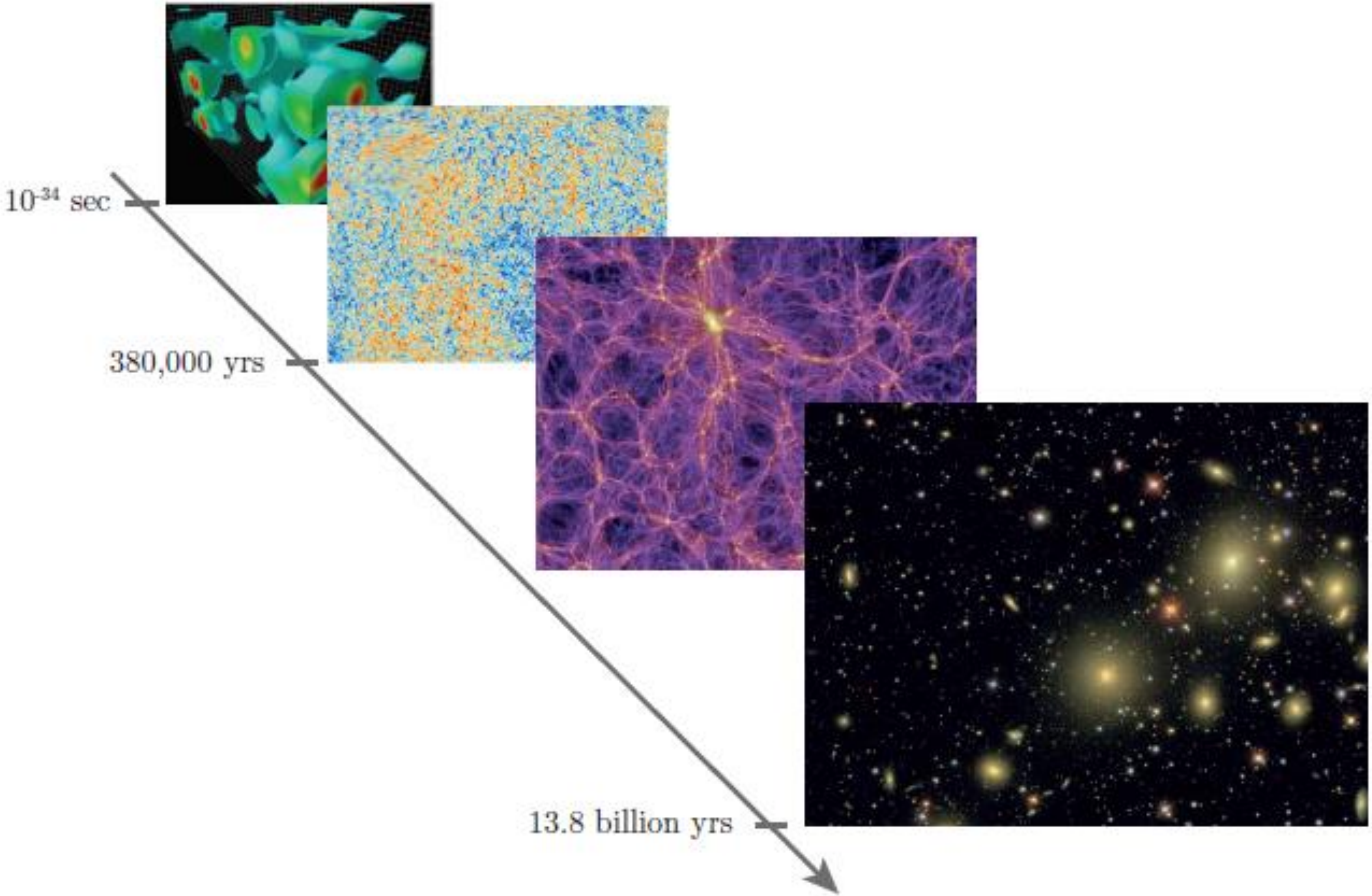
The darkness of the night sky is one of the pieces of evidence for a dynamic universe, such as the Big Bang model.

The Universe had a beginning!

The extremely successful BIG BANG theory!



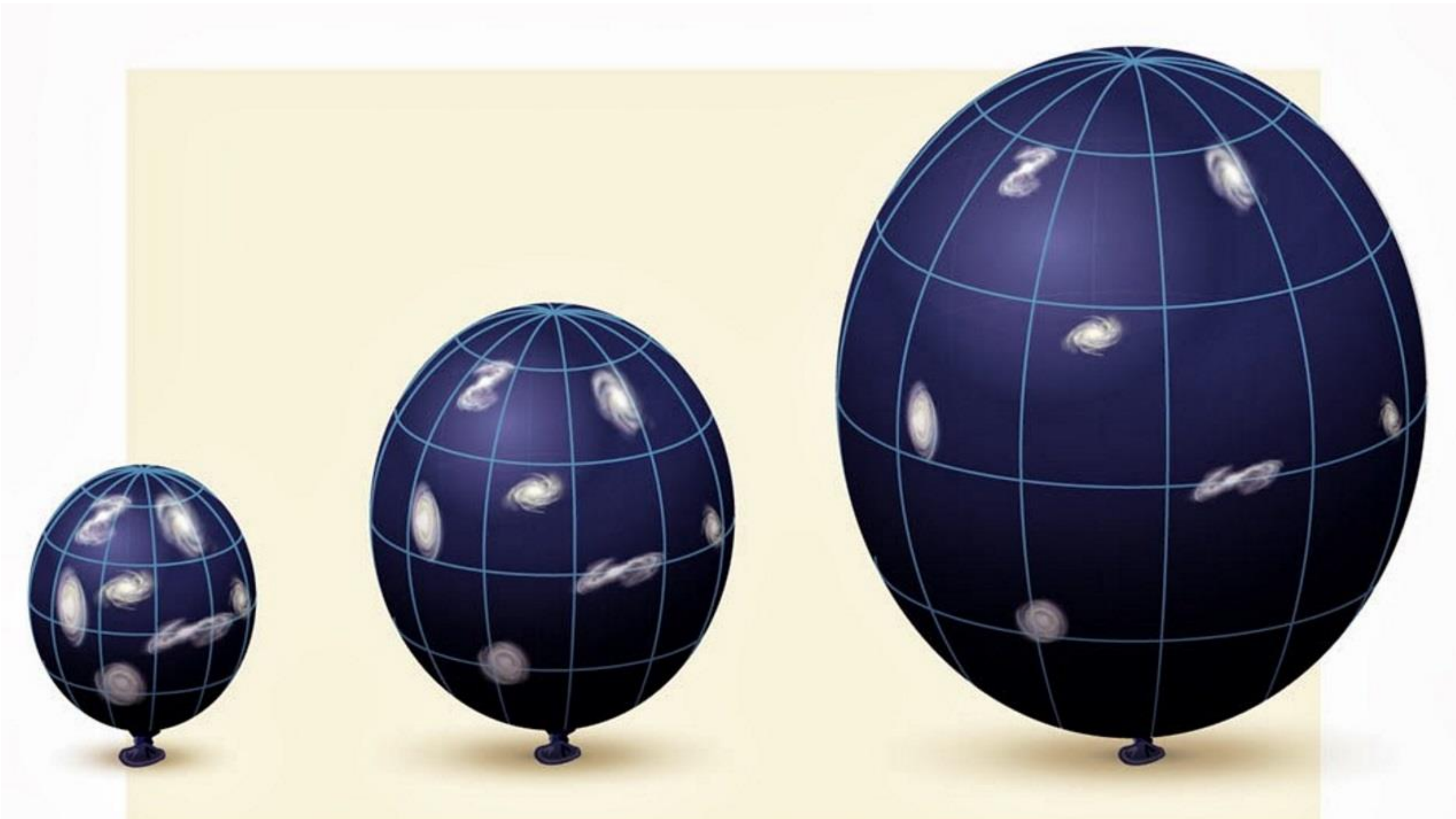
NATIONAL GEOGRAPHIC



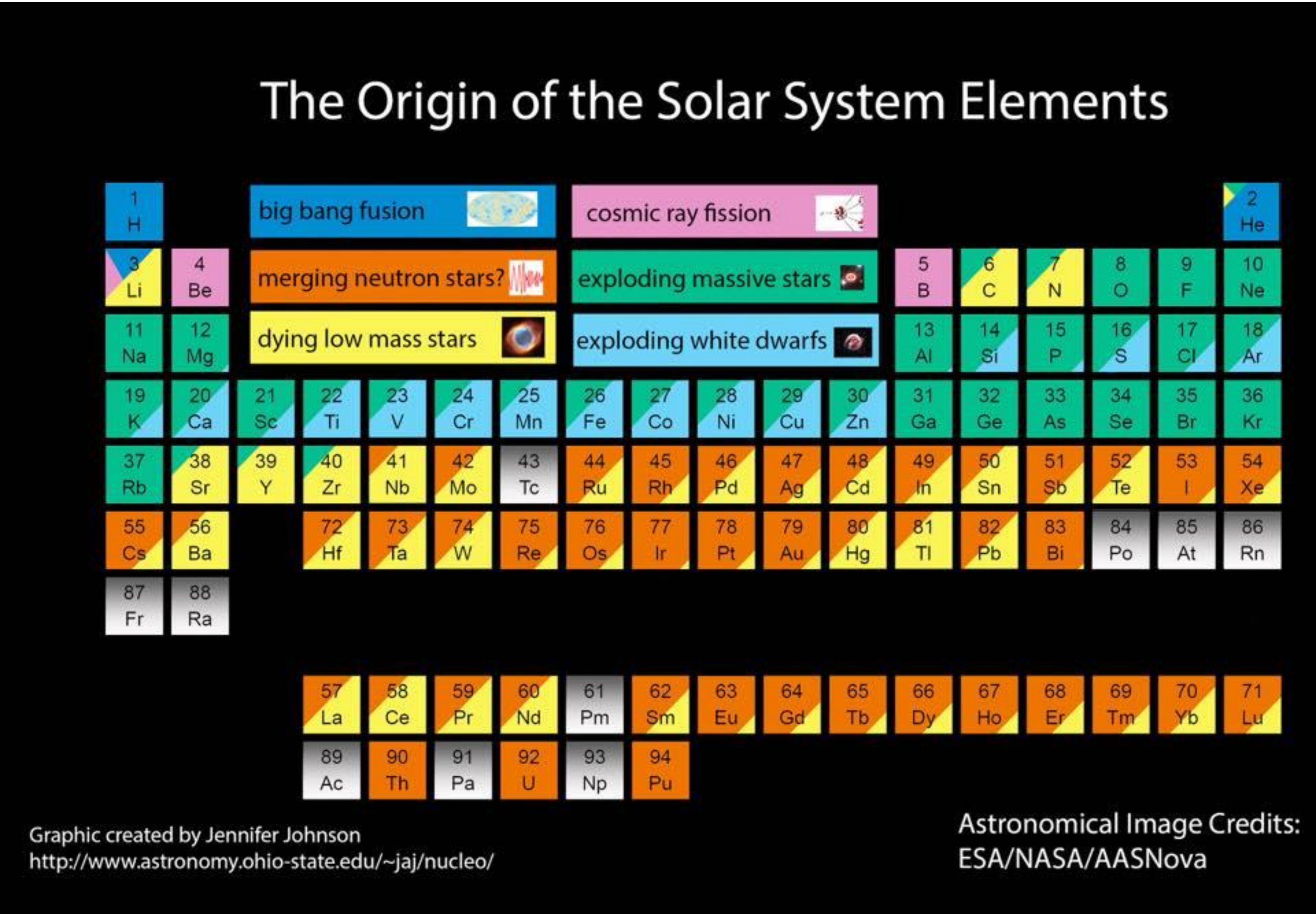
10^{-34} sec
380,000 yrs
13.8 billion yrs

Evidences of Big Bing

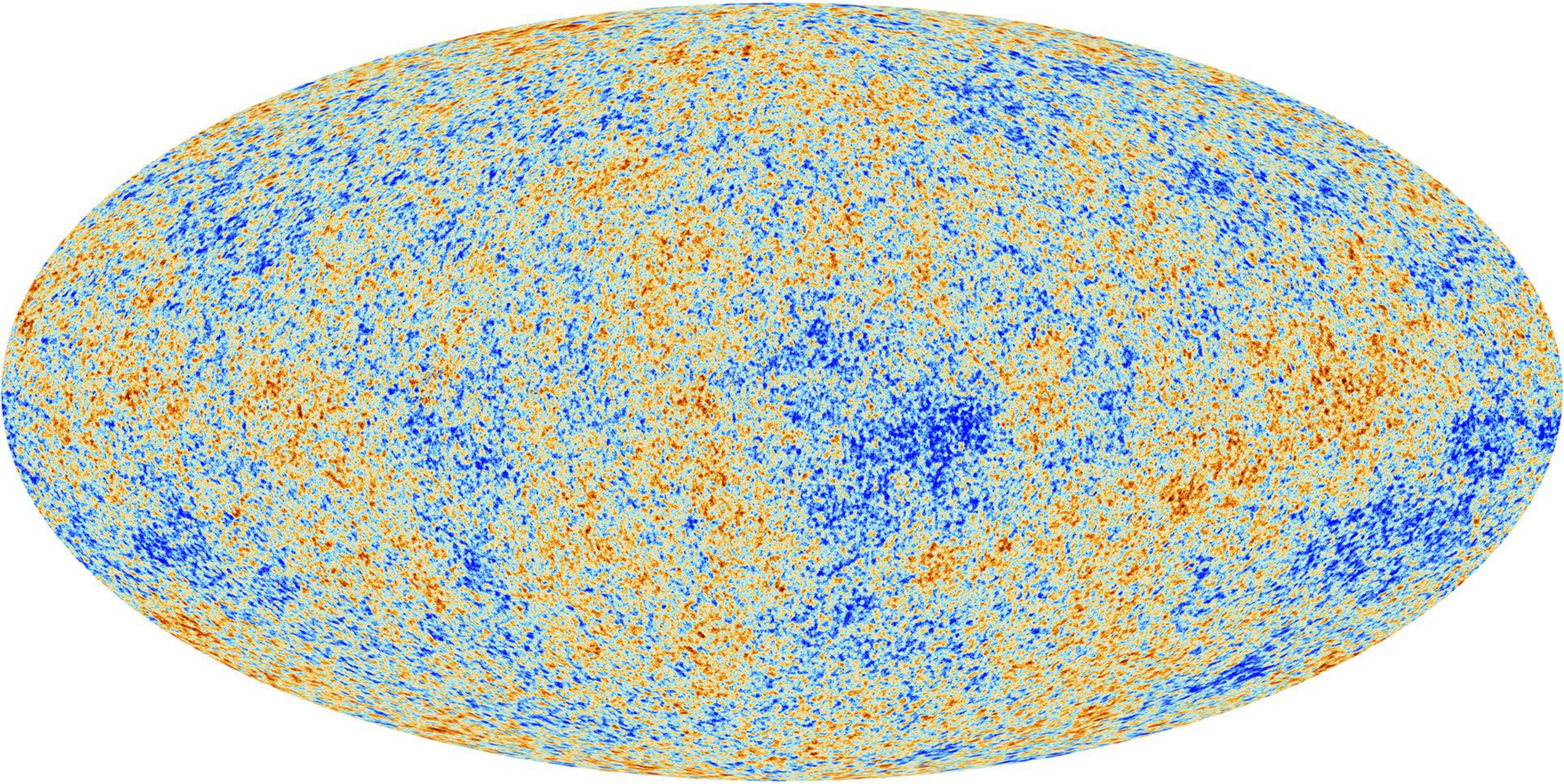
The Expansion of the Universe



Credit: Eugenio Bianchi, Carlo Rovelli & Rocky Kolb.

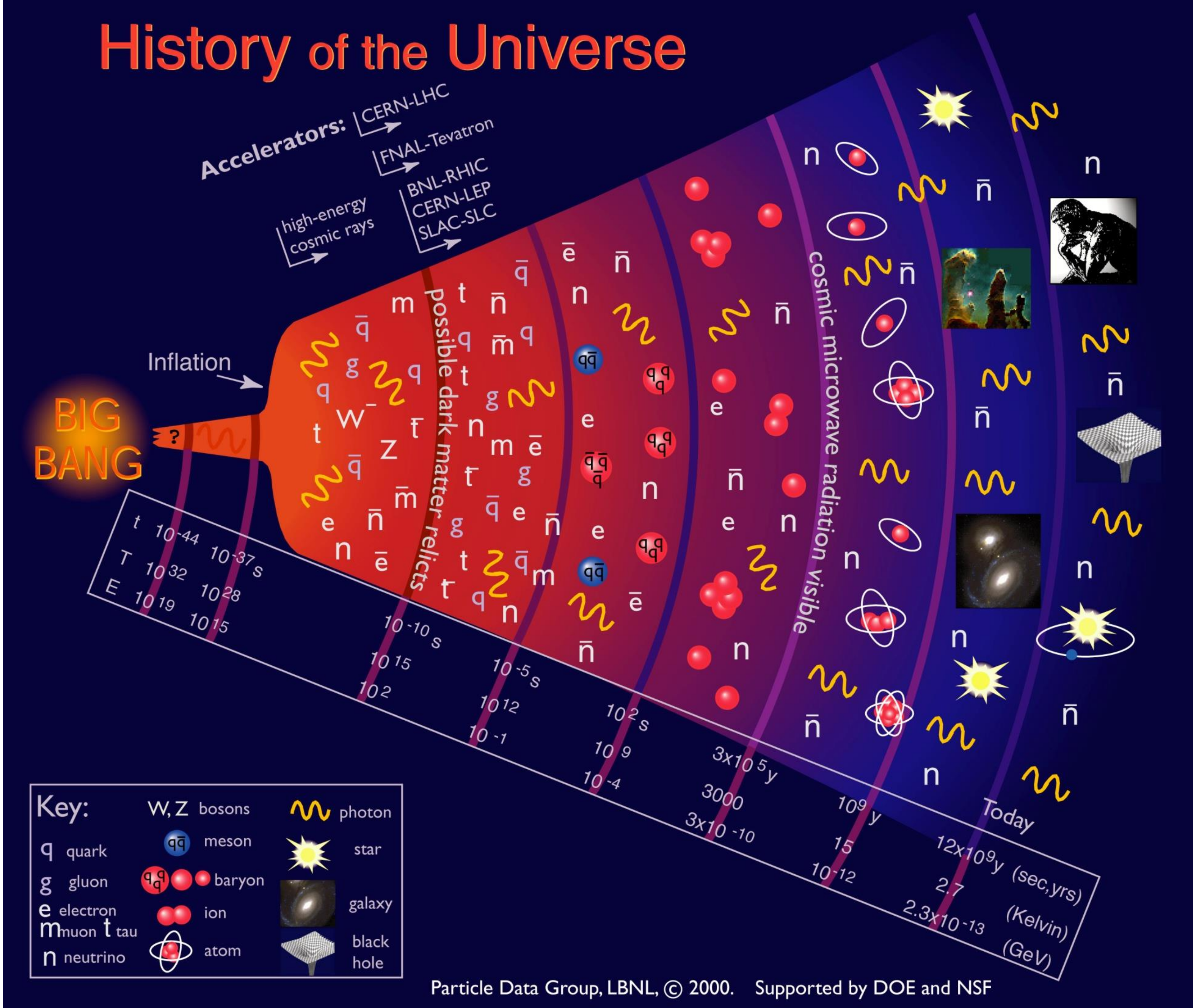

The Origin of the Solar System Elements
big bang fusion
cosmic ray fission
merging neutron stars?
exploding massive stars
dying low mass stars
exploding white dwarfs
Graphic created by Jennifer Johnson
http://www.astronomy.ohio-state.edu/~jaj/nucleo/
Astronomical Image Credits:
ESA/NASA/AASNova

Cosmic Microwave Background (CMB)

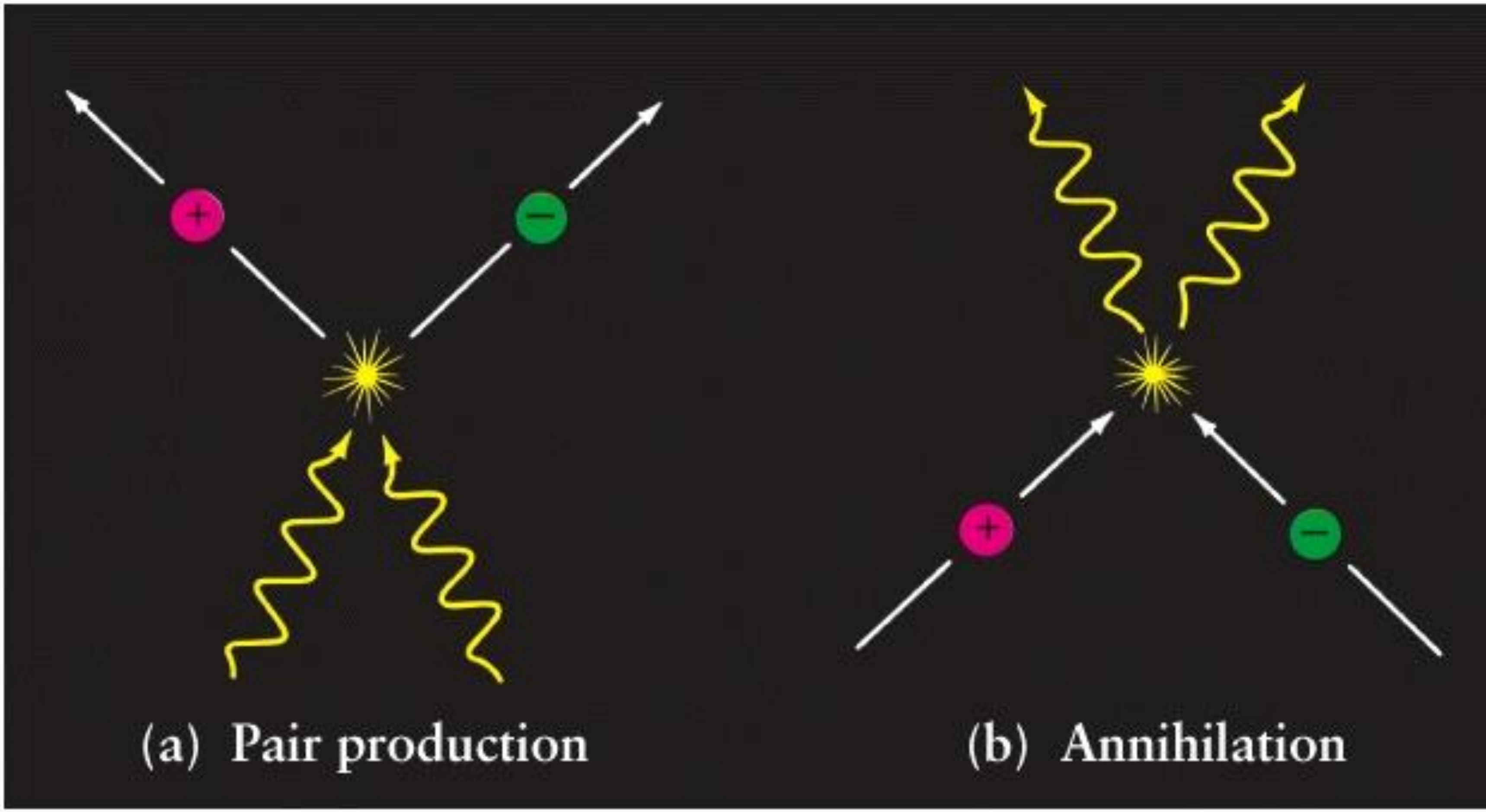


Credit: Planck

The Hot Big Bang


History of the Universe
Accelerators:
CERN-LHC
FNAL-Tevatron
BNL-RHIC
CERN-LEP
SLAC-SLC
high-energy cosmic rays
Inflation
BIG BANG
possible dark matter relicts
cosmic microwave radiation visible
t 10-44 10-37 s
T 1032 1028
E 1019 1015
10-10 s
1015
102
10-5 s
1012
10-1
102 s
109
10-4
3x105 y
3000
3x10-10
109 y
15
10-12
Today
12x109 y (sec,yrs)
2.7 (Kelvin)
2.3x10-13 (GeV)
Key:
q quark
g gluon
e electron
m muon t tau
n neutrino
W, Z bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL, © 2000. Supported by DOE and NSF

Pair Production and Annihilation



https://sites.ualberta.ca/~pogosyan/teaching/ASTRO_122/lect32/lecture32.html

Timeline of the Big Bang: 10^{-9} s

- The universe was filled with a gas of fundamental particles: quarks and antiquarks, leptons and antileptons, neutrinos and antineutrinos, and gluons and photons.
- When the temperature fell below 10^{14} K, the quarks, antiquarks and gluons disappeared, annihilating and transforming into less massive particles.

Timeline of the Big Bang: 10^{-9} s

- Fortunately, because the number of quarks slightly exceeded the number of antiquarks, a few quarks were left behind to form the protons and neutrons present in today's universe.
- The heavier leptons and antileptons were also annihilated as the temperature fell.

Timeline of the Big Bang: 10^{-3} -1 s

- The universe consisted of a gas of neutrons and protons, electrons and positrons, neutrinos and antineutrinos, and photons.
- As the temperature fell, the density of the universe became too low for the neutrinos to interact effectively with matter; this occurred when the temperature was about 10^{10} K.

Timeline of the Big Bang: 10^{-3} -1 s

- These non-interacting, decoupled neutrinos now form a universal gas which, because of the expansion of space, has cooled to a temperature of about 2 K.
- As yet it has not been possible to detect this universal background of neutrinos.

Timeline of the Big Bang: 10^{-3} -1 s

- Soon after the decoupling of the neutrinos, the annihilation of electron–positron pairs removed all of the positrons and most of the electrons.

Timeline of the Big Bang: 100 s

- Neutrons combined with protons to form light nuclei, ultimately leading to a universe in which approximately 75% of the mass consists of hydrogen and 25% is helium.

Timeline of the Big Bang: 300,000 Y

- The temperature fell to 4000 K, low enough for the formation of stable atoms.
- Hydrogen and helium nuclei combined with electrons to form neutral hydrogen and helium atoms.

Timeline of the Big Bang: 300,000 Y

- It is the cosmic microwave background radiation which was first detected by Penzias and Wilson.
- This radiation is slightly warmer than the as yet undetected neutrino background at 2 K because, unlike neutrinos, photons were warmed by the heat generated by electron–positron annihilation in the early universe.

Timeline of the Big Bang: Today

- The universe continued to expand and cool until it reached its present lumpy condition with most of the matter assembled in stars, galaxies and clusters of galaxies.

Two key concepts:

- Particles are in *local thermal equilibrium* as long as

$$\boxed{\underset{\textit{interaction rate}}{\Gamma} \gg \underset{\textit{expansion rate}}{H}}$$

– relativistic particles dominate

– non-relativistic particles are "Boltzmann-suppressed": $n \propto e^{-m/T}$.

- Particles *decouple* from the thermal bath when

$$\boxed{\Gamma \lesssim H}$$

Non-equilibrium phenomena make the world interesting:

– dark matter freeze-out

– Big Bang nucleosynthesis

– recombination

For particles in *kinetic equilibrium* (maximum entropy), the distribution functions are

$$\boxed{f(p) = \frac{1}{e^{(E(p)-\mu)/T} \pm 1}}$$

$+$ fermions $\quad T(t)$: temperature $(k_B \equiv 1)$

$-$ bosons $\quad \mu(T)$: chemical potential

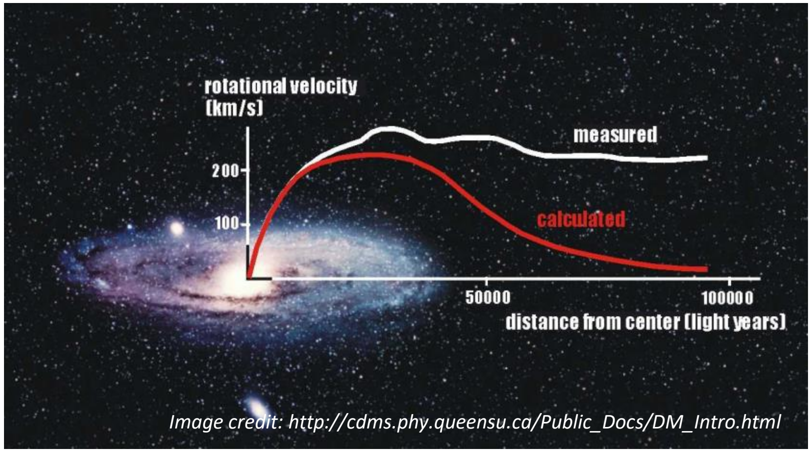
$$\boxed{\mathrm{d}S = \frac{\mathrm{d}U + P\mathrm{d}V - \mu\mathrm{d}N}{T}}$$

Each particle species i (with m_i, μ_i, T_i) has its own $f_i \Rightarrow n_i\,, \rho_i\,, P_i\,.$

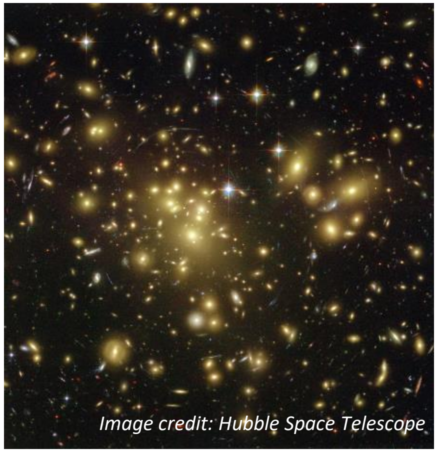
Event	time t	redshift z	temperature T
Inflation	10^{-34} s (?)	–	–
Baryogenesis	?	?	?
EW phase transition	20 ps	10^{15}	100 GeV
QCD phase transition	20 μs	10^{12}	150 MeV
Dark matter freeze-out	?	?	?
Neutrino decoupling	1 s	6×10^{9}	1 MeV
Electron-positron annihilation	6 s	2×10^{9}	500 keV
Big Bang nucleosynthesis	3 min	4×10^{8}	100 keV
Matter-radiation equality	60 kyr	3400	0.75 eV
Recombination	260–380 kyr	1100–1400	0.26–0.33 eV
Photon decoupling	380 kyr	1000–1200	0.23–0.28 eV
Reionization	100–400 Myr	11–30	2.6–7.0 meV
Dark energy-matter equality	9 Gyr	0.4	0.33 meV
Present	13.8 Gyr	0	0.24 meV

Dark matter freeze out

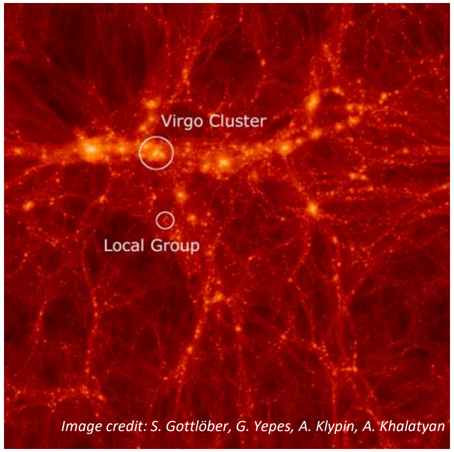
Cosmological Evidences for DM



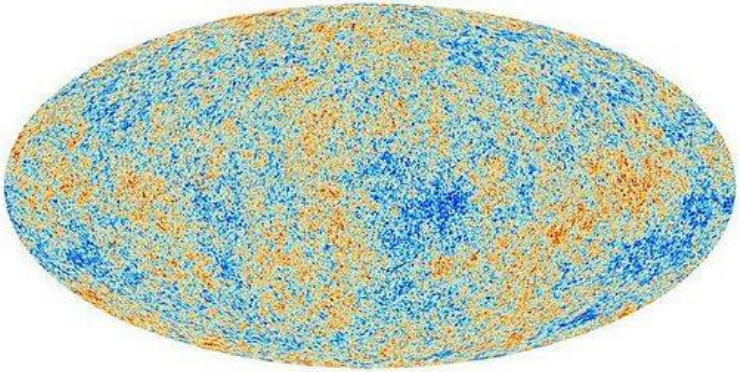
Rotation curves of galaxies



Gravitational lensing



Large scale structure



Total amount of DM

Planck Collaboration et al. 2013

Consider **W**eakly **I**nteracting **M**assive **P**articles:

$$X + \bar{X} \leftrightarrow \ell + \bar{\ell}$$

$$\Omega_{\rm WIMP} \sim \frac{x_f T_0^3}{\rho_c M_{\rm Pl}} \langle \sigma_{\rm ann} v \rangle^{-1}$$

$$x_f = m_{\rm WIMP}/T_f \approx 20$$

$$\Omega_{\rm WIMP} h^2 = 0.1 \frac{3 \times 10^{-26}\,{\rm cm}^3\,{\rm s}^{-1}}{\langle \sigma_{\rm ann} v \rangle}.$$

$$\Omega_{\rm WIMP} \sim \Omega_{\rm DM}$$

$$\langle \sigma_{\rm ann} v \rangle \approx 3 \times 10^{-26}\,{\rm cm}^3\,{\rm s}^{-1}$$

WIMP Miracle

Neutrino Decoupling

Neutrinos are coupled to the thermal bath by *weak interactions*:

$$\nu_e + \bar{\nu}_e \leftrightarrow e^+ + e^- \ ,$$
$$e^- + \bar{\nu}_e \leftrightarrow e^- + \bar{\nu}_e \ .$$

with *interaction rate*

$$\Gamma \sim \underbrace{G_F^2}_{\text{Fermi's constant}} \times \overbrace{T^5}^{\text{dimensional analysis}}$$

$G_F \approx 10^{-5}\,\mathrm{GeV}^{-2}$

Decoupling occurs when $\Rightarrow \boxed{T_{dec} \sim 1\,\mathrm{MeV}}$.

Electrons and Positrons Annihilation

- Shortly after neutrino decoupling, electrons and positrons annihilate.

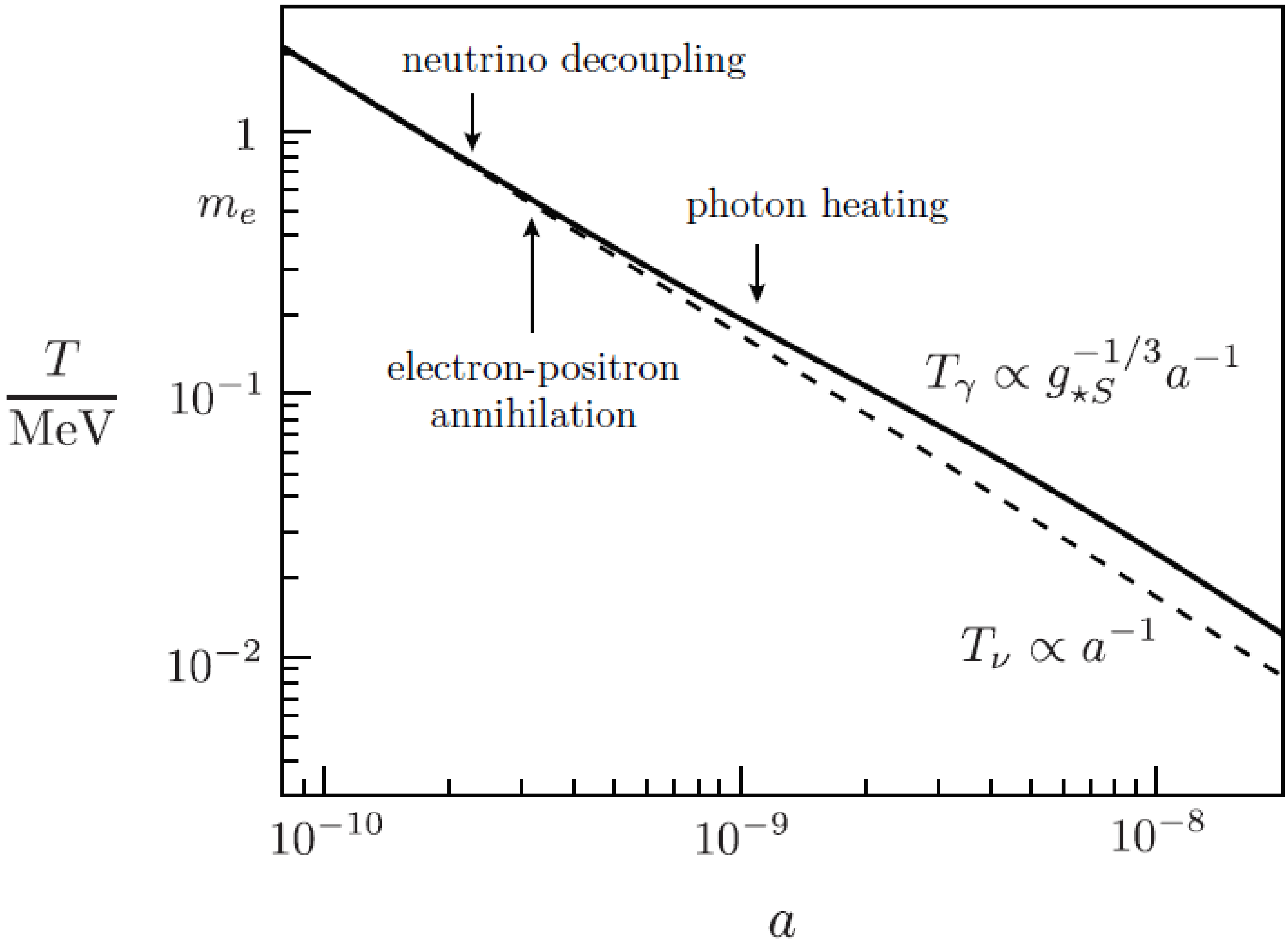
$$e^+ + e^- \leftrightarrow \gamma + \gamma \, .$$

- This transfers entropy to the photons,

but not to the decoupled neutrinos.

- Photons are heated (relative to neutrinos).

$$\boxed{T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma}$$



neutrino decoupling
photon heating
electron-positron annihilation
$T_\gamma \propto g_{\star S}^{-1/3} a^{-1}$
$T_\nu \propto a^{-1}$
$\frac{T}{\mathrm{MeV}}$
1
m_e
10^{-1}
10^{-2}
10^{-10}
10^{-9}
10^{-8}
a

Cosmic Neutrino Background

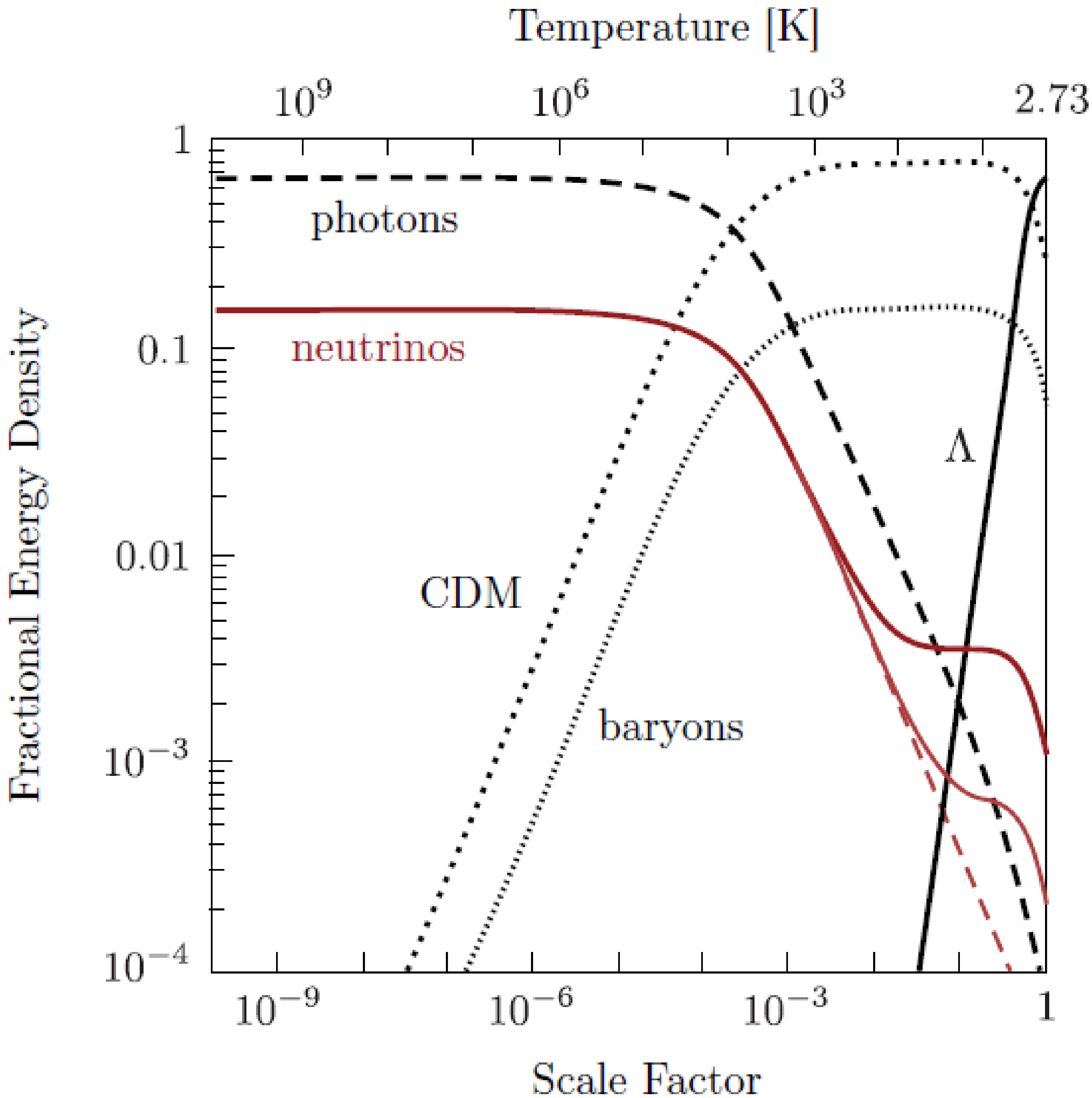
$$\boxed{T_{\nu,0} = 1.95 \text{ K} = 0.17 \text{ meV}}$$

Observational constraints on neutrino masses,

$$\boxed{0.05 \text{ eV} \le \sum m_\nu \le 1 \text{ eV}}$$

imply

$$\boxed{0.001 < \Omega_\nu < 0.02} \ .$$



Temperature [K]
10^9
10^6
10^3
2.73
1
0.1
0.01
10^{-3}
10^{-4}
Fractional Energy Density
photons
neutrinos
CDM
baryons
Λ
10^{-9}
10^{-6}
10^{-3}
1
Scale Factor

Recombination

Above 1 eV, photons are in equilibrium with electrons and protons:

$$\boxed{e^- + p^+ \leftrightarrow \mathrm{H} + \gamma},$$

Using the Saha equation, we find

$$T_{rec} \approx 0.3\,\mathrm{eV} \ll B_{\mathrm{H}}$$

$$z_{rec} \approx 1320 \ll z_{eq}$$

$$t_{rec} \approx \frac{t_0}{(1+z_{rec})^{3/2}} \sim 290\,000\,\mathrm{yrs}$$

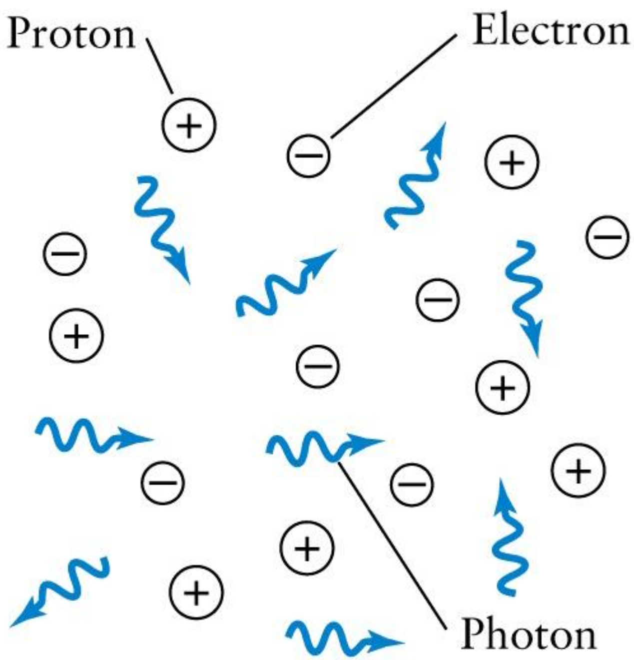
Photon decoupling

Thomson scattering, $\boxed{e^- + \gamma \leftrightarrow e^- + \gamma}$, occurs with rate

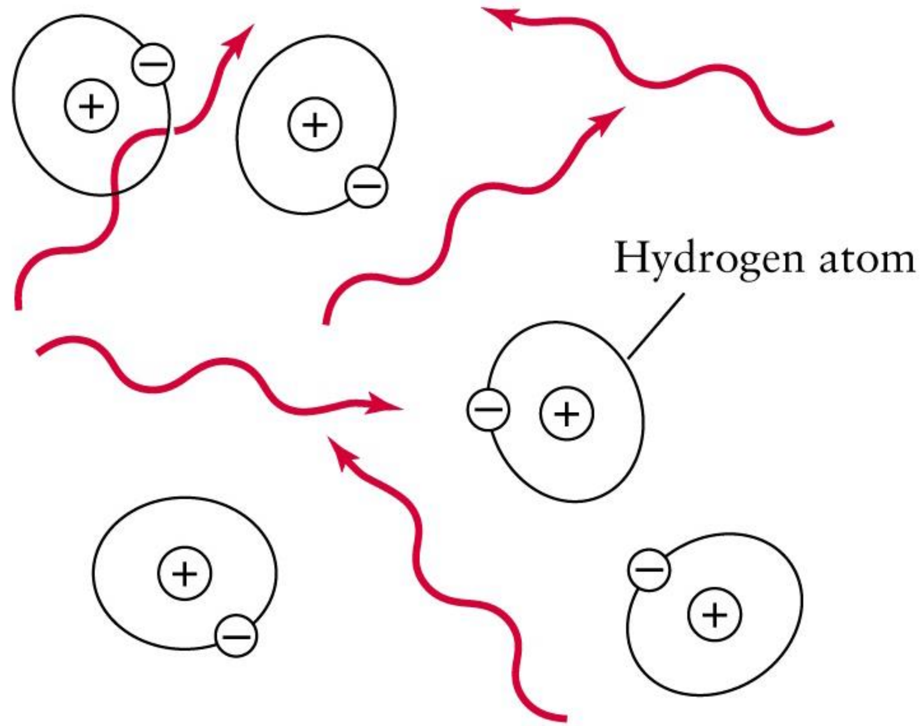
$$T_{dec} \approx 0.27\,\mathrm{eV}$$

$$z_{dec} \approx 1100$$

$$t_{dec} \approx 380\,000\,\mathrm{yrs}$$



a Before recombination



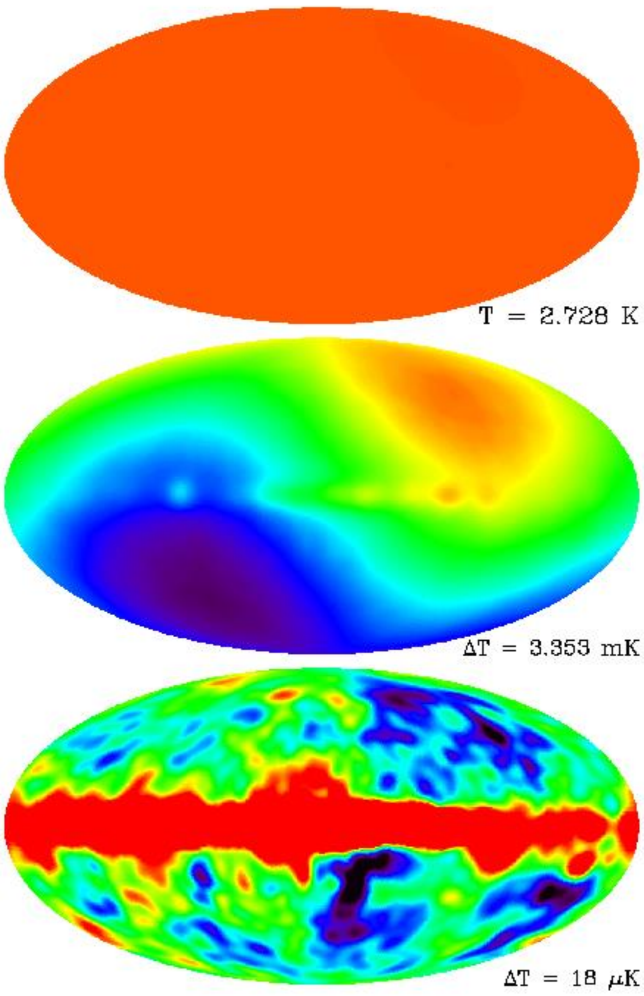
b After recombination

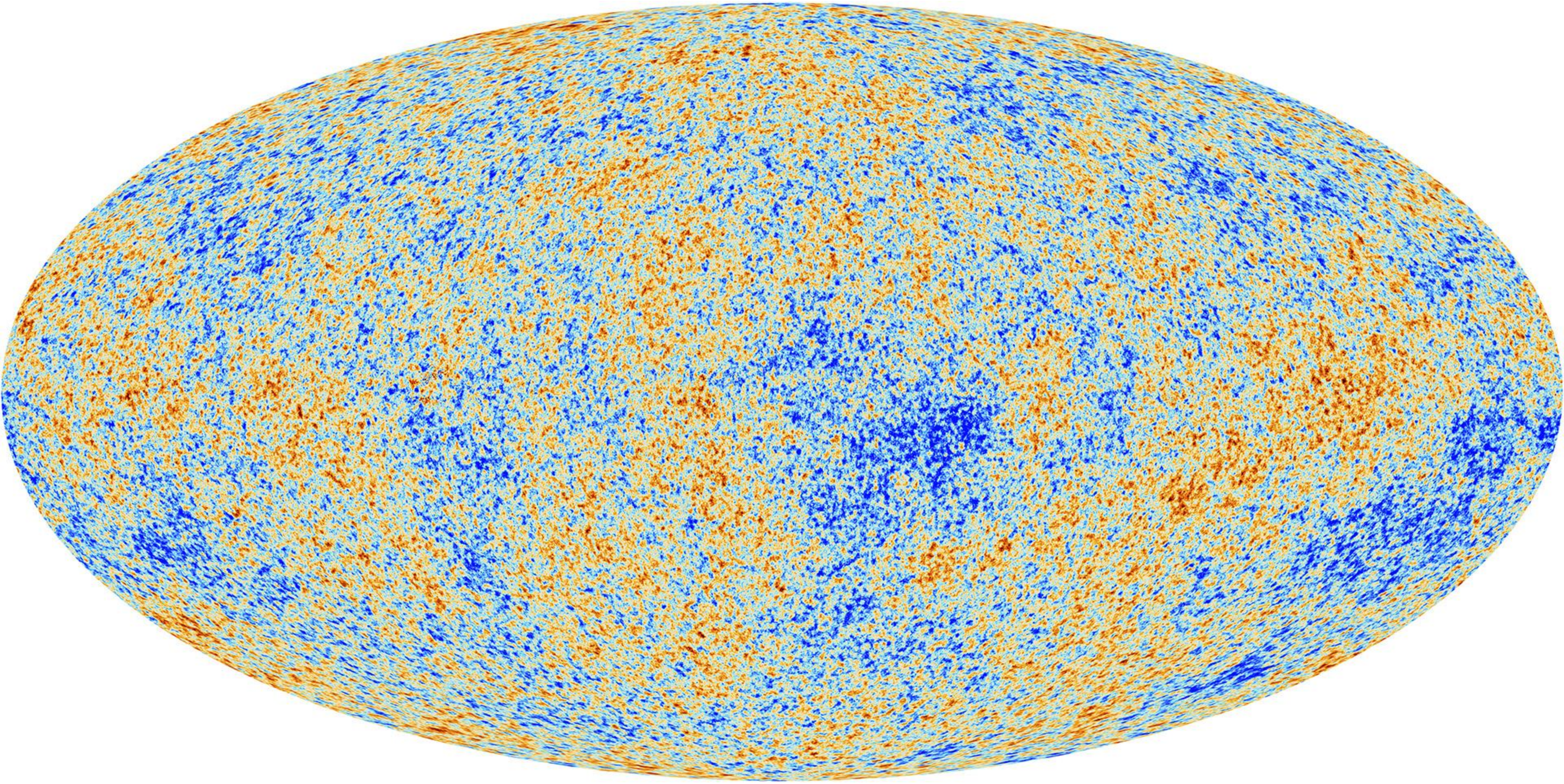
The Universe is opague to light

The Universe is transparent

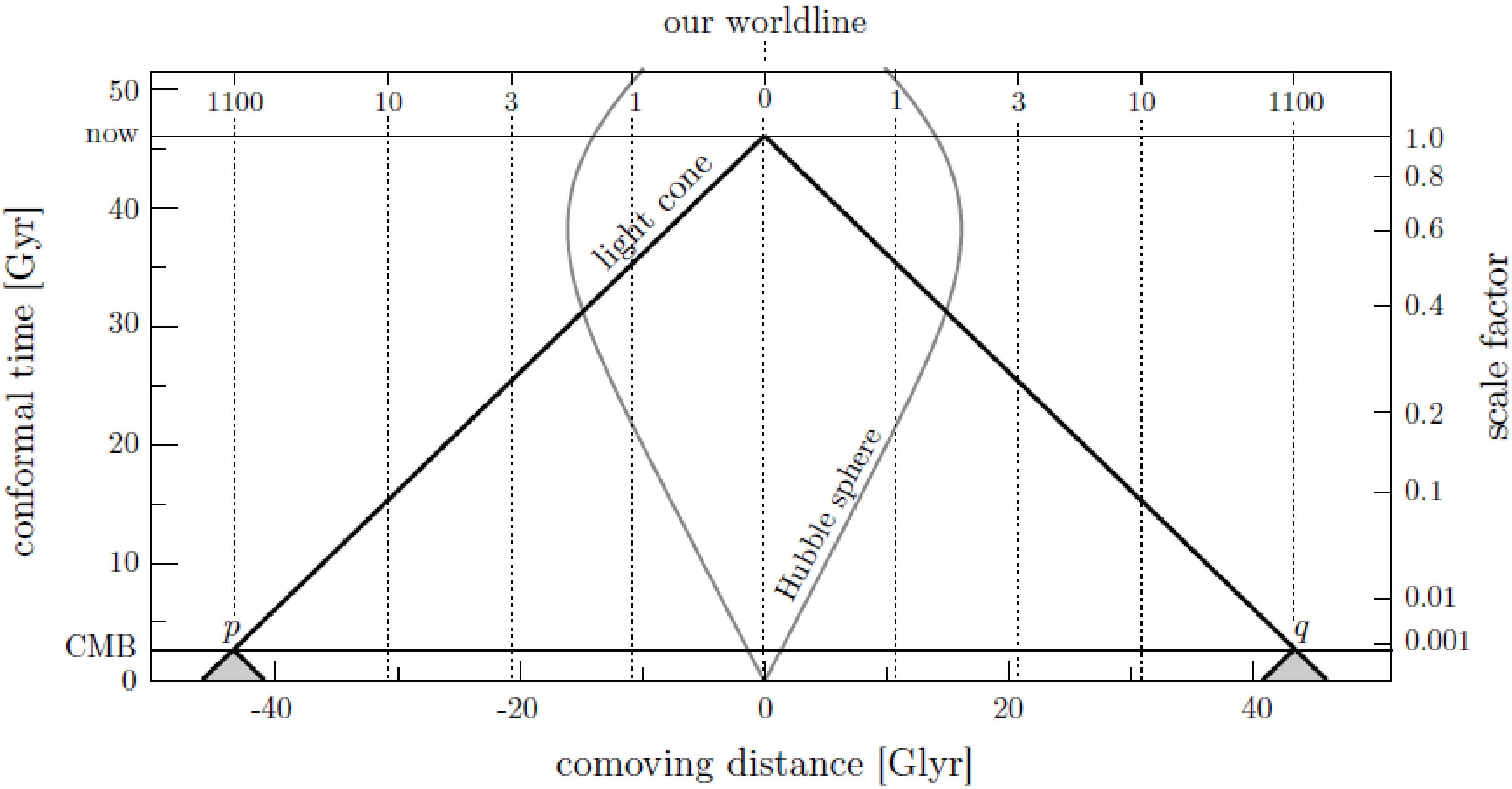
Three Important Results

- Penzias & Wilson saw only a uniform glow
- Later, Doppler shift from Milky Way's motion seen; the Galaxy is moving towards Hydra/Centaurus at 620 km/s
- In 1992 was any nonuniformity in the CMB observed—and then only about 10^{-5} K worth.





Why is the CMB so uniform?


our worldline
50
now
40
30
20
10
CMB
0
conformal time [Gyr]
1100
10
3
1
0
1
3
10
1100
light cone
Hubble sphere
p
q
1.0
0.8
0.6
0.4
0.2
0.1
0.01
0.001
scale factor
-40
-20
0
20
40
comoving distance [Glyr]

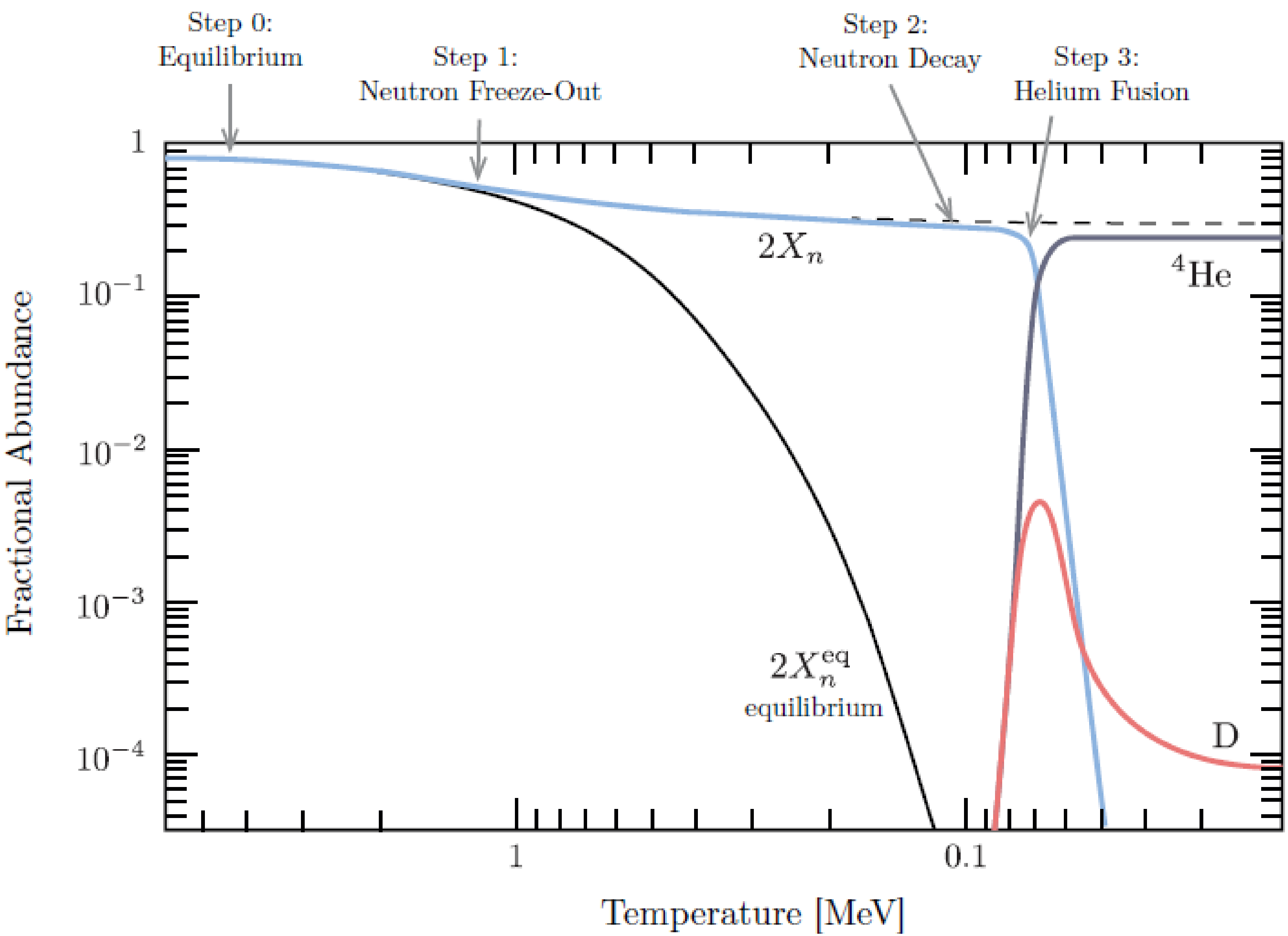
- Consider two opposite directions on the sky.
- *The CMB photons* that we receive from these directions were emitted at the points labelled *p* and *q*.
- We see that the photons were emitted sufficiently close to the Big Bang singularity that the past light cones of *p* and *q* don't overlap.
- This implies that no point lies inside the particle horizons of both *p* and *q* .
- So the puzzle is: ***how do the photons coming from p and q "know" that they should be at almost exactly the same temperature?***

- The same question applies to *any two points* in the CMB that are separated by more than 1 degree in the sky.
- The *homogeneity* of the CMB spans scales that are much larger than the particle horizon at the time when the CMB was formed.
- In fact, in the standard cosmology the CMB is made of about 10^4 disconnected patches of space.
- If there wasn't enough time for these regions to communicate, *why do they look so similar*?
- This is the **horizon problem**.

Big Bang Nucleosynthesis

Light elements (H, He, Li) were synthesised in the Big Bang.

Goal: explain one number: $\dfrac{n_{\rm He}}{n_{\rm H}} \sim \dfrac{1}{16}$


Step 0:
Equilibrium
Step 1:
Neutron Freeze-Out
Step 2:
Neutron Decay
Step 3:
Helium Fusion
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
Fractional Abundance
$2X_n$
^{4}He
$2X_n^{\rm eq}$
equilibrium
D
1
0.1
Temperature [MeV]

Step 0: EQUILIBRIUM

$$\boxed{n + \nu_e \leftrightarrow p^+ + e^-}$$

Step 1: NEUTRON FREEZE-OUT

It is convenient to define the *neutron fraction* as

$$X_n \equiv \frac{n_n}{n_n + n_p} \qquad \Rightarrow \qquad X_n^{\rm eq}(T) = \frac{e^{-Q/T}}{1 + e^{-Q/T}} \ .$$

- Neutrons freeze-out when the weak interactions become inefficient (cf. neutrino decoupling).
- We can estimate the relic abundance of neutrons by their equilibrium abundance at neutrino decoupling

$$\boxed{X_n^\infty \sim X_n^{\rm eq}(0.8\,{\rm MeV}) = 0.17 \sim \frac{1}{6} \ .}$$

Step 2: NEUTRON DECAY

Neutrons are unstable:

$$\boxed{X_n(t) = X_n^{\infty} e^{-t/\tau_n} = \frac{1}{6} e^{-t/\tau_n} \,, \qquad \tau_n \sim 900\,\mathrm{sec}\ .}$$

Step 3: HELIUM FUSION

Helium can only form **after** deuterium is produced

$=$ **deuterium bottleneck**

$$
\begin{array}{l}
n + p \rightarrow \mathrm{D} + \gamma \\
\qquad\qquad\quad \downarrow \\
\qquad\qquad\quad \mathrm{D} + p \rightarrow {}^3\mathrm{He} + \gamma \\
\qquad\qquad\quad \mathrm{D} + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + p
\end{array}
$$

- Since virtually all neutrons go into ^{4}He, we get $n_{\rm He}(t_{\rm nuc}) = \frac{1}{2} n_n(t_{\rm nuc})$, or

$$\boxed{\frac{n_{\rm He}}{n_{\rm H}} = \frac{n_{\rm He}}{n_p} \simeq \frac{\frac{1}{2} X_n(t_{\rm nuc})}{1 - X_n(t_{\rm nuc})} \sim \frac{1}{2} X_n(t_{\rm nuc}) \sim \frac{1}{16}}\ ,$$

Sometimes, the result is expressed as the mass fraction of helium,

$$\boxed{\frac{4 n_{\rm He}}{n_{\rm H}} \sim \frac{1}{4}}\ .$$

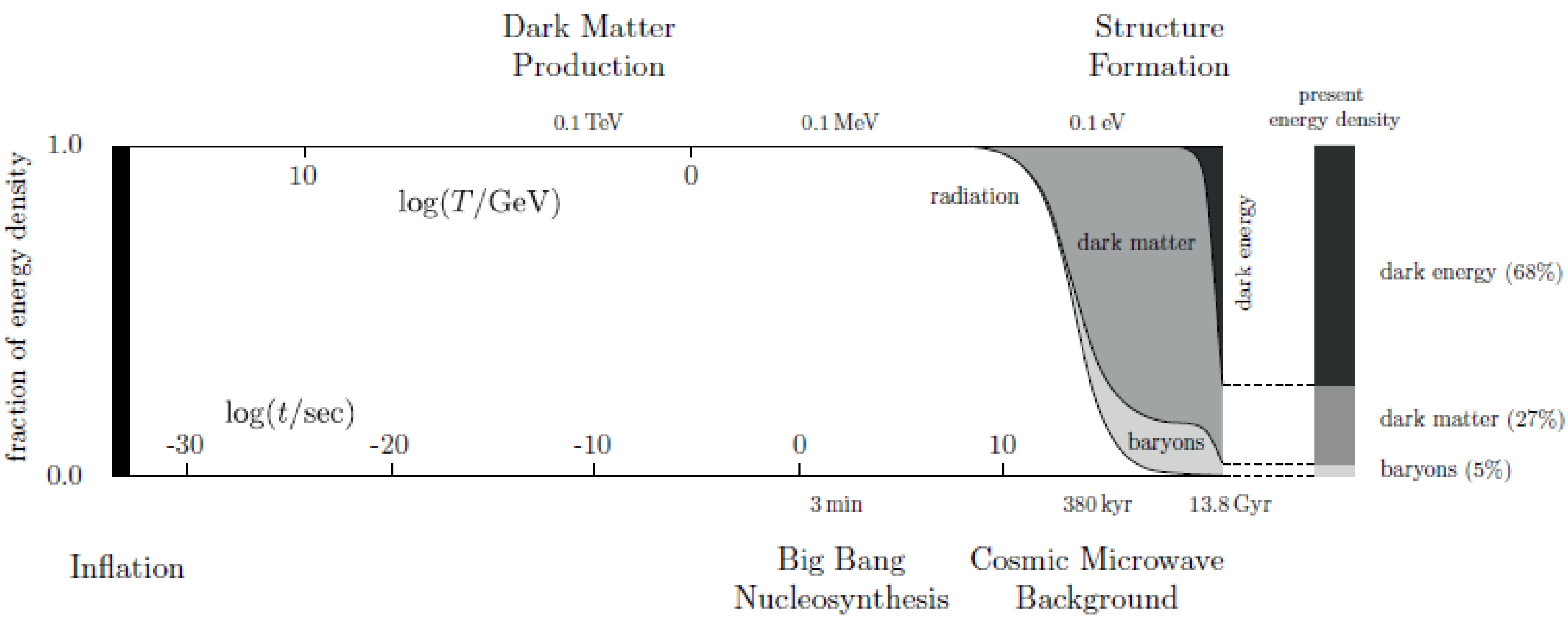
The Origin of the Solar System Elements

Graphic created by Jennifer Johnson
http://www.astronomy.ohio-state.edu/~jaj/nucleo/

Astronomical Image Credits:
ESA/NASA/AASNova

Outline

- The expanding universe
- A brief history of time
- Geometry and dynamics


Dark Matter Production
Structure Formation
present energy density
0.1 TeV
0.1 MeV
0.1 eV
1.0
10
0
log(T/GeV)
radiation
dark matter
dark energy
dark energy (68%)
fraction of energy density
log(t/sec)
dark matter (27%)
baryons
baryons (5%)
-30
-20
-10
0
10
0.0
3 min
380 kyr
13.8 Gyr
Inflation
Big Bang Nucleosynthesis
Cosmic Microwave Background

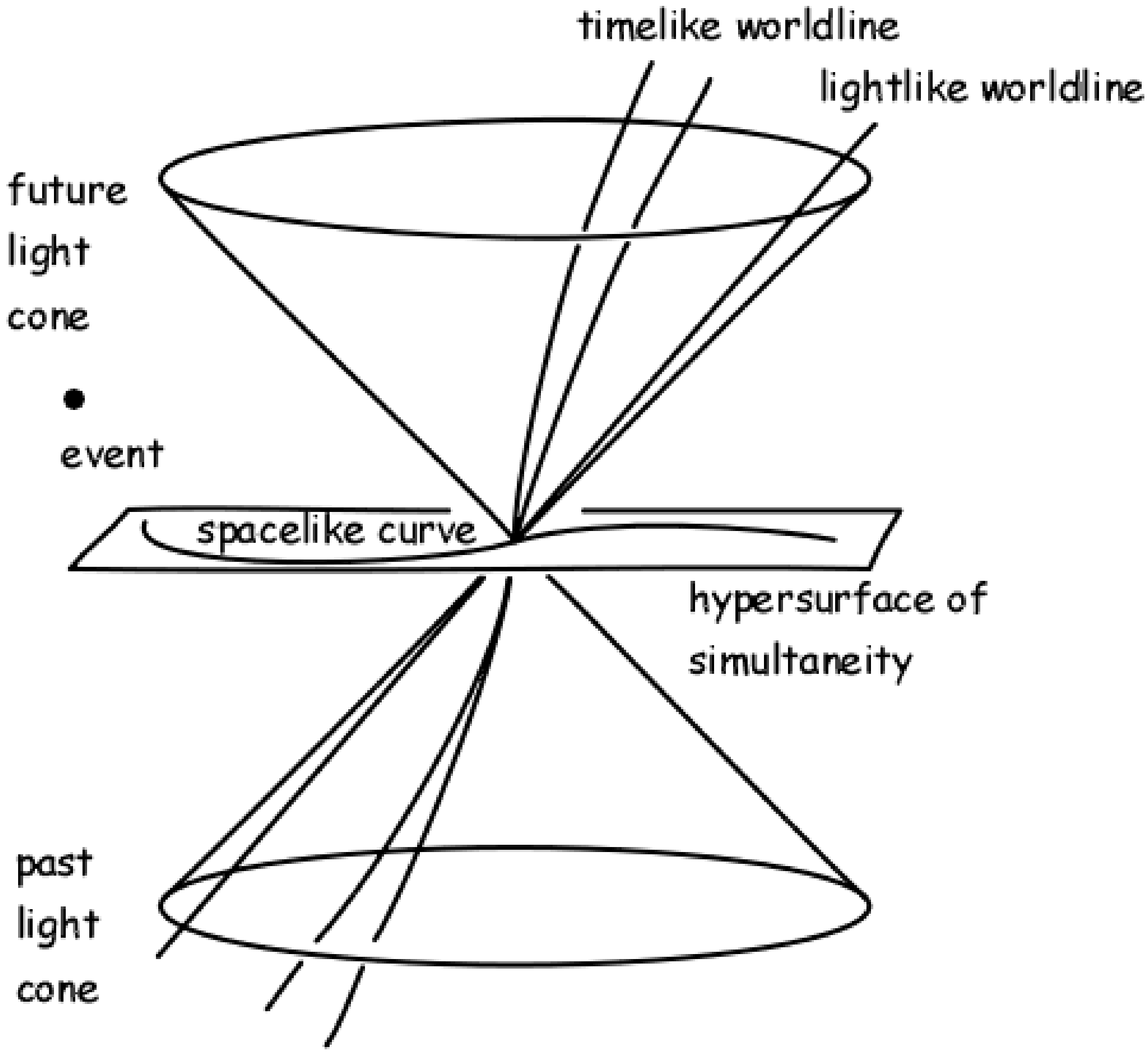
4D Flat Spacetime

- The metric of the Minkowski space of special relativity in Cartesian coordinates is

$$\boxed{ds^2 = -dt^2 + dx^2 + dy^2 + dz^2}$$

4D Flat Spacetime

- There are three kinds of "spacetime intervals" or "distance intervals,
  - **timelike** $ds^2 < 0$
  - **lightlike** $ds^2 = 0$
  - **spacelike** $ds^2 > 0$


timelike worldline
lightlike worldline
future
light
cone
event
spacelike curve
hypersurface of
simultaneity
past
light
cone

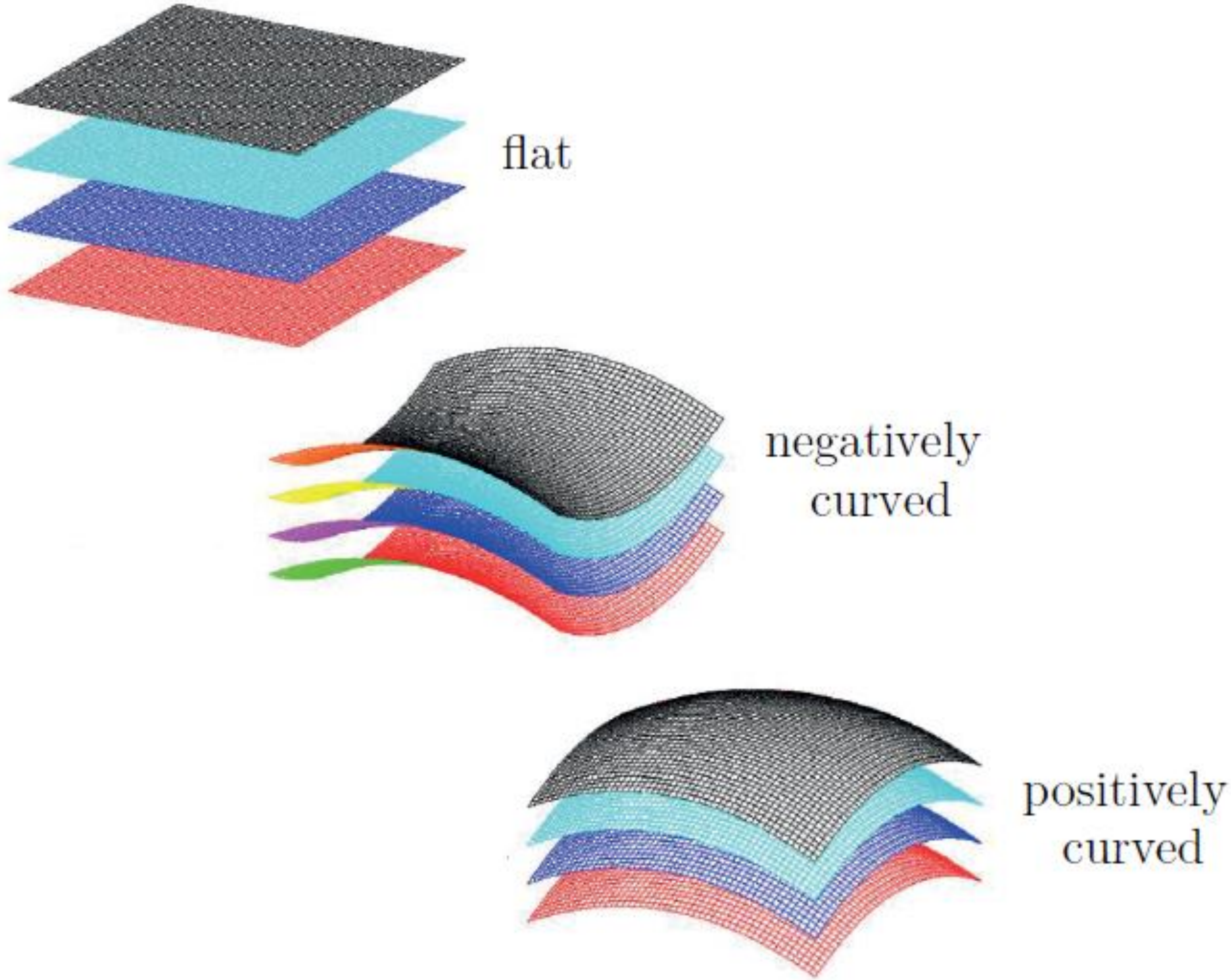
$$\boxed{\mathrm{d}s^2 = \sum_{\mu,\nu=0}^{3} g_{\mu\nu}\mathrm{d}X^\mu \mathrm{d}X^\nu \equiv g_{\mu\nu}\mathrm{d}X^\mu \mathrm{d}X^\nu}$$

- The Minkowski metric is the same everywhere in space and time,

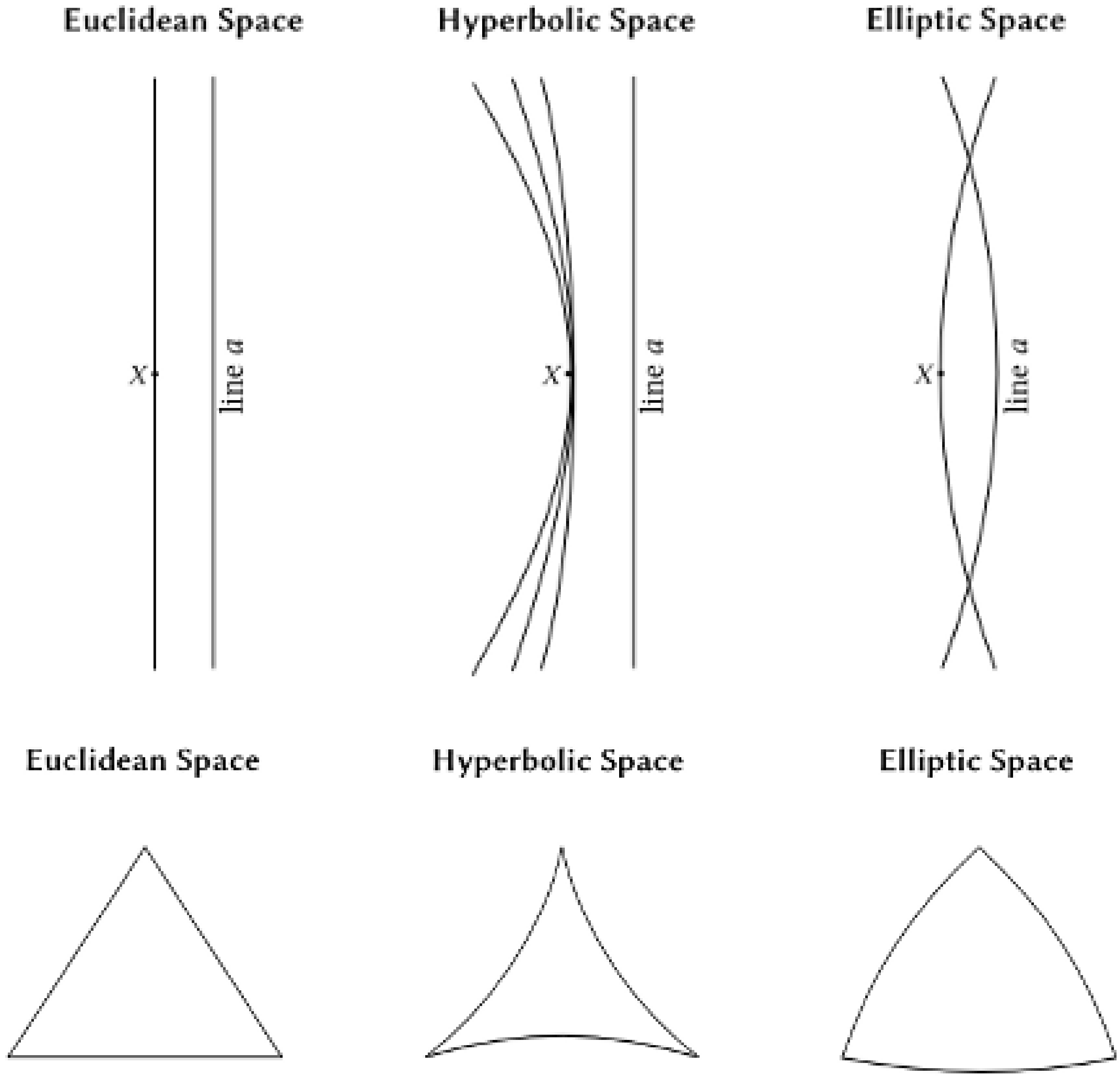
$$\boxed{g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)}$$

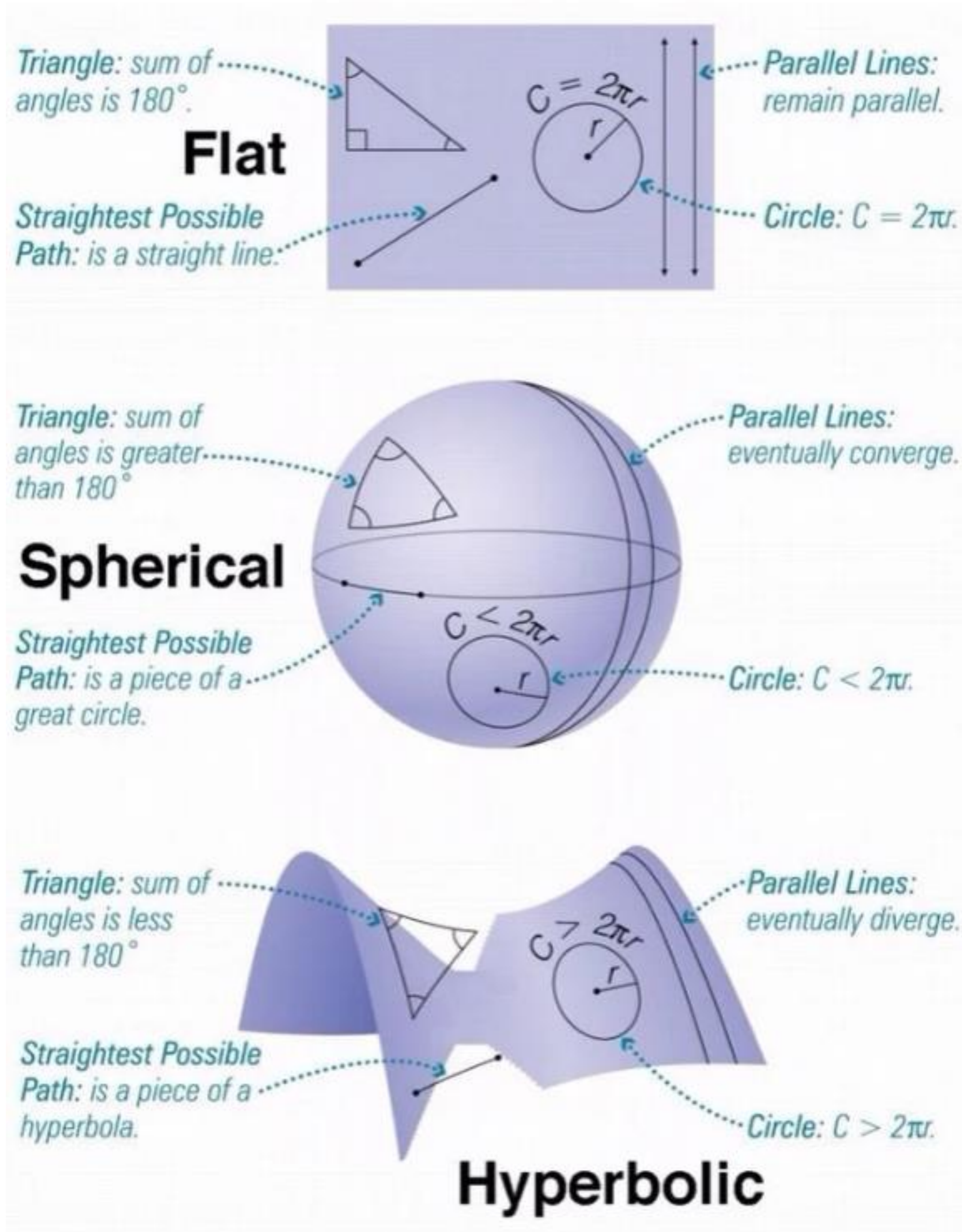
- In general relativity, the metric will depend on where we are and when we are,

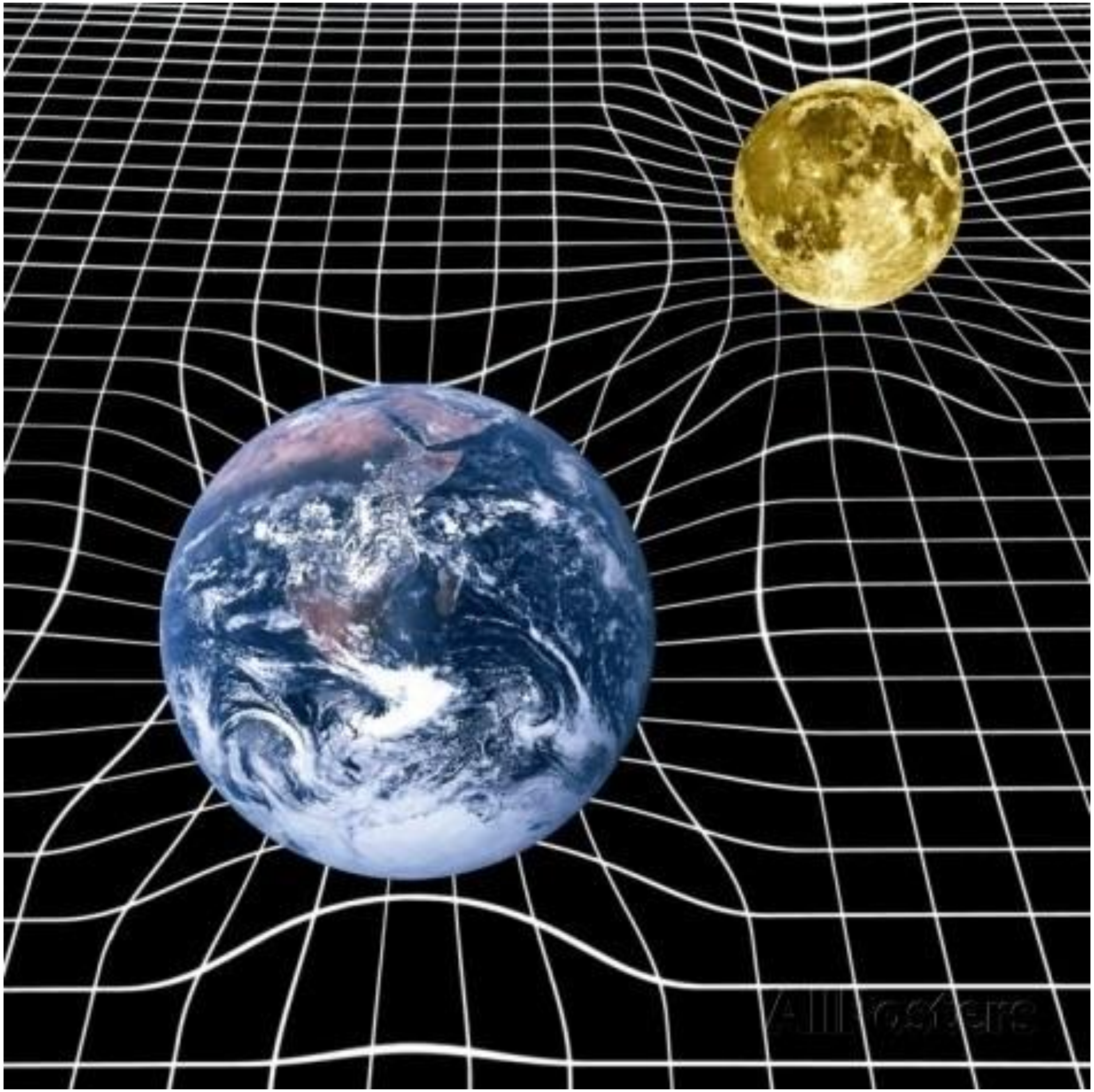
$$\boxed{g_{\mu\nu}(t, \boldsymbol{x})}$$

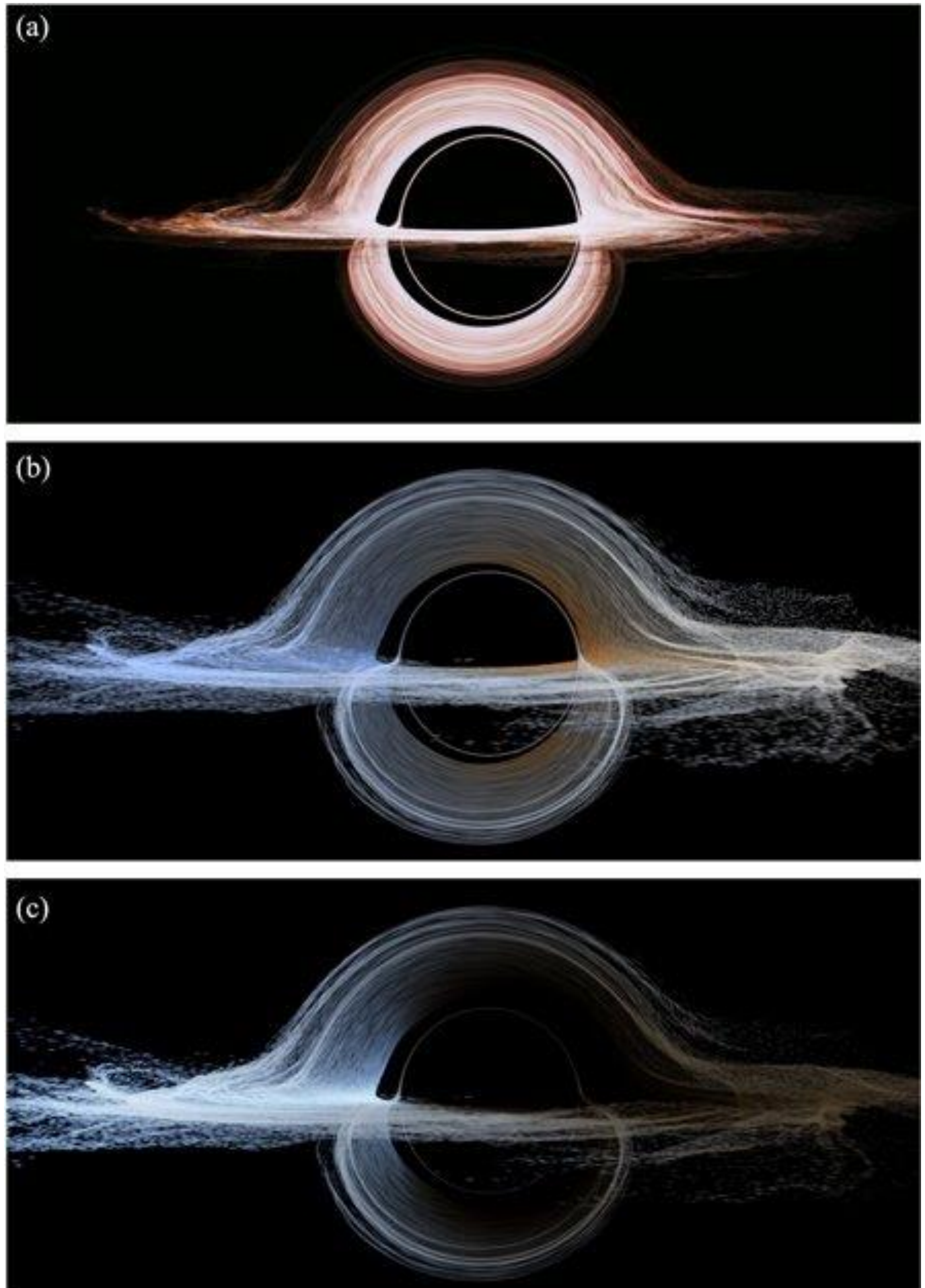


The spacetime of the universe can be foliated into at, positively curved or negatively curved spatial hypersurfaces.


Euclidean Space
Hyperbolic Space
Elliptic Space
X
line a
X
line a
X
line a
Euclidean Space
Hyperbolic Space
Elliptic Space


Triangle: sum of angles is 180°.
Flat
Straightest Possible Path: is a straight line.
C = 2πr
r
Parallel Lines: remain parallel.
Circle: C = 2πr.
Triangle: sum of angles is greater than 180°
Spherical
Straightest Possible Path: is a piece of a great circle.
C < 2πr
r
Parallel Lines: eventually converge.
Circle: C < 2πr.
Triangle: sum of angles is less than 180°
Straightest Possible Path: is a piece of a hyperbola.
C > 2πr
r
Parallel Lines: eventually diverge.
Circle: C > 2πr.
Hyperbolic




(a)
(b)
(c)

- The dynamics of the universe is determined by the **Einstein equation**

$$\boxed{G_{\mu\nu} = 8\pi G T_{\mu\nu}}$$

- This relates the Einstein tensor $\boldsymbol{G_{\mu\nu}}$ (a measure of the *"spacetime curvature"* of the FLRW universe) to the stress-energy tensor $\boldsymbol{T_{\mu\nu}}$ (a measure of the *"matter content"* of the universe).
- We will first discuss possible forms of cosmological stress-energy tensors $\boldsymbol{T_{\mu\nu}}$, then compute the Einstein tensor $\boldsymbol{G_{\mu\nu}}$ for the FLRW background, and finally put them together to solve for the evolution of the scale factor *a(t)* as a function of the matter content.

Stress-energy tensor

- The stress-energy tensor of a perfect fluid (homogeneity and isotropy univesre) as seen by a comoving observer is

$$T^{\mu}{}_{\nu} = g^{\mu\lambda}T_{\lambda\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -P & 0 & 0 \\ 0 & 0 & -P & 0 \\ 0 & 0 & 0 & -P \end{pmatrix}$$

- The ***continuity equation***

$$\boxed{\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0}$$

- The universe is filled with a mixture of *different matter components*.

- It is useful to classify the different sources by *their contribution to the pressure*.

Matter

- We will use the term *"matter"* to refer to all forms of matter for which the pressure is much smaller than the energy density,

$$\boxed{|P| \ll \rho}$$

- This is the case for a gas of *non-relativistic particles* (where the energy density is dominated by the mass).
- Setting *P = 0* in *the continuity equation* gives

$$\boxed{\rho \propto a^{-3}}$$

- This dilution of the energy density simply reflects the expansion of the volume $V \propto a^3$.

Matter

- ***Dark matter***.
- Most of the matter in the universe is in the form of invisible dark matter.
- This is usually thought to be a new heavy particle species, but what it really is, we don't know.

- ***Baryons***.
- Cosmologists refer to ordinary matter (*nuclei and electrons*) as baryons.

Radiation

- We will use the term “radiation" to denote anything for which the pressure is about a third of the energy density,

$$\boxed{P = \frac{1}{3}\rho}$$

- This is the case for a gas of *relativistic particles*, for which the energy density is *dominated by the kinetic energy* (i.e. the momentum is much bigger than the mass).
- In this case, *the continuity equation* implies

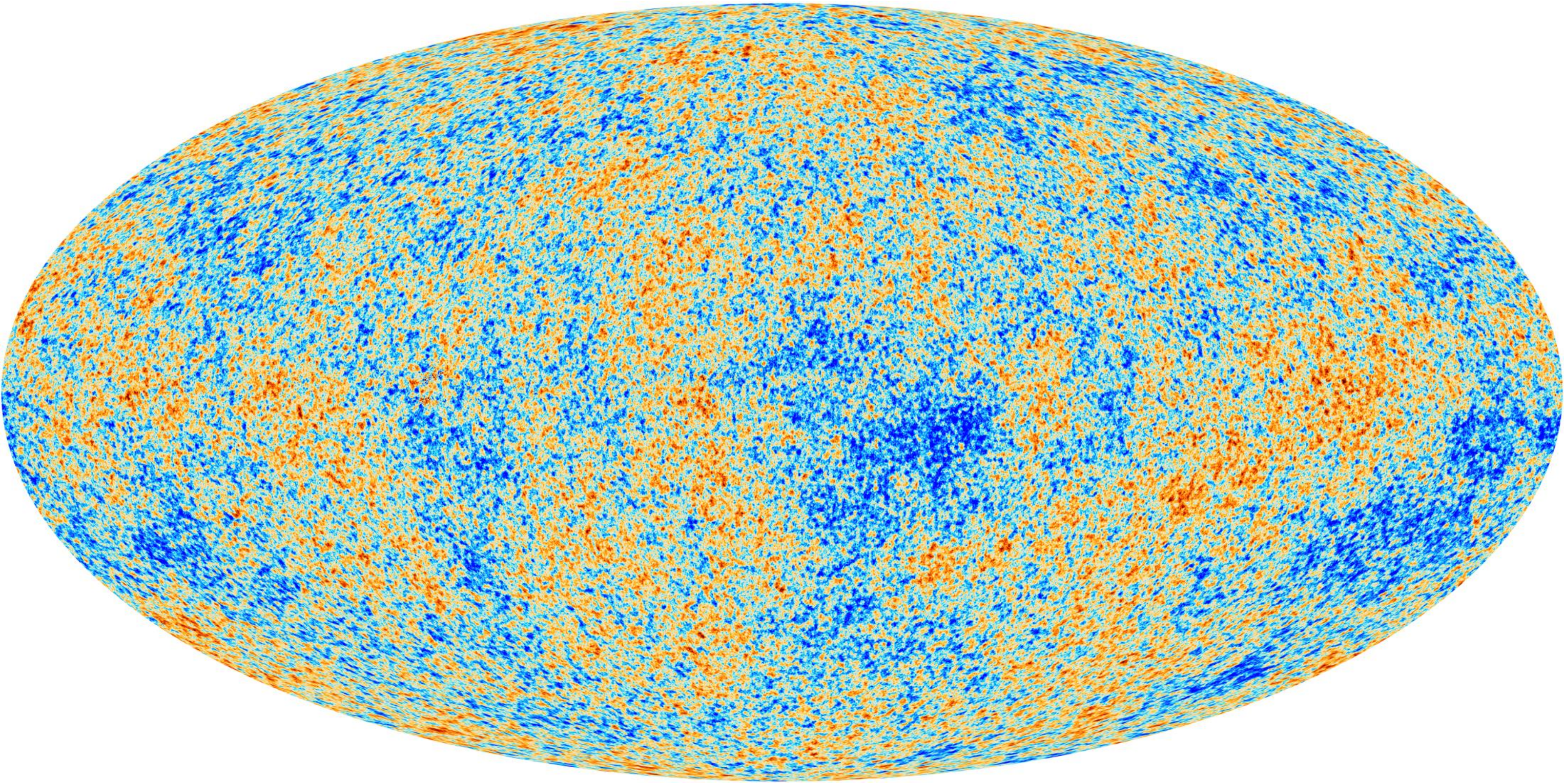
$$\boxed{\rho \propto a^{-4}}$$

- The dilution now includes the redshifting of the energy, $E \propto a^{-1}$.

Radiation

- ***Photons.*** The early universe was dominated by photons.
- Being massless, they are always relativistic.
- Today, we detect those photons in the form of the cosmic microwave background.
- ***Neutrinos.*** For most of the history of the universe, neutrinos behaved like radiation.
- Only recently have their small masses become relevant and they started to behave like matter.
- ***Gravitons.*** The early universe may have produced a background of gravitons (i.e. *gravitational waves*).
- Experimental efforts are underway to detect them.

Radiation



Dark Energy

- We have recently learned that *matter and radiation aren't enough* to describe the evolution of the universe.
- Instead, the universe today seems to be dominated by a mysterious negative pressure component,

$$\boxed{P = -\rho.}$$

- This is unlike anything we have ever encountered in the lab. In particular, from *the continuity equation*, we find that the *energy density is constant*,

$$\boxed{\rho \propto a^0}$$

- Since the energy density doesn't dilute, *energy has to be created as the universe expands*.

Dark Energy

- ***Vacuum energy***. In *quantum field theory*, this effect is actually predicted! The ground state energy of the vacuum corresponds to the following stress-energy tensor

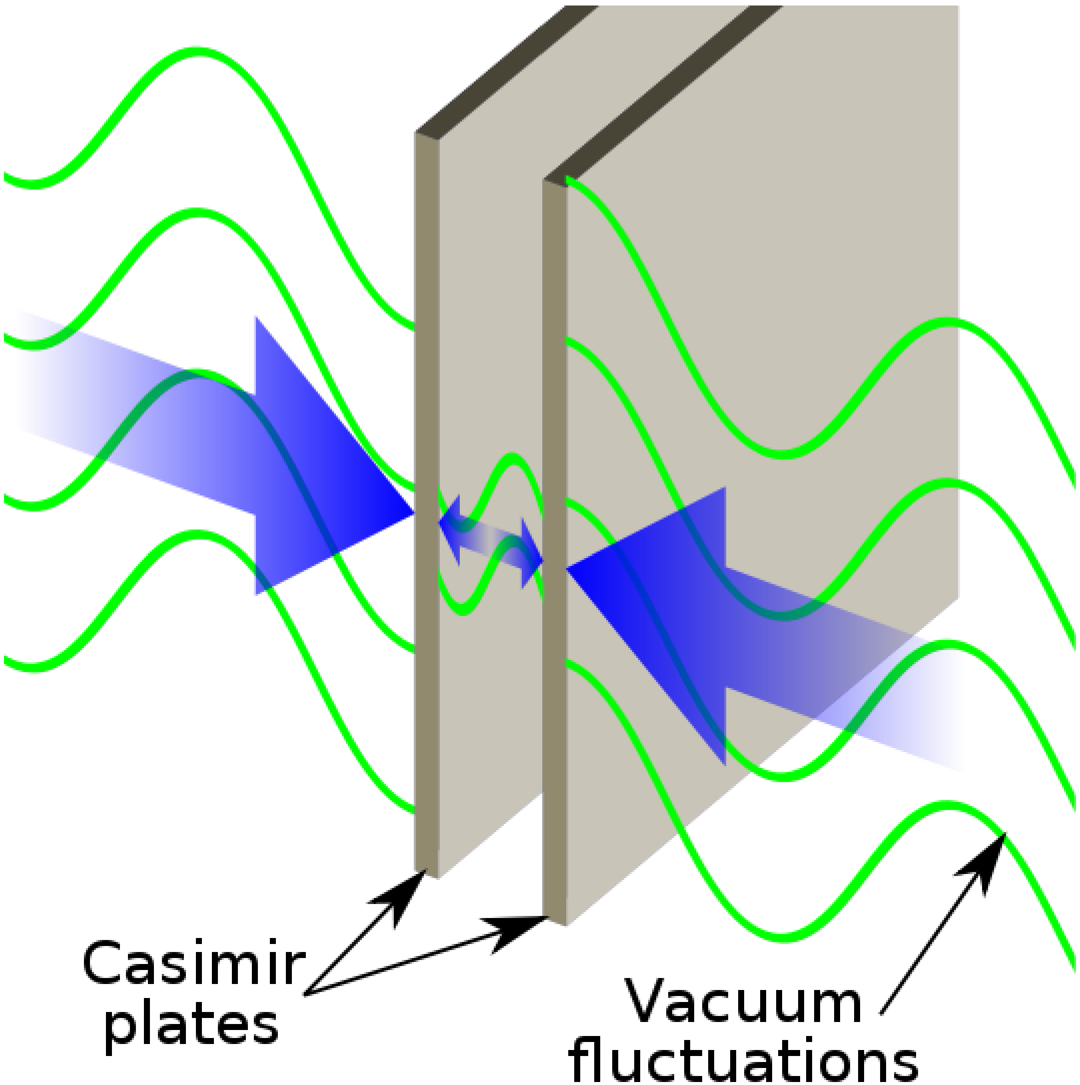
$$\boxed{T^{\text{vac}}_{\mu\nu} = \rho_{\text{vac}}\, g_{\mu\nu}}$$

- show that this indeed implies ➡ $\boxed{P_{\text{vac}} = -\rho_{\text{vac}}}$

- Unfortunately, the predicted size of ρ_{vac} is completely off,

$$\boxed{\frac{\rho_{\text{vac}}}{\rho_{\text{obs}}} \sim 10^{120}}$$

Dark Energy



Dark Energy

- ***Something else?***
- The failure of quantum field theory to explain the size of the observed dark energy has lead theorists to consider more exotic possibilities (such as time-varying dark energy and modifications of general relativity).

- **None of these ideas works very well.**

Dark Energy

- ***Cosmological constant***.
- The left-hand side of the *Einstein equation* isn't uniquely defined.

$$\boxed{G_{\mu\nu} = 8\pi G T_{\mu\nu}}$$

- We can add the term $-\Lambda g_{\mu\nu}$, for some constant , without changing the conservation of the stress tensor, $\nabla^{\mu} T_{\mu\nu} = 0$.
- In other words, we could have written the Einstein equation as

$$\boxed{G_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}}$$

- Einstein, in fact, did add such a term and called it the *cosmological constant*.

Summary

- Most cosmological fluids can be parameterised in terms of a constant *equation of state* ⇒ $\boxed{w = P/\rho}$

$$\boxed{\rho \propto a^{-3(1+w)}}$$

- dark matter ⇒ $w = 0$
- radiation ⇒ $w = 1/3$
- vacuum energy ⇒ $w = -1$

Summary

$$\rho \propto \begin{cases} a^{-3} & \text{matter} \\ a^{-4} & \text{radiation} \\ a^{0} & \text{vacuum} \end{cases}$$

Spacetime Curvature

- We want to relate these matter sources to the evolution of the scale factor in the **FLRW metric**

$$\boxed{\mathrm{d}\ell^2 = a^2\left[\mathrm{d}\chi^2 + S_k^2(\chi)\,\mathrm{d}\Omega^2\right]}$$

- To do this we have to compute the Einstein tensor on the l.h.s. of the **Einstein equation**,

$$\boxed{G_{\mu\nu} = 8\pi G T_{\mu\nu}}$$

$$\boxed{G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}}$$

- We will need the **Ricci tensor**

$$\boxed{R_{\mu\nu} \equiv \partial_\lambda \Gamma^\lambda_{\mu\nu} - \partial_\nu \Gamma^\lambda_{\mu\lambda} + \Gamma^\lambda_{\lambda\rho}\Gamma^\rho_{\mu\nu} - \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\rho}}$$

- and the **Ricci scalar**

$$\boxed{R = R^\mu{}_\mu = g^{\mu\nu} R_{\mu\nu}}$$

$$R_{00} = -3\,\frac{\ddot{a}}{a}\;,$$

$$R_{ij} = -\left[\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + 2\,\frac{k}{a^2}\right] g_{ij}$$

- The Ricci scalar is

$$\boxed{R = -6\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right]}$$

- The non-zero components of the Einstein tensor

$$G^{\mu}{}_{\nu} \equiv g^{\mu\lambda} G_{\lambda\nu}$$

- are

$$\boxed{\begin{aligned} G^0{}_0 &= 3\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right] , \\ G^i{}_j &= \left[2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right]\delta^i_j . \end{aligned}}$$

Friedmann Equations

- From the Einstein equation $G_{\mu\nu} = 8\pi G T_{\mu\nu}$
- We combine equations

$$
\begin{aligned}
G^0{}_0 &= 3\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right], \\
G^i{}_j &= \left[2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right]\delta^i_j .
\end{aligned}
$$

- with stress-tensor

$$
T^\mu{}_\nu = g^{\mu\lambda} T_{\lambda\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -P & 0 & 0 \\ 0 & 0 & -P & 0 \\ 0 & 0 & 0 & -P \end{pmatrix}
$$

- to get the Friedmann equations.

- The Friedmann equations are

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} ,$$
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

- Here, ρ and P should be understood as the sum of all contributions to the energy density and pressure in the universe.

- We write:
- ρ_r for the contribution from radiation (with for photons
- ρ_ν for neutrinos
- ρ_m for the contribution by matter (with ρ_c for cold dark matter and ρ_b for baryons)
- ρ_Λ for the vacuum energy contribution.

- The first Friedmann equation is often written in terms of the Hubble parameter, $H \equiv \dot{a}/a$

$$\boxed{H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}}$$

- Let us use subscripts "0" to denote quantities evaluated today, at $t = t_0$.
- A flat universe ($k = 0$) corresponds to the following *"critical density"* today

$$\boxed{\begin{aligned} \rho_{\mathrm{crit},0} = \frac{3H_0^2}{8\pi G} &= 1.9 \times 10^{-29}\, h^2\, \mathrm{grams\,cm^{-3}} \\ &= 2.8 \times 10^{11}\, h^2\, M_\odot\, \mathrm{Mpc^{-3}} \\ &= 1.1 \times 10^{-5}\, h^2\, \mathrm{protons\,cm^{-3}} \end{aligned}}$$

- We use the *critical density* to define dimensionless density parameters

$$\boxed{\Omega_{I,0} \equiv \frac{\rho_{I,0}}{\rho_{\mathrm{crit},0}}}$$

- The Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

can then be written as

$$\boxed{H^2(a) = H_0^2\left[\Omega_{r,0}\left(\frac{a_0}{a}\right)^4 + \Omega_{m,0}\left(\frac{a_0}{a}\right)^3 + \Omega_{k,0}\left(\frac{a_0}{a}\right)^2 + \Omega_{\Lambda,0}\right]}$$

- where we have defined a *"curvature"* density parameter, $\boxed{\Omega_{k,0} \equiv -k/(a_0H_0)^2}$

- It should be noted that in the literature, the subscript *"0"* is normally dropped,

  so that e.g. Ω_{m} usually denotes the matter density *today* in terms of the critical density *today*.
- From now on we will follow this convention and drop the *"0"* subscripts on the density parameters.
- We will also use the conventional normalization for the scale factor, $a_0 = 1$.
- Then it becomes

$$\boxed{\frac{H^2}{H_0^2} = \Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda}$$

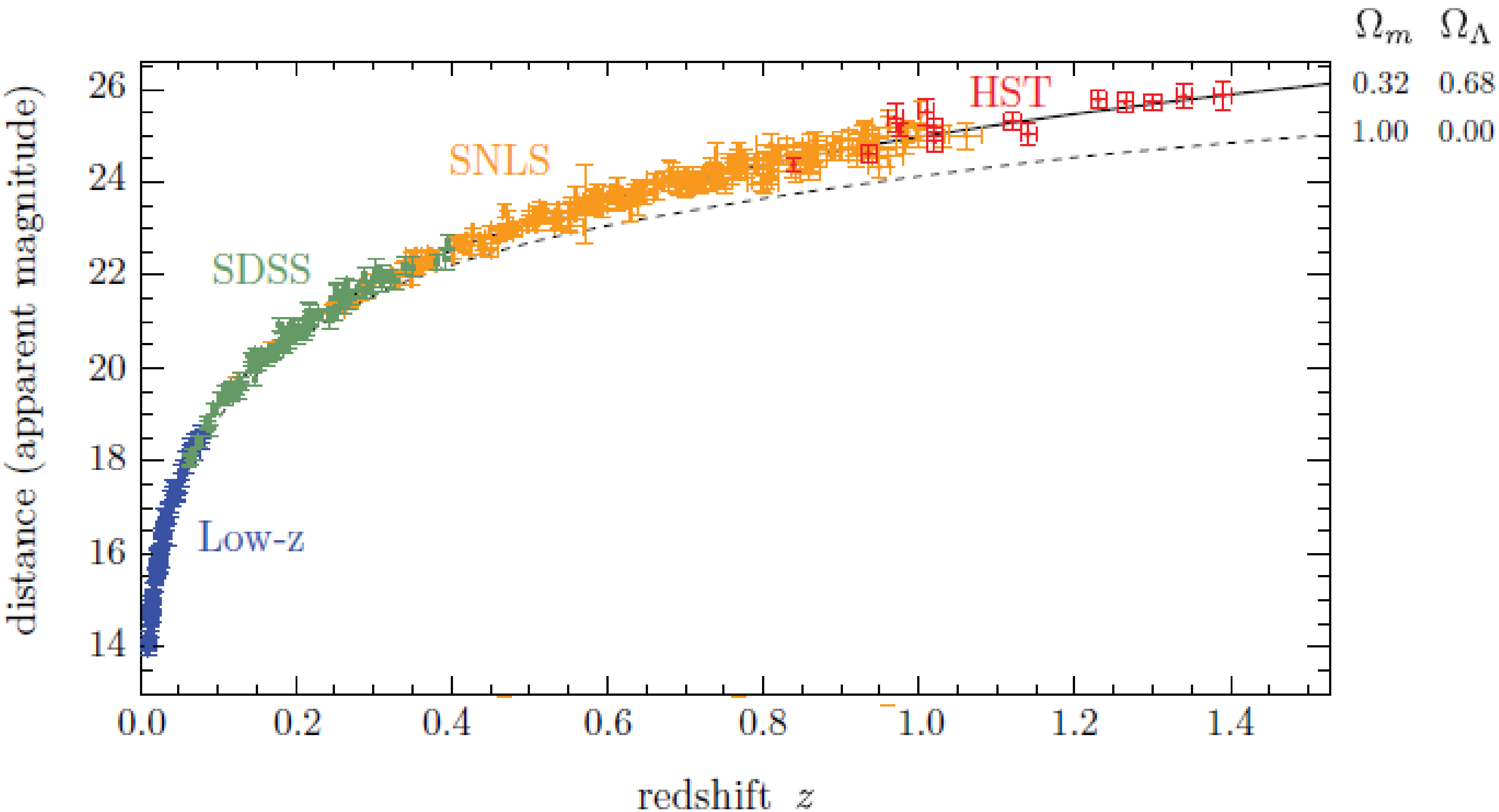
ΛCDM

- Observations show that the universe is filled with radiation ("*r*), matter (*m*) and dark energy (*Λ*):

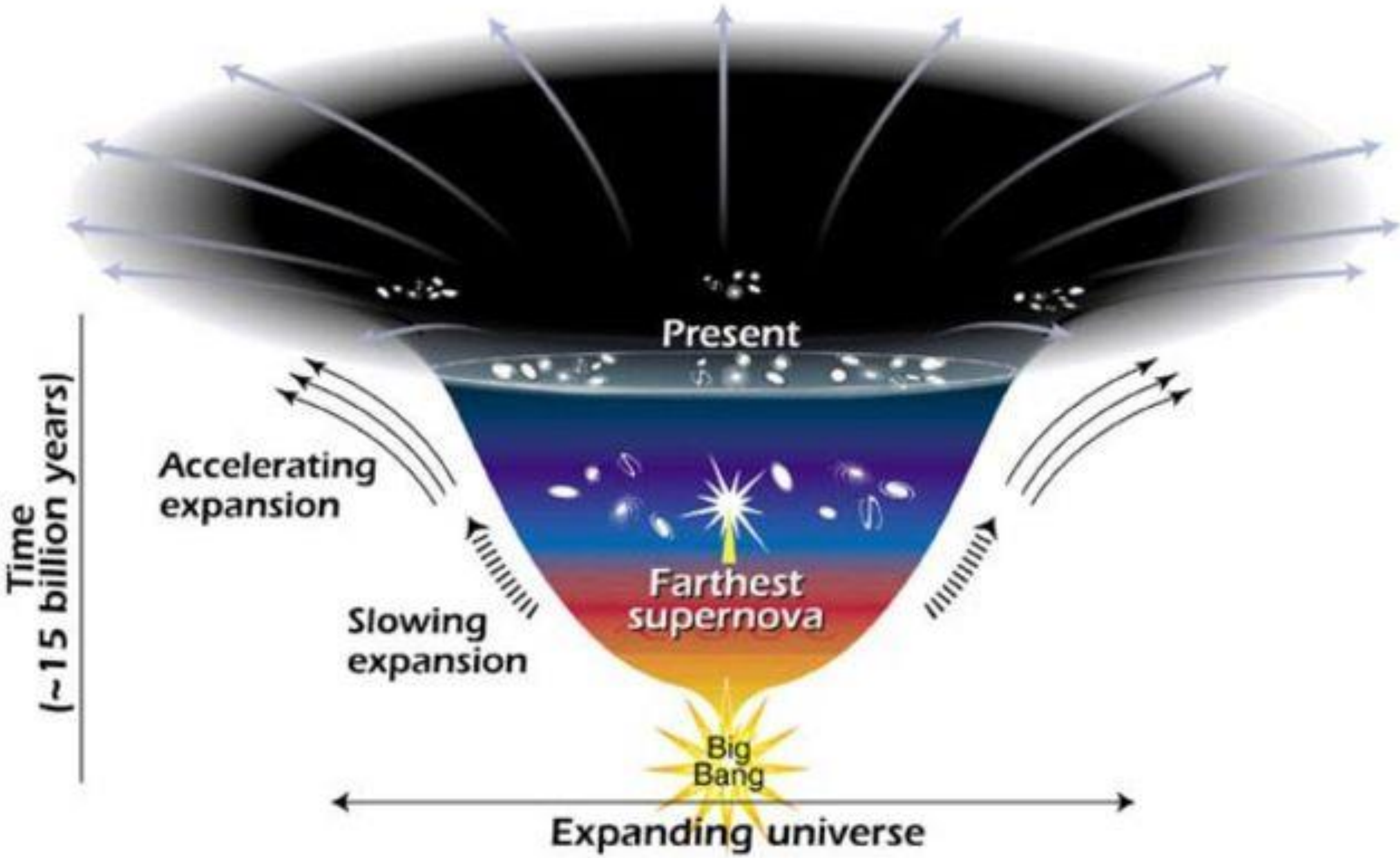
$$|\Omega_k| \leq 0.01\,, \quad \Omega_r = 9.4 \times 10^{-5}\,, \quad \Omega_m = 0.32\,, \quad \Omega_\Lambda = 0.68$$

- The equation of state of dark energy seems to be that of a cosmological constant, $w_\Lambda \approx -1$.
- The matter splits into 5% ordinary matter (baryons, "*b*") and 27% (cold) dark matter (CDM, "*c*"):

$$\Omega_b = 0.05\,, \quad \Omega_c = 0.27$$



Type IA supernovae and the discovery dark energy. If we assume a flat universe, then the supernovae clearly appear fainter (or more distant) than predicted in a matter-only universe ($\Omega_m = 1$).
(SDSS = Sloan Digital Sky Survey; SNLS = SuperNova Legacy Survey; HST = Hubble Space Telescope.)


Present
Time
(~15 billion years)
Accelerating
expansion
Farthest
supernova
Slowing
expansion
Big
Bang
Expanding universe